Hybridized Fuzzy and GAs Distributed Optimization Method

H. Kawarada and H. Suito

Faculty of Engineering, Chiba University, Chiba, 263-8522, Japan
Tel&Fax +81-43-290-3505, Email:kawarada@applmath.tg.chiba-u.ac.jp

Abstract

In this paper, hybridized algorithm with fuzzy optimization method (FOM) and GAs is presented to solve nonlinear optimization problems. FOM is a modification of steepest descent method (SDM), which plays a role of local minimizer search algorithm. For small size problems of global optimization, Single-start FOM is enough to solve them. For large size problems of global optimization, Multi-start FOM is presented, which is a hybridized one stated in the beginning and is found to be of high performance from a lot of numerical experiments. Furthermore, extremely large size problems of global optimization are often encountered. Hierarchical algorithm is presented to solve them, which is also a hybridization of domain decomposition of searching domain and Multi-start FOM.

1 Introduction

In optimization problems arising in industry and applied sciences, nonlinear optimization problems are frequently treated. The difficulties in solving them are;

1. Existence of plural local minimizers.
2. Need to search for a global minimizer among plural local minimizers.

It is well known that it is very expensive to solve such difficulties. The optimization algorithm discussed here aims at searching for a global minimizer in as low computational cost as possible.

2 Optimization Algorithm

In this section, hybridized optimization algorithm is summarized.

2.1 Fuzzy Optimization Method[2]

FOM is a modified version of steepest descent method (SDM), in which searching direction vector at nth step is constructed by use of convex conjugation between $(n-1)th$ searching direction vector and nth searching direction vector used in SDM. The coefficient of convex conjugation is computed by use of stochastic fuzzy estimation based on data resulting from $(n-1)th$ and $(n-2)th$ searching direction vectors. A lot of numerical experiments by means of FOM have shown that its convergence domain is wide and low computational costs.

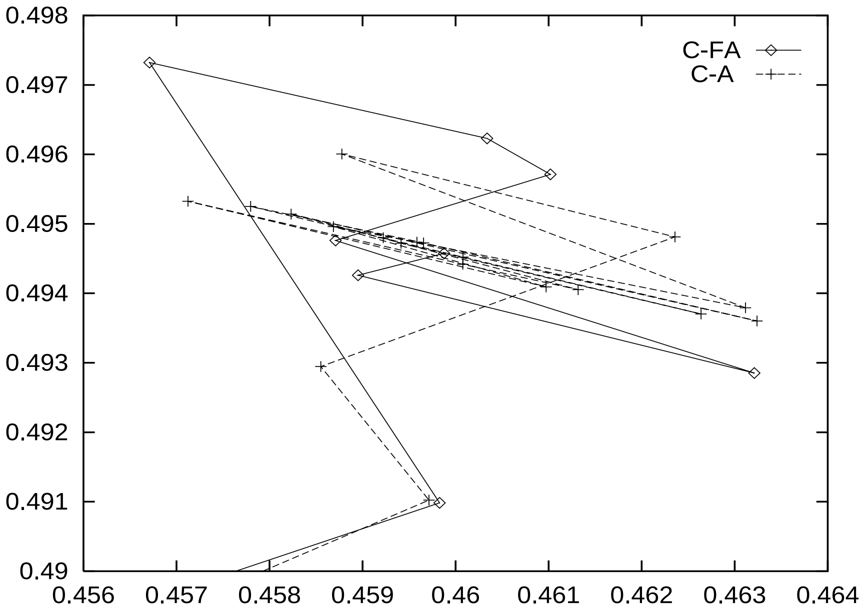

Figure 1: FOM

2.2 Single-start FOM[2]

Single-start FOM is implemented by repeating up- and down-hill procedure in the following steps;

1. Search for a local minimizer by FOM, i.e., down-hill algorithm.
2. Climb a hill of the target manifold up to the top of hill or cross the ridge by using golden section algorithm along straight line started stochastically from a local minimizer already obtained, i.e., up-hill algorithm.
3. Choose a quasi-local maximizer obtained in step 2 as an initial point for the next down-hill procedure.
4. Continue to find local minimizers by repeating steps 2 and 3. Sometimes revisit to old minimizer occurs.
5. If the number of successive revisits to old minimizers is beyond the preset one, stop step 2.
6. Scan all local minimizers, compare all local minima and set the lowest one as a global minimum.

The main causes to fail in search for a global minimizer of nonlinear minimization problem with multi-parameters (> 5) by use of Single-start FOM are;

1. The positioning of an initial point for next down-hill procedure after up-hill is wrong, in other words, the quasi-local maximizer, i.e., the restarting point for down-hill procedure, remains in the middle way of the hill and does not climb up to the top of hills or not cross completely the ridge.
2. The lack of the number of the searching vectors, which mean the patrols to search for local minimizers.

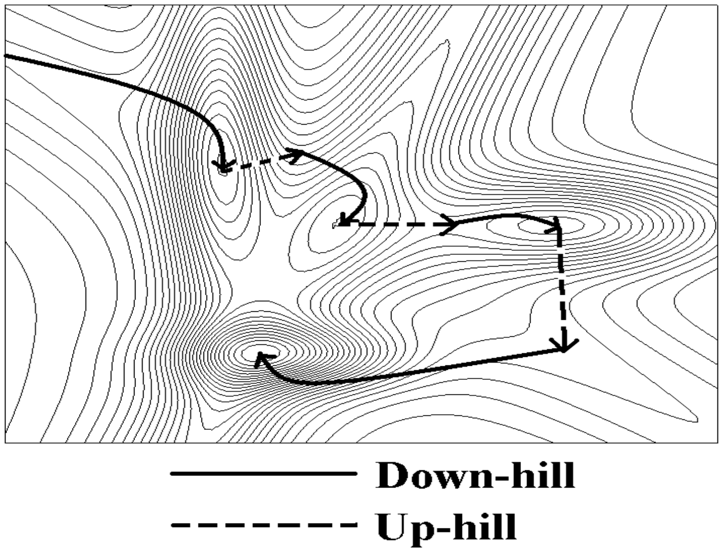

Figure 2: Single-start FOM

2.3 Multi-start FOM[3]

2.3.1 Motivation for Multi-start FOM

Multi-start FOM is proposed in order to improve the defects mentioned above. The first one of these improvements is an increase of the number of patrols, which are arranged in the target manifold. The second one is to let the up-hill patrols completely climb to the top or cross the ridge.

In order to realize the second improvement after the first one, GAs is used to scatter the set of quasi-local maximizers, i.e., restarting points to the next down-hill procedure, into much higher positions and to let the set of quasi-local maximizers escape completely from the hole.

The three fundamental functions in GAs play an important role in the following way;

The *selection* operation in GAs aids the complete climb up to the summits of the target manifold or cross the ridge. In other words, it prevents the patrol from remaining in the same hole. The *cross over* and *mutation* operations contribute to the optimal rearrangement of the set of restarting initial points, i.e., the down-hill patrols.

That is to say, we give individuals the intelligence to exchange information through *cross over* and *mutation* operations about the way to get the good positioning for looking for other local minimizers. Here, the *individuals* of population in GAs mean the *patrols*.

2.3.2 Prospect Positioning

Let $f(\mathbf{x})$ be a smooth cost function defined in a bounded domain $\mathcal{A} \subset R^D$ (D is the number of dimensions). Here, $\mathbf{x} = (x_1, x_2, \cdots, x_D) \in \mathcal{A}$.

If we apply GAs to search for a global minimizer of $f(\mathbf{x})$, the positive constant C is chosen so as to satisfy $C - f(\mathbf{x}) > 0$ in $\mathcal{A}$, and the fitness for the individuals, i.e., the restarting initial points are computed by use of $C - f(\mathbf{x})$.

Here, the positive constant C' is chosen so as to satisfy $C' + f(\mathbf{x}) > 0$ in $\mathcal{A}$ and then the fitness are computed by use of $C' + f(\mathbf{x})$. If we use the fitness selection rule due to

$C' + f(\mathbf{x})$, the positions of the patrols obtained by use of MCA are scattered into much higher places than the original one, which allow to take the fine *prospect positioning* for the restarting patrols.

2.3.3 Algorithm

Let operators F, M and R define as follows;

- F : Algorithm due to FOM,
- M : Mountain crossing algorithm,
- R : Rearrangement algorithm by GAs.

Then the algorithm of Multi-start FOM is stated in the following way;

1. Give a initial population W^0 (the set of patrols)
2. Compute $U^n := FW^n$ (the set of local minimizers obtained)
3. Compute $V^n := MU^n$ (the set of quasi-local maximizers obtained)
4. Compute $W^n := RV^n$ (the set of rearranged patrols)
5. Increase generation number $n := n + 1$ and repeat the steps from 2 to 4 until the generation number n is beyond the preset one.

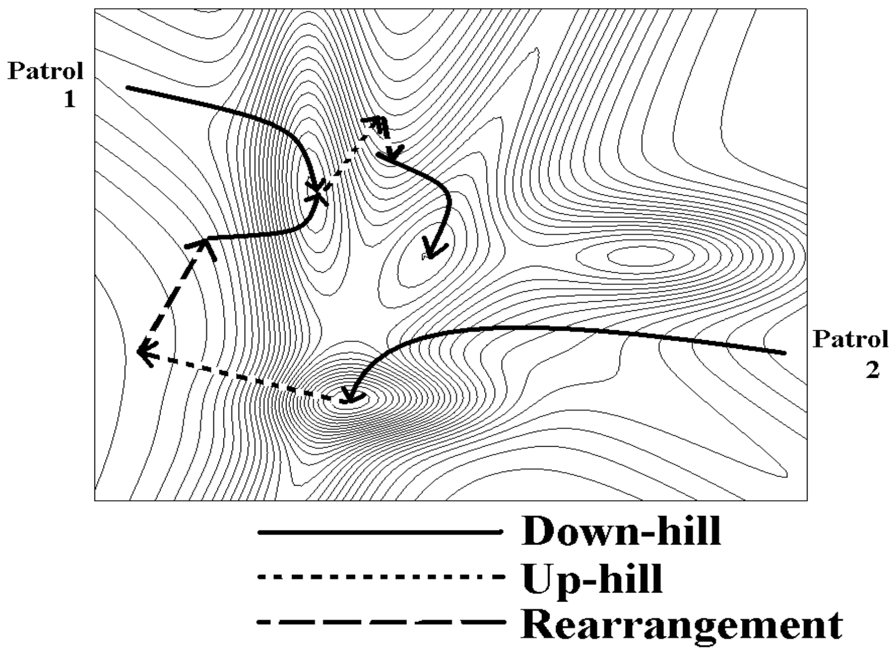

Figure 3: Multi-start FOM

In this study, operation R is applied in order to obtain *prospect positioning*, which is taken by the survived patrols through the fitness selection rule. It is observed through our numerical experiments that the *prospect positioning* for the restarting initial points are rather effective.

2.4 Hierarchical Multi-start FOM

Hierarchical algorithm is described in the following;

1. Put an appropriate number of islands in searching domain.
2. Find a subglobal minimizer in each island by use of Multi-start FOM.
3. Regard a subglobal minimizer as a representative point in each island.
4. Scatter all representative points into another positions by means of an idea of GAs.
5. Reconstruct each island by regarding removed representative point as center point for it.
6. Return to step 2 and repeat this procedure N times, which is the preset one.

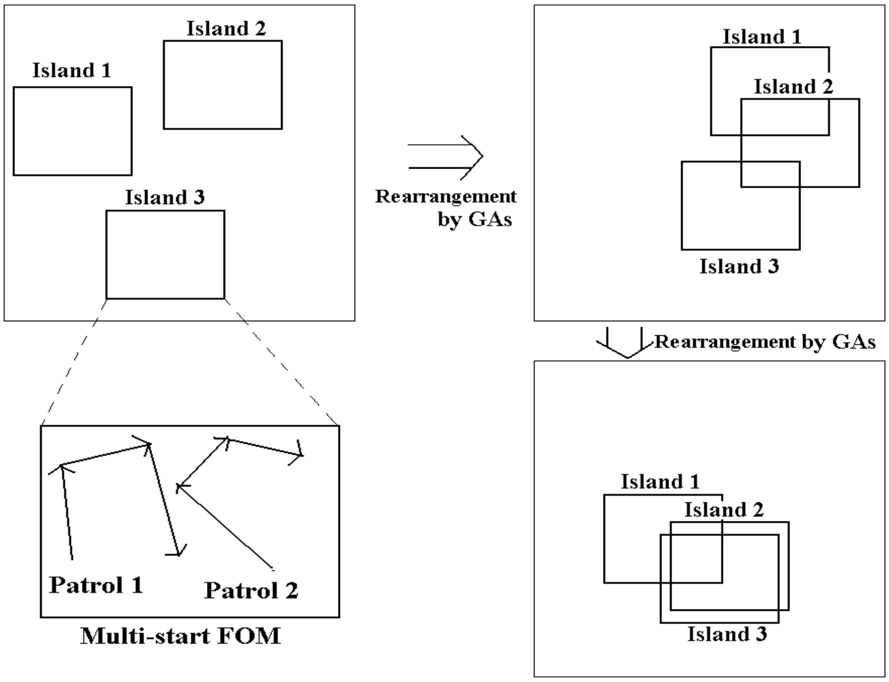

Figure 4: Hierarchical Multi-start FOM

References

[1] Kowalik, J. and Osborne, M. R., *Methods for Unconstrained Optimization Problems*, Elsevier, 1968.

[2] Kawarada, H. and Suito, H., "Fuzzy Optimization Method", *Computational Science for the 21st Century*, pp 642-651, John Wiley & Sons, 1997.

[3] Kawarada, H., Ohtomo, T., Periaux, J. and Suito, H., "Multi-Start Fuzzy Optimization Method", *GAKUTO International Series, Mathematical Sciences and Applications*, Vol.11,*Recent Developments in Domain Decomposition Methods and Flow Problems*, 1997.