PARAMETRICAL IDENTIFICATION FOR A COMPLEX MATHEMATICAL MODEL OF INDEXING CAM MECHANISMS BY MEANS OF A GENETIC ALGORITHM

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KEYWORDS

indexing cam mechanisms, genetic algorithms, parametrical identification

ABSTRACT

The authors describe the application of a genetic algorithm to a mechanical design problem: the dynamic characterisation of indexing cam mechanisms.
During the working phase of indexing mechanisms, vibration phenomena have been experimentally observed. Such a behaviour has been well reproduced by a lumped parameters elasto-dynamic model developed by the authors. The most ticklish operation in the adjustment of the model is the identification of the parameters, because they are not directly inferable from the mechanical characteristics of the system. An automatic technique to identify the best set of parameters, which optimise a goal function based on experimental results, is needed to be developed: a genetic algorithm seems the right approach to solve the problem.
Besides the stability and the opportunity to fairly quick gain the solutions, genetic algorithms offer not only one set of identified parameters, but a whole “family” of solutions, which are close to the goal. So that among them it is possible to select the one which appears the best in the design of the specific indexing mechanism.

INTRODUCTION

The object of this memory are indexing drives, mechanisms which transform a uniform rotating motion into an indexing motion [1, 2, 6].
The more demanding requirements of today’s manufacturing industry include ever increasing production rates. The operators are, so, driven in the direction of using cam mechanisms, which are the best way to minimise cycle-time in high speed machines, incorporating everything into a compact and efficient design to build automated machines that perform regularly at high speed.
In particular, the roller gear cam mechanism is in high demand for industrial machines, because it possesses high degree of rigidity, positive motion mechanism to ensure no backlash and the flexibility to adapt to various applications.
The operation of these indexing drives is marked out by the onset of vibration phenomena, especially during the deceleration phase. Such a problem is not, normally, considered designing roller gear cam mechanisms: the way usually followed, is to make an over-scantling of the main dimensions of the device. The research group to which the authors belong, on the contrary, have faced the question in its complexity. After several tests, a dependence of the dynamical behaviour on the characteristics of the motion generators and transformers systems of the indexing drive has been observed.
On this basis, the authors developed an elasto-dynamic model of the ‘indexing system’, with a global view of the situation, including both the actuating system and the load, elements that cannot be ignored if the aim is to carry out a designing phase which optimise the indexing cam mechanism [3, 4].

With reference to the model complexity, the need to identify a high number of parameters, not all known a-priori, arise. A genetic algorithm appeared as the best tool to develop the parametrical identification, owing to the high number of the parameters staked and to the use of functions with not very nice mathematical properties, matters that well arrange the problem in the sphere of the Soft Computing. By the use of a genetic algorithm, we can find a whole family of solutions, everyone with good performances.
When the parametrical identification has been carried out, the developed elasto-dynamic model appears an efficient instrument to simulate the real dynamical behaviour of the devices, to be actually used to optimise the indexing cam mechanisms during the designing phase.

THE ELASTO-DYNAMIC MODEL OF THE INDEXING SYSTEM

By ‘indexing system’ we mean the whole of a motor (usually an A.C. asynchronous motor) represented by its characteristic curve and with, eventually, a converter for the speed regulation, of a speed reducer (represented by its velocity ratio), of the indexing drive and, finally, of the load (rotary table), as shown in Fig. 1.

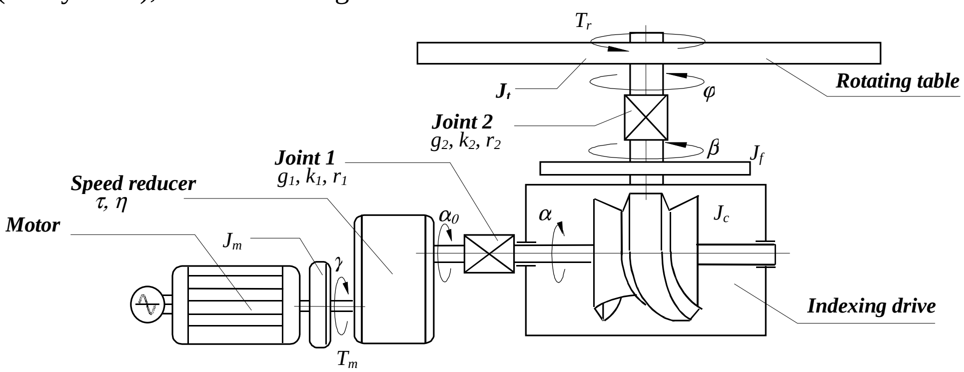

Fig. 1 – Elasto-dynamic model of the indexing system

For the analysis of the system, an angular accelerometer has been mounted on the table, for different configurations of the indexing system. We observed that under high inertial loads or for particular values of the actuating velocity, the real acceleration is considerably unequal to the theoretical one, achieving deceleration values sensibly higher (see Fig.2).

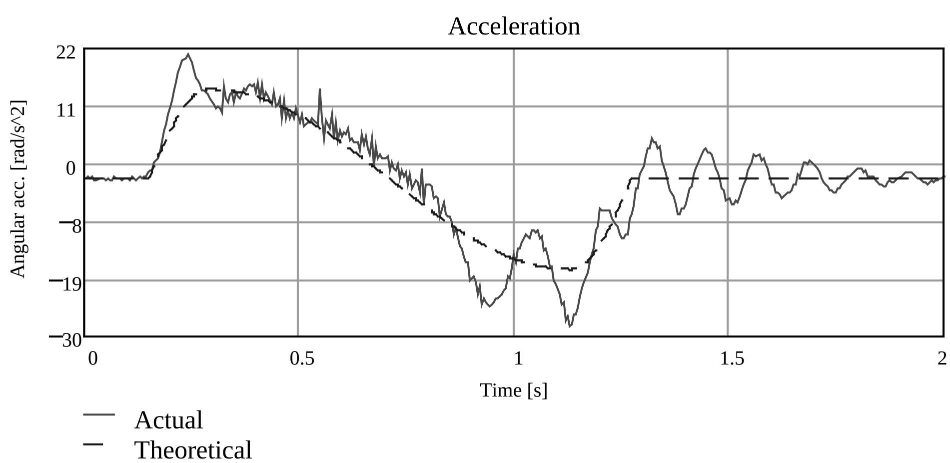

Fig. 2 – Comparison between the real and the theoretical dynamical behaviours.

Such a behaviour is, meanly, caused by the non perfect rigidity of the mechanical members used for the motion transformation (the speed reducers have backlash inside as well as compliance the transmission systems).
Accepting a margin of approximation (that is necessary if we do not want to highly increase the model complexity), the elastic behaviour of the mechanical organs (clearly distributed), has been modelled with two elastic joints (Fig. 1), both characterised by their stiffness, damping and clearance (k, r and g) values.
The kinematic relationships between α and β, in accordance with the well known formulas (see [1]) are:

$$\beta = \beta(\alpha) \quad \dot{\beta} = \beta'(\alpha)\dot{\alpha} \quad \ddot{\beta} = \beta''(\alpha)\dot{\alpha}^2 + \beta'(\alpha)\ddot{\alpha}$$

The torques transmitted by the coupling joints (T_1 and T_2) are made by an elastic component and by a damping one and can be mathematically described as follow:

$$T_1 = \begin{cases} k_1(\alpha_0 - \alpha + g_1/2) + r_1(\dot{\alpha}_0 - \dot{\alpha}) & \alpha_0 - \alpha < -g_1/2 \\ 0 & |\alpha_0 - \alpha| \le g_1/2 \\ k_1(\alpha_0 - \alpha - g_1/2) + r_1(\dot{\alpha}_0 - \dot{\alpha}) & \alpha_0 - \alpha > g_1/2 \end{cases} \qquad T_2 = \begin{cases} k_2(\varphi - \beta + g_2/2) + r_2(\dot{\varphi} - \dot{\beta}) & \varphi - \beta < -g_2/2 \\ 0 & |\varphi - \beta| \le g_2/2 \\ k_2(\varphi - \beta - g_2/2) + r_2(\dot{\varphi} - \dot{\beta}) & \varphi - \beta > g_2/2 \end{cases}$$

The two degrees of freedom mathematical model developed is, therefore, described by the equations below written:

$$\begin{cases} \ddot{\alpha} = \dfrac{T_1 + \beta' T_2 - J_f \beta' \beta'' \dot{\alpha}^2}{J_c + J_f \beta'^2} \\ \ddot{\varphi} = -\dfrac{T_r + T_2}{J_t} \\ \ddot{\gamma} = \dfrac{T_m - \tau T_1}{J_m} \end{cases}$$

In the model proposed, the parameters which are not directly measurable on the mechanical system are g_1, k_1, r_1, g_2, k_2 and r_2. Being the aim of the authors to develop an automatic technique for the indexing drive modelling, it is clear that the correct valuation of the above mentioned parameters is a critical phase and plays a central role in the use of the model.

THE PARAMETERS IDENTIFICATION AND THE GENETIC ALGORITHM USED

An experimental approach has appeared the best way to develop the parameters identification, given the great number of series of data picked up on various mechanisms.
Such a method consists in the research of a particular set of the mentioned coefficients which make the model able to simulate as best as possible the actual acceleration behaviour of the rotating-table. The comparison between the actual and the simulated signals has been made mathematically quantifiable, by the introduction of an original “goodness-index”. Its valuation consists of the three following phases: to define a pre-fixed tolerance stripe around the plot that reproduces the experimental observation, to sample the real and the simulated acceleration curves and, finally, to evaluate the percentage of simulated points which fall inside the defined band (see Fig. 3).
Such an index (afterwards indicated as *PGP*, that is ***P***ercentage of “***G***ood ***P***oints”) will assume higher and lower values when the two graphs will tend to superimpose one another. Consequently, when it will obtain a good resemblance between the real and the simulated plot, the points of the latter will lie completely inside the tolerance area and the *PGP* will assume the value *100%*; on the contrary, if an inadequate set of coefficients is given to the mathematical

model, the simulated results will be very far from the actual ones. In this case, just a low number of real acceleration points will fall near the simulated ones, so that the goodness-index will assume a very low value.
As our purpose is to better simulate the signals amplitude than their phase-displacement, the described property of *PGP* is the main reason for which a non-conventional goodness-index has been used; in fact if we compare two signals with the same wave-form but with a little time-shift, a conventional index (for example "Root Mean Square" or Euclidean norm) gives low values, whereas *PGP* is high (as we need).

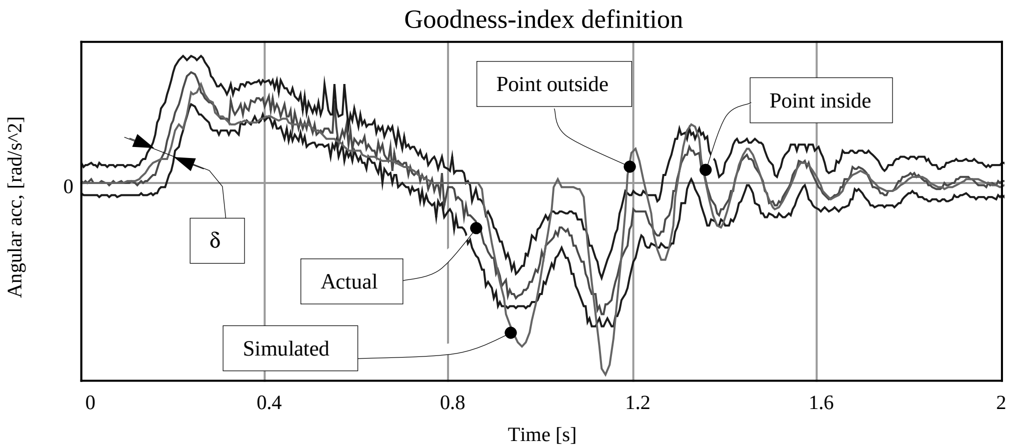

Fig. 3 – Simulated plot and the prefixed area around the actual plot for the *PGP*-index definition.

Mathematically, the goodness-index function, is built on two data series of values: we thus write $PGP = f(\mathbf{y}, \hat{\mathbf{y}}, \delta)$, where $\mathbf{y}$ and $\hat{\mathbf{y}}$ are known and represent the simulated and actual data arrays respectively, and δ is the width of the tolerance stripe. By the use of this function, we can transform the parametric identification problem in the maximisation of a functional which is defined on the parameters of the model to be estimated and on a given data series to reproduce. More precisely, let $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$ denote the set of these parameters. During the identification procedure followed, the sampled graph regarding the real acceleration ($\hat{\mathbf{y}}$) may be kept constant, whereas the simulated results directly depend on λ parameters given to the model.
If the width of the tolerance area δ is kept constant too, the goodness-index becomes a functional defined only on the λ parameters which must be identified, as shown by

$$PGP = f(\mathbf{y}, \hat{\mathbf{y}}, \delta) = f(\mathbf{y}(\lambda), \hat{\mathbf{y}}_{const}, \delta_{const}) \qquad \Rightarrow \qquad PGP = PGP(\lambda)$$

From this viewpoint the parametric identification consists in establishing the particular set of values $\lambda^* = (\lambda_1^*, \lambda_2^*, ..., \lambda_n^*)$ which maximise the *PGP* functional, that is:

$$\lambda^* \quad | \quad PGP(\lambda^*) = \max PGP(\lambda)$$

To face this mathematical problem, we have thought to use a Genetic Algorithm [9, 10, 11], because the cardinality of λ could be high, and *PGP* functional generally does not possess "nice" properties such as continuity and differentiability on λ variable: this makes the use of conventional approaches critic. In this context, a generic set of parameters λ**,** representing a potential solution of the maximisation, is assimilated to one individual belonging to an animal species, and the k-th element λ_k of λ becomes the value of a particular character of the

individual. The optimisation procedure operates on groups of λ-vectors correlated with *PGP* functional in the same way in which the natural evolution process adapts and improves individual features of the members of the species to permit their survival in particular environment conditions.
Precisely, after an opportune definition of an initial population $\Lambda^0 = (\lambda^0_0, \lambda^0_1, \ldots, \lambda^0_m)$, the algorithm recombines its members and manipulates their characters by applying in sequence some genetic operators hereafter explained. The procedure is reiterated several times defining a succession of populations $(\Lambda^0, \Lambda^1, \Lambda^2, \Lambda^3, \ldots)$ whose best individuals are closer and closer to the optimum (non-divergence of the method is guaranteed by reinserting the best member of previous generation in every new population created). The simulated evolution will stop when a pre-defined goodness target is attained, or when the functional to be optimised no longer meets substantial increments.
The last population found during the process, generally, represents a family of different solutions of the problem proposed, everyone of which offers high values of the functional *PGP*. Therefore, it is possible to operate a further selection of the correct solution (as above mentioned we would reward the signal amplitude than the phase-displacement).

Going into details, first generation of individuals Λ^0 is built by setting every character $(\lambda^0_{i,1}, \lambda^0_{i,2}, \ldots, \lambda^0_{i,n})$ of its *i*-th member to a random value drawn from a pre-fixed range (some parameters may assume a physical mean in respect of which they must lie within bounds).
Some genetic operators named selection, cross-over and mutation are then sequentially applied.
The *selection* operator supplies a new population Λ^p by operating an extraction of individuals from an existing one Λ^{p-1}:the selection is carried out by means of a geometric probability distribution in which the best members of population Λ^{p-1} have a higher chance of being selected. In particular, let q denote a pre-fixed value of the probability of selecting the best individual, after a descending sorting of the population elements by goodness index *PGP*, the probability P of selection assigned to the *i*-th individual is computed by the following expression:

$$P = \frac{q}{1-(1-q)^m} \cdot (1-q)^{i-1}$$

where m denotes the population size.

In this context, *PGP* becomes a fitness index, and bad individuals of Λ^{p-1} are substituted by copies of the best ones in Λ^p (see Fig. 4).

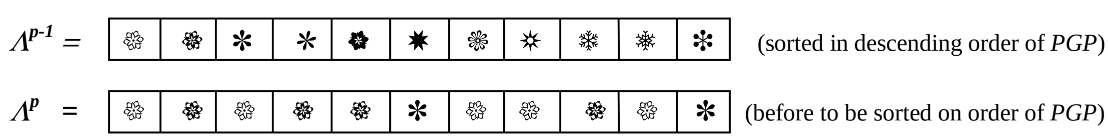

Fig. 4 – The selection operator: in Λ^p only the three better individuals of Λ^{p-1} survive, and their number of presence is "proportional" to their *PGP*-values.

The *crossover* works combing groups of characters between two individuals randomly selected by existing population to generate new member with best characteristics. Let λ_a and λ_b denote the two individuals selected. In the algorithm proposed, two kinds of crossover are operated: the first one generates a partition on the vectors λ_a and λ_b in some random cut-points taken in interval *[1..n-1]*, and then exchanges the parts between them as shown in Fig.5 producing two new individuals λ_a' and λ_b'.

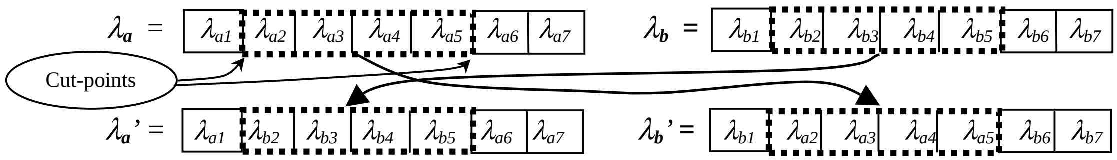

Fig. 5 – An example of crossover operation on individuals composed by seven characters.

The second one produces a linear extrapolation of the characters values of the two individuals in the direction of the best one. If λ_a is better than λ_b, the new individual is given by

$$\lambda = \lambda_a + r \cdot (\lambda_a - \lambda_b)$$

where r is a random number in the range $[0,1]$. A check and an eventual adjustment must be made on the new values generated to keep them pre-defined bounds.

The *mutation* operator is concerned with changing some random characters of a randomly selected individual, with the purpose being to insert in the population elements which cannot be generated with the other described mechanisms. The following four types of mutation have been adopted:

1. *Boundary*: the value of the selected character is exchanged with one bound of its variation field.
2. *Uniform*: a random value uniformly drawn by its variation field is given to character variable.
3. *Non-uniform*: the selected element is varied with a non-uniform distribution depending on which individual goodness-rank it belong to, so that characters selected by nearest goal individuals are varied in a more restricted range.
4. *Percentage*: the character value is increased or decreased of a pre-fixed percentage (usually small) of its value.

Generally, the described operators are not simultaneously applied, therefore a new individual may result very similar to a previous one as well as totally different from it.

RESULTS OBTAINED AND CONCLUSIONS

The computer program implementing the identification algorithm exposed, running with the parameters listed in Tab. 1, gave very good results.
Fig. 6 shows the goodness-index graph evaluated on the best individual inside each population during the evolution process. As we can see, the method shows high performances specially in the first phase of the program running. The stepping trend of the diagram is due to the reinsertion of the best element in every new generation.

Number of generations simulated	*250*
Number of individuals in a population (m)	*30*
Number of characters for each individual (n)	*6*
Number of every cross-over for each iteration	*5*
Number of every mutation for each iteration	*5*
Number of randomly selected characters for each mutation	*3*
Percentage of variation for percentage mutation	*10%*
Probability of selecting the best individual (q)	*0.1*
Sampling interval for *PGP*-index computation	*0.001*
Amplitude of tolerance stripe (δ)	*0.01*

Tab. 1 – The parameters of the genetic algorithm.

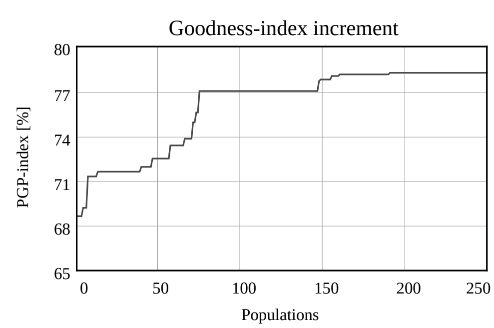

Fig. 6 – *PGP*-index vs populations created.

The result of the application of the described identification method is shown in the following picture: on the left appears a simulated graph where the parameters of the model have been defined by a “genetic identification”; on the right the actual behaviour of the acceleration of the device table is reproduced.
The performances appear remarkable, being curve (a) globally very similar to experimental data (b): in particular the amplitude of the various zones fits very well.
The parametric identification has been performed on the six parameters g_1, k_1, r_1, g_2, k_2, r_2 and the results obtained for the best ten solutions are summed up in Tab. 2.

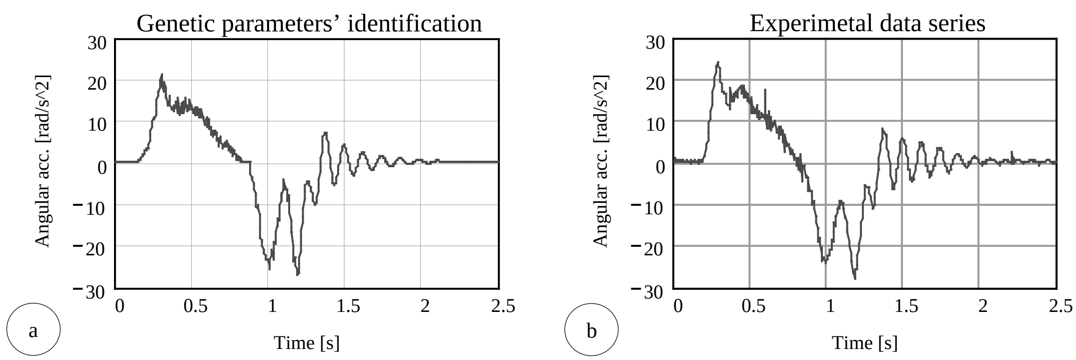

Fig. 7 – Simulation with genetic identification (a) and actual data series (b).

It is possible to note by the table that we obtained more or less equal *PGP* values with different combination of the six parameters. At this point it will be the user of the model who will “manually” select the solutions having the parameters which appear closer to the “physics” of the system. The variety of the results and the multiple solutions suggest also that the use of another kind of optimisation method would probably lead to instability.

In conclusion, we can state that parameters identification based on the genetic algorithm illustrated has given good results as for our problem. Moreover, the procedure implemented can be generalised to every identification problem in which target data series, like experimental ones, are available. Actually, we are developing variants of the procedure described in order to improve the *PGP*-index, i. e. making it auto-adjusting during the evolution process.

g_1 [deg]	k_1 [Nm/rad]	r_1 [Nms/rad]	g_2 [deg]	k_2 [Nm/rad]	r_2 [Nms/rad]	**PGP**[%]	**Quality**
0.1107	385989	9.53	0.00018	38852697	127894	78.775	Very good
0.0979	424212	238	0.00307	37692523	84329	78.05	Very good
0.0979	424212	238	0.00026	37663886	85949	77.975	Very good
0.0849	408817	445	0.00001	38703740	114387	77.925	Very good
0.0441	343879	2569	0.00034	40637651	88257	77.3	Good
0.1342	408817	445	0.00001	38703740	114387	77.15	Good
0.0794	420972	331	0.00045	38262075	126390	76.675	Acceptable
0.0798	461085	331	0.00201	38262075	126390	74.575	Rejected
0.2863	432164	246	0.00000	38508514	126853	72.925	Rejected
0.0000	548988	2296	0.00047	40826950	119269	72.9	Rejected

Tab. 2 – A family of solutions offered by G.A. are shown.

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