Heat Exchanger Networks (HENs) Design:
Topology Optimization

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1. Introduction

In chemical processes it is often necessary to transfer heat between two sets of fluid streams: the first one is the set of hot streams to be cooled, while the latter contains cold streams to be heated. HENs are designed in order to avoid, or at least minimize, the use of external utilities to heat up or cool down the temperature of each stream.

A variety of industries, as petrochemical and chemical, pharmaceutical, brewery and diary, need 'thermodynamically-based tools for the synthesis of cost-optimum energy-integrated systems', with the goal of significant cost benefits. It's important to underline that these tools are not based on a well organized theory and methodology, probably because the vast literature on industrial HENS (*HEN Synthesis*) presents the results obtained with heuristics and models developed by researchers in different specific applications and cases.

The problem of HENS involves graph theory, thermodynamics, combinatorial and continuous variables optimization, combined with heuristics and experience-based approaches, in order to satisfy one or more of the following objectives (in addition to the satisfaction of the target temperatures): fixed number of heat exchangers, minimum total area (connected to capital cost), minimum heat exchange with external devices (connected to operating cost) and then flexibility (which is a function of the topology of the network), good controllability, operability and safety. Moreover, all these objectives have to satisfy constraints due the laws of thermodynamics and plant layout.

In our work we have focused only on a subset of problems, trying to build a model not affected by heuristics to exploit in full the capabilities of Evolutionary Algorithms in finding not only optimal solutions, but also typical experience-based rules 'embedded' in the configurations.

2. HENS: base-theory

To design a HEN we need first of all data related to the streams. It's need specifying the following information:

- Number of hot streams (*hs*) and cold streams (*cs*);
- Input temperature (*Tin*) and output (or target) temperature (*Tou*) for each stream;
- Heat capacity flow rate (*Mcp* = *m'* Cp*, where *m'*= flow rate and *Cp*= specific heat) and heat transfer coefficients (*h*) of the streams, eventually with the relative functions of temperature;
- Characteristics of the available utilities (*Tin*, *Tou*, *Mcp*, *h* + costs);
- Plant life, rate of interest and cost of heat exchange area.

With these data we can already know if the problem admits solutions. In fact, under the hypothesis of constant *Mcp* (where *M* is the mass flowrate, *cp* the specific heat at constant pressure) , each stream can be represented in a T-Q (Temperature - Heat) plot by his temperature-enthalpy profile (a straight-line segment), being $dQ=Mcp*dT$. Instead of looking at each stream as an isolated entity, disconnected from the others, it's possible to build two particular curves for the hot and the cold set, called *Composite Curves*; they are indicative of the feasibility of the problem. These curves combine the profiles of each group of streams in a unique curve, leading to two curves (*HotCC* and *ColdCC*) in the same Q-T plane. CCC and HCC can be thought as a single cold stream in counter-flow with a single hot stream. Obviously, the problem is feasible only if HCC is above CCC, because we cannot exchange heat spontaneously from a lower to a higher temperature. Composite curves are the result of summing up the *Mcp* values of streams belonging to the same set, within each temperature interval. Note that CCC exists independently by HCC and *vice versa*, so their relative horizontal position, when drawn in the same Q-T plane, is not defined. Obviously, more the heat hot streams transfer to cold streams, more the overlapping between the two curves increases. If we fix the position of one curve respect to the other, there is the possibility that the right part of CCC and/or the left part of HCC is 'uncovered'. We can extend CCs into two curves having the same width, incorporating cold and/or utilities into CCC and HCC; the obtained curves are the 'so-called' *Balanced CCs* (*BCCC* and *BHCC*).

We can observe now that each time there is a change in the slope of balanced curves, a vertex is generated and surely the minimum distance between the curves occurs in correspondence to (at least) a vertex (Fig.2.1). This positive temperature difference (*Dtmin*) is named *Heat Recovery Approach Temperature* (*HRAT*) and it represents the driving force for heat transfer between hot and cold streams, being able to win the 'thermal inertia' of heat exchange.

To start the design of the HEN we lastly define *ERAT*, that is the *Exchange Minimum Approach Temperature*. It defines the minimum temperature difference that must exist between hot and cold streams in each exchanger of the network. Surely will be *EMAT≤HRAT*. A lower limit to *EMAT*, due to heuristic, is usually set to: *EMAT≥HRAT/3* [5Sh95].

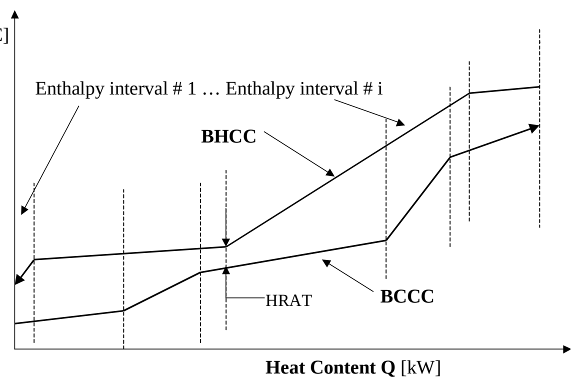


Fig.2.1

Note that if a dashed vertical line is traced in correspondence to each vertex, we subdivide the BCCs into enthalpy intervals (Fig.2.1). It is simple to prove that the maximum number of intervals (*nh*) is a function of the total number of streams, that is:

$$1 \le nh \le 2*(cs+hs) - 3 \quad \text{(where } cs,hs>1\text{)}.$$

The technique of the '*spaghetti design*' uses a number of matches equal to the product of the number of cold and hot streams present in each enthalpy interval, leading to networks with many units, impossible to realize. Other techniques, as MINLP (Mixed Integer NonLinear Programming, [10.3Sh95]) make use of the concept of '*network superstructure*', which is similar to a spaghetti design, but differs from it essentially in the number of subdivisions (*ns*), much smaller than the number of enthalpy intervals (because some contiguous enthalpy intervals are joined together to form a unique interval). We will use a model similar to superstructure, with the possibility to vary the number of intervals from a minimum of one to *nh*.

Since, given the initial data and chosen an *EMAT* value, we are actually able to know if there exists at least a solution, it is necessary to define a set of significant objective functions. A hypothetical solution (a cost-efficient network) should own the following features:

- Minimum total area (hard to find when heat transfer coefficients h_i of streams are different)
- Minimum number of units

- Minimum utility requirements.

Being the operating cost (*OC*) a function of utility requirements and capital cost (*CC*) connected with the number of units and the total heat transfer area, the total annual cost (*TAC*) of a HEN is then given by:

$$TAC = OC + CC*A_f\,,$$

where $A_f = (1+r)^t/t$ = *annualization factor* (*r = return of capital interest*, *t = expected plant life*) [Sh95].

3. An Evolutionary Approach: How & Why

We need first of all to choose the genetic representation of a HEN. Representations based on matrices are limited by the law that connects the size of the matrix to the number of streams (in one of the models of [Baeck98] size is proportional to hs^2*cs^2, decreasing the performance of the search process for large problems). To overcome this problem a vector-based representation has been used. Each individual of the population is divided in a number of parts (as in superstructure), from a minimum of one to a maximum equal to the number of enthalpy intervals that characterize our problem. It is necessary to fix the number of stages as a constant, equal for the whole population. The generic portion of an individual is further divided into triplets; each triplet represents a heat exchanger: the first two numbers indicate the cold and the hot streams in input, while the third value is reserved for the power of heat exchanged in [kW]. So, for example, the HEN with *ns=1* in Fig.3.1, is represented by the following vector:

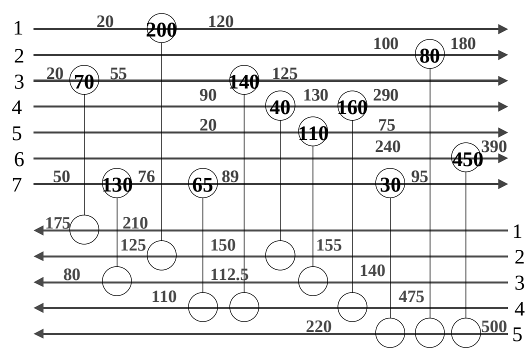


Fig.3.1

(1) *{3,1,70 /6,5,450 /2,5,80 /4,2,40 /4,4,160 /5,3,110/7,3,130 /1,2,200 /3,4,140 /7,4,65 / 7,5,30 }*

When *ns>1*, the *k-part* of the chromosome corresponding to the *k-interval* will be built, obviously, only with the streams present in the interval itself.

Note that if (1) represents a HEN, the same vector with all zeros in the third value of each triplet represents only the topology of the HEN, that is his structure, saying nothing about the heat exchanges. We could look at this vector as to a class of HENs because, from theory, a topology could generate an infinity of HENs too. So we have also to find a strategy to associate, to a particular topology, a specific HENs (when and if it is possible). The solution we have adopted is: to make a parallel development of topology complexity and well-dimensioned HEN. In practice, given a topology, there is a subroutine which make it collapsing (completely or in part) in a dimensioned-HEN, that is: in a vector with no third-values zero.

The algorithm HENS99 shows the following structure:

1. Checking of the feasibility of the problem: BCCC and BHCC are built and enthalpy intervals are determined
2. Choice and specification of the number of intervals (*ns*), with $1 \le ns \le nh$; specification of *EMAT* value

3. Randomized generation of the population (an individual chromosome is composed filling its *ns* parts with a triplet, each triplet having third value zero)
4. A subroutine verifies if the heat exchange between hot and cold stream is possible respecting *EMAT*. If so, it determines the load of heat exchanger (in [kW]), choosing randomly a value between the minimum thermodynamically possible and the minimum load of the two streams. If third values of some triplets are still zero, triplets are deleted from the chromosome.
5. Fitness calculation. When *ns=1*, the objectives are: target temperatures and fixed number of units. The first one is zero only when all streams have reached target temperatures. The second one, on the contrary, is based on Euler's Network Theorem for graphs, which becomes, in HENS theory [Sh95]:

   Nheat_exch.s = (cs + hs + numb. utility streams) + Nloops – Nsubsystems

   With no loops and only one subsystem (that is, the network itself), the solution HENs have to be made up of *Nheat_exch.s = (cs + hs + numb. utility streams) – 1* heat exchangers and the objective value will be zero only when the number of units of the individual is equal to *Nheat_exch.s*.
   When *ns>1*, the objectives are: target temperatures and minimum total area. The second objective is obtained using $Q = A*U*LMTD$ (*LMTD* = logarithmic mean temp. difference) and $1/U = 1/h_{cs} + 1/h_{hs}$, and calculating *A*. *LMTD* is here approximated by the sum of two-thirds of the geometric mean and one-third of the arithmetic mean (*Paterson's approximation*).
6. Tournament selection (based on Pareto dominance [Pol95])
7. Crossover. It is the classical multipoint crossover used in EA. It exchanges portions of two chromosomes A and B to form new strings A' and B'. The sites for exchange can only correspond to whole intervals in our vector representation, so there are *ns* different position where chromosomes can be cut. If *ns=1* crossover operator loses one's sense.
8. Mutation. Given a chromosome, all his portion (*ns*) are subjected to this operator with a low probability. If a portion has to be mutated, a random choice is done between two kinds of mutation. The first one fixes two sites and flip triplets included between them. The latter chooses randomly one site of the portion and inserts a new triplet (obviously with third value zero). Before returning to the main program, subroutine of point (4) is called; it recalculates all loads starting from the position of mutation and proceeding to the end of the chromosome's portion. Triplets with loads equal to zero at the end of previous operation are deleted.
9. Fitness calculation
10. Point (6) for *n* generations or until the achievement of the goal

4. Results

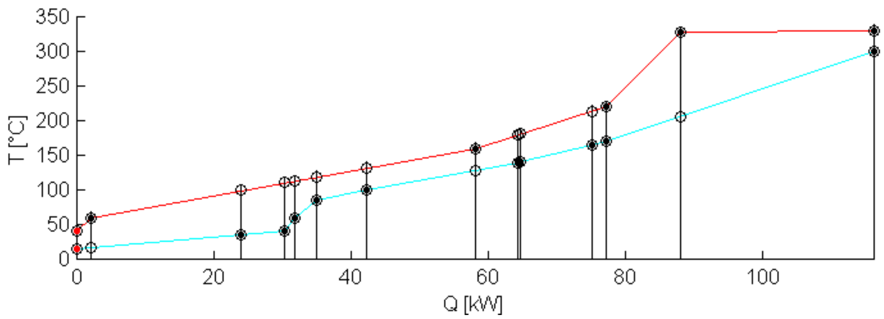
HENS99 has been applied to a particular problem obtained from a case study proposed in [AppxB Sh95] (case study 9A2, *Aromatics Plant*); its formulation follows:

```
cs = 7;
hs = 6;
Tinpc =[100,35,85,60,140,15,15];                [°C]
Toutc =[300,164,138,170,300,40,40];             [°C]
Mcp_cold =[0.1,0.07,0.35,0.06,0.2,0.6,0.6];     [kW/°C]
h_cold =[0.4,0.2,0.82,0.16,0.02,1.15,1.15];     [kW/(m2 °C)]
Tinph =[327,220,220,160,330,330];               [°C]
Touth =[40,160,60,60,327,327];                  [°C]
```

```
Mcp_hot =[0.1,0.16,0.06,0.4,4.7133,4.7133];
h_hot =[0.3,0.12,0.011,1.58,0.16,0.16];
```

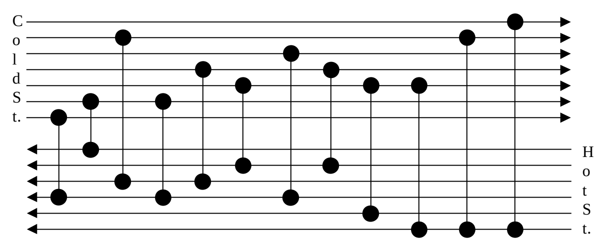
[kW/°C]
[kW/(m^2 °C)]

All streams are process streams, except 6-7 cold and 5-6 hot (utility streams). Note that we have already found the amount of utility consumption to build BCCC and BHCC. Once fixed temperatures of utility streams, we have at first imposed energetic balance and then varied Mcp values until the reaching of HRAT=25 °C, obtaining the following Q-T plot:



There are 14 enthalpy intervals, determined by Q = (0; 2.00; 24.00; 30.35; 31.75; 35.00; 42.20; 58.00; 64.24; 64.70; 75.02; 77.18; 77.20; 87.90; 116.18); HRAT shows itself for Q = 0. We have considered the two cases with *nh=1* and *nh=4* (the second one consists of the union of enthalpy intervals 1-4, 5-8, 9-12, 13-14), keeping a constant value for EMAT=20 °C. About the first one, the best solution obtained in 5 runs of 50 generations with *npop=30*, is composed from *cs+hs-1* heat exchangers (therefore no loops are present) and undergoes the constraint of minimum load *minkw=3 [kW]* (figure below reports its topology):

best-chromosome:{1,1,20 / 4,3,3.6 / 4,2,3 /5,2,6.6 /2,3,6 /3,4,18.55/2,6,3.03 / 5,5,14.14 / 6,1,8.7 / 6,4,6.3 / 7,4,15 / 5,6,11.11}

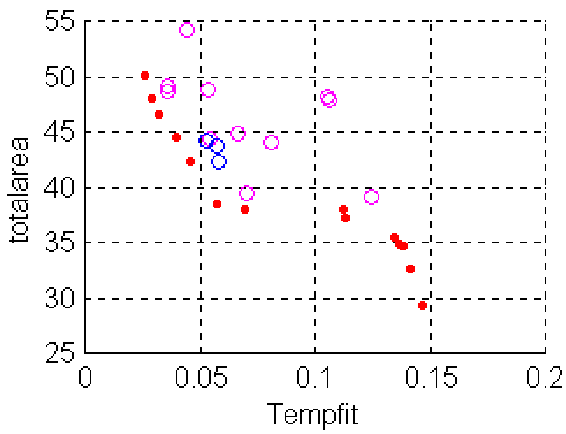


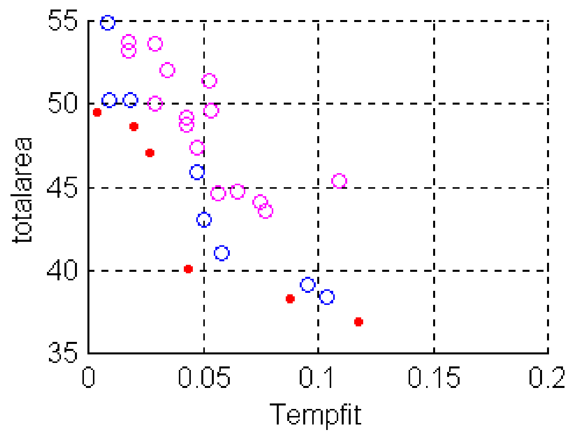
The second series of runs consists of 30 runs of 60 generations with *npop=30*. All streams are here considered as process streams. The portions of the chromosomes corresponding to the four intervals are divided by '///'. The HEN with minimum total area obtained is represented by:

best-chromosome:{6,3,3.19 / 6,1,7.28 / 6,4,4.53 / 7,1,0.03 / 2,4,1.75 / 7,4,14.97/// 1,1,3.8 / 4,1,2.84 / 2,3,3.98 / 2,2,1.48 / 4,2,1.64 / 3,4,18.55 /// 5,3,2.43 / 5,2,3.58 / 1,1,3.21 / 4,1,0.84 / 2,2,1.82 / 4,2,0.85 ///1,1,10.7 /1,5,2.29 /5,5,11.85 /5,6,14.14}

Minimum Total Area found = 49.48 m^2

Obviously, the last result has only a theoretical interest, since too many heat exchangers with low loads form the network. We also report two snapshots of the objective-functions space, the first one corresponding to the early phase of evolution, the second one to the population at the achievement of the best result. Pareto-front is determined by individuals indicated with a point. Note that objective functions are in conflict, because the HEN with minimum total area is the HEN without heat exchangers, that is the network with area equal to zero.





References

[Baeck98] Baeck, T. et al., in *'Genetic Algorithms and Evolution Strategies In Engineering & Computer Science'*, Quagliarella, D. et al. editors J. Wiley and Sons (1998)

[Pol95] Poloni, C. 'Hybrid GA for Multi Objective Aerodynamic Shape Optimization', on *'Evolution Algorithms in Engineering and Computer Science'*, (J.Periaux, et Al. ed.) John Wiley & Sons (1995)

[Sh95] Shenoy, U.V. 'Heat Exchanger Network Synthesis', Gulf Publishing Company (1995)