Application of Multi-start Fuzzy Optimization Method to Shape Optimization Problems

H. Suito and H. Kawarada

Faculty of Engineering, Chiba University, Chiba, 263-8522, Japan
Tel&Fax +81-43-290-3505, Email:suito@applmath.tg.chiba-u.ac.jp

Abstract

Multi-start Fuzzy Optimization Method (MSFOM) is applied to solve shape optimization problems arising in advanced engineering. MSFOM is a hybrid algorithm with fuzzy logic and GAs. This algorithm is able to find global minimum in low computational cost. An optimization problem for sound absorbing plate is treated by use of MSFOM. This problem needs to solve Helmholtz equation in deformed region. Numerical algorithm to solve Helmholtz equation and several numerical results are presented.

1 Introduction

In this paper, we apply the Multi-start Fuzzy Optimization Method to a shape optimization problem for sound absorbing plate. This problem deals with the phenomena of sound propagation near pine in the water. Plane incident wave with frequency ω and incident angle (α, β) comes from right hand side through medium 1(water) toward absorbing plate made of medium 2 (pine). Some part of the incident wave is transmitted into the pine and the other is reflected into the water. In fact, pine has been used as one of typical absorbing materials from old times.

In the first step, the boundary matching problem is studied; in other words, the sound field of total system is obtained by using techniques of domain decomposition type. The sound field composed of the incident and reflected waves in the water and the transmitted wave in the pine is computed separately. The total solution is obtained by coupling two solutions computed in each domain to satisfy transmission conditions defined on an interfacial boundary. Here, let us construct the cost function of Dirichlet-Neumann type on the boundary, which is directly minimized by means of Fuzzy Optimization Method (FOM)[1, 2].

In order to evaluate the shape of interfacial boundary in pursuit of better absorption of sound field into the pine, we proceed to the shape optimization. The minimization problem, the cost of which is represented by the amplitude defined on the fixed flat surface in the pine, is proposed to regard Γ as a control variable. Here, the deformation of Γ is represented by a superposition of the product of two scaling functions for wavelet. Finally, the minimization problem constructed is also directly solved by means of FOM.

2 Deformation of the Interfacial Boundary

2.1 Scaling functions for wavelet

Wedge-shaped patterns often arise in wavelet. A scaling function for wavelet is introduced as follows;
Let

$$\eta_0(x) = \begin{cases} 1 & x \in [0,1], \\ 0 & else, \end{cases} \tag{1}$$

$$
\begin{aligned}
f_0(x) &= (-0.585x^2 + 1.867x)\eta_0(x), && (2)\\
f_1(x) &= (1.170x^2 - 2.734x + 1.282)\eta_0(x), && (3)\\
f_2(x) &= (0.585x^2 + 0.867x - 0.282)\eta_0(x). && (4)
\end{aligned}
$$

and

$$\phi(x) = f_0(x) + f_1(x-1) + f_2(x-2). \tag{5}$$

Define

$$\phi_{L,m}(x) = \sqrt{N_L} \cdot \phi(N_L x - m) \ \ (m \in Z) \tag{6}$$

where $N_L = 2^L$.

Then $\{\phi_{L,m}\}$ constitutes an orthonormal set, i.e.,

$$\int\limits_R \phi_{L,m}\phi_{L,m'}dx = \delta_{m,m'}. \tag{7}$$

2.2 Admissible Set for the Deformed Surface

Represent the deformed surface by means of a superposition of $\phi_{L,m}(y)\phi_{L,m'}(z)$, i.e.,

$$\Gamma = \sum_{m,m'} \gamma_{mm'} \cdot \phi_{L,m}(y)\phi_{L,m'}(z). \tag{8}$$

Admissible set for the deformation of the interfacial boundary is defined by

$$\mathcal{A}_1 = \{\gamma_{mm'} \in R | \, |\gamma_{mm'}| \leq K \ (m, m' = 1, 2, 3, \cdots, M_1)\} . \tag{9}$$

3 Shape optimization

3.1 Configuration

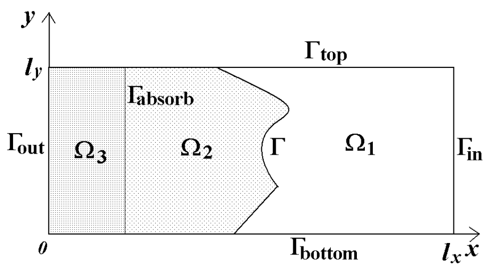

Figure 1: Projection of computed domain into $x - y$ plane

- $\Gamma_{top}, \Gamma_{side}$ and Γ_{bottom} are rigid boundaries, i.e., density of these walls is infinity. Hence, sound wave is completely reflected at these boundaries.
- Γ_{in} is a vibrating plate which makes sound wave.

- Ω_1 is occupied with water.
- Ω_2 is made of pine, the role of which is to absorb the sound wave coming through Ω_1.
- Γ is the boundary between Ω_1 and Ω_2. We will try to optimize its shape to absorb the sound wave into Ω_2 as much as possible.
- Ω_3 is a so-called Fictitious Domain, i.e., artificial domain to approximate the boundary condition at the infinity. In this domain, Helmholtz eq. with complex wave number is assumed, which is derived from Navier-Stokes eq. including viscosity term. Sound wave transmitted from Ω_2 is almost completely damped in this domain and is not reflected into Ω_2.
- Γ_{absorb}, on which the amplitude of absorbed sound wave in Ω_2 is computed.
- Γ_{out}, on which sound wave does not exist because of the damping effect in the domain Ω_3.
- $\Omega = \Omega_1 \cup \Gamma \cup \Omega_2 \cup \Omega_3 = (0, l_x) \times (0, l_y) \times (0, l_z)$
- $u^{(i)}(x, y, z)$ $(i = 1, 2, 3)$: Complex sound pressure.
- k_i $(i = 1, 2, 3)$: Wave number.
- λ_i $(i = 1, 2, 3)$: Wave length.
- c_i $(i = 1, 2, 3)$: Sound speed.
- ω : Angular velocity of the incident wave.
- ρ_i $(i = 1, 2, 3)$: Density of medium.
- $\mathbf{n}$: Outward normal vector on the boundaries.
- Γ : Interfacial boundary between Ω_1 and Ω_2.
- α, β : An incident angle of plane wave,

where $i = 1$ means water, $i = 2$ means pine and $i = 3$ means the fictitious domain.

3.2 Minimization with the Constraint

Define the state equation;

$$\begin{cases} (\triangle + k_i^2)\, u^{(i)}(\Gamma, a) = 0 & \text{in } \Omega_i, \quad (i = 1, 2, 3), \\ u^{(1)}(\Gamma, a) = u^{(2)}(\Gamma, a) = a & \text{on } \Gamma, \\ \dfrac{\partial u^{(i)}(\Gamma, a)}{\partial n} = 0 & \text{on } \Gamma_{top} \cup \Gamma_{bottom}, \\ & \quad \cup \Gamma_{side} (i = 1, 2, 3), \\ u^{(1)}(\Gamma, a) = e^{ik_1 \cos\beta l_x} e^{ik_1 \sin\beta \cos\alpha y} e^{ik_1 \sin\beta \cos\alpha z} & \text{on } \Gamma_{in}, \\ u^{(2)}(\Gamma, a) = 0 & \text{on } \Gamma_{out}, \end{cases} \tag{10}$$

and the cost function;

$$J_c(\Gamma,a) = -\int\limits_{\Gamma_{absorb}} \left|u^{(2)}(\Gamma,a)\right|^2 d\Gamma$$
$$+\frac{1}{\varepsilon}\int\limits_{\Gamma}\left|\frac{1}{\rho_1}\frac{\partial u^{(1)}(\Gamma,a)}{\partial n} - \frac{1}{\rho_2}\frac{\partial u^{(2)}(\Gamma,a)}{\partial n}\right|^2 d\Gamma, \ \ (\varepsilon > 0). \tag{11}$$

The Dirichlet datum a is defined on Γ by

$$a = \sum_{mm'} a_{mm'} \cos(\frac{\pi m}{l_y} y) \cos(\frac{\pi m'}{l_z} z). \tag{12}$$

Admissible set for a is represented by

$$\mathcal{A}_2 = \{a_{mm'} \in \mathbf{C} \ \ (m, m' = 0, 1, 2, \cdots, M_2) || a_{mm'}| \leq L\}. \tag{13}$$

Minimization problem is;
$[Pr_c]$: Minimize $J_c(\Gamma, a)$ for $(\Gamma, a) \in \mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2$.

4 Numerical Solution of Helmholtz Eq. in the Deformed Region

In order to compute the sound field in the domain with complicated boundary, the coordinate transformation is used as follows;

1. Generate mesh system in the deformed domain, which is the transformation from physical domain to computational one;

$$x = x(\xi,\eta,\zeta), \ y = y(\xi,\eta,\zeta), \ z = z(\xi,\eta,\zeta). \tag{14}$$

2. Transform differential operators by use of (14).

3. Transform Helmholtz eq. by use of (14).

Transformed Helmholtz equation is discretized by use of finite difference method. Discretized Helmholtz eq. constitutes a large scale of linear system of equations. In order to solve this system of equations, GPBi-CG method[3] is adopted.

5 Hybrid Algorithm by FOM and GAs

In this section, Fuzzy optimization Method (FOM) is briefly summarized.

1. Fuzzy Optimization Method[2]

   Modification of the steepest descent method by means of stochastic Fuzzy ruling.

2. Single-start FOM[2]

   Combination of FOM and Mountain Crossing Algorithm (MCA).

3. Multi-start FOM[1]

   An increase of number of initial points. Let operators F, M and R define as follows;

- F : Algorithm due to single-start Fuzzy optimization method,
- M : Mountain crossing algorithm,
- R : Rearrangement algorithm by GAs.

Then the algorithm of Multi-start FOM is stated in the following way;

(a) Give an initial population W^0 (the set of patrols).

(b) Compute $U^n := FW^n$ (the set of local minimizers obtained).

(c) Compute $V^n := MU^n$ (the set of quasi-local maximizers obtained).

(d) Compute $W^n := RV^n$ (the set of rearranged patrols).

(e) Increase generation number $n := n + 1$ and repeat steps from (2) to (4) until the generation number n is beyond the preset one.

6 Numerical Results

Incident angle is chosen $(\alpha = 0, \beta = 30 (> \beta_c))$ in all cases. β_c is a critical angle at which the complete reflection occurs in the case of flat interfacial boundary;

$$\sin \beta_c = \frac{k_2}{k_1}. \tag{15}$$

First, boundary matching problem is discussed. In this case, interfacial boundary is flat and cost function is defined as follows;

$$J_b(a) = \int_\Gamma \left| \frac{1}{\rho_1} \frac{\partial u^{(1)}(a)}{\partial n} - \frac{1}{\rho_2} \frac{\partial u^{(2)}(a)}{\partial n} \right|^2 d\Gamma. \tag{16}$$

Figure 2 shows an absolute value of complex sound pressure after optimization process terminates.

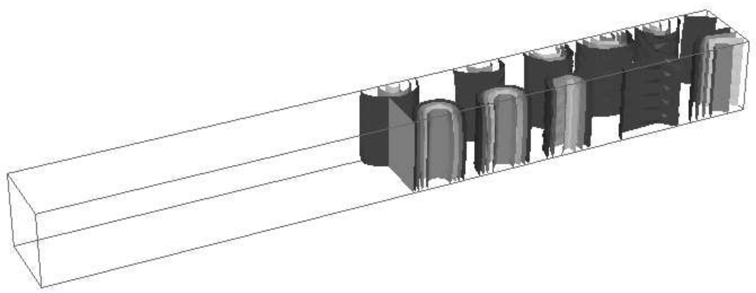

Figure 2: Boundary matching problem (absolute value)

Since incident angle β is larger than the critical angle β_c, numerical results show that sound wave is almost completely reflected into water region.

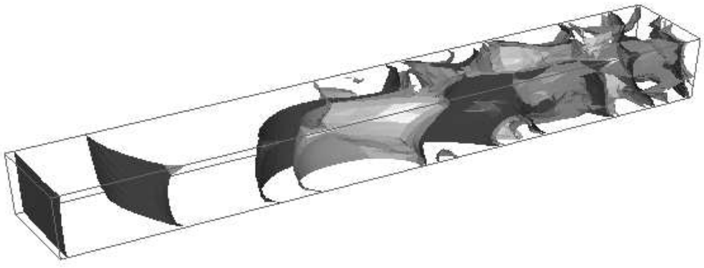

Figure 3: Shape optimization (absolute value)

Second we optimize the shape of the interfacial boundary under the constraint of the matching of transmission condition, i.e., Shape Optimization Problem. Figure 3 shows an absolute value of complex sound pressure after optimization process terminates.

As a result of shape optimization of the interfacial boundary, numerical results show that a large portion of the sound wave is transmitted into the pine region.

7 Conclusions

One of the difficulties we had to overcome is to construct Helmholtz solver in the deformed region. The deformation of interfacial boundary is represented by a superposition of the scaling functions in wavelet, that makes it not only easy to compute the deformed boundary in each iterative step of the optimization, but also catches the characteristic of the present problem. The use of FOM as the minimization technique manifests the validity and the utility of itself from satisfactorily numerical results. Thanks to these combined uses of the above-mentioned techniques, it seems that reasonable results were obtained from a numerical point of view.

References

[1] H.Kawarada, T.Ohtomo, J.Periaux and H.Suito, "Multi-start Fuzzy Optimization Method", GAKUTO International Series, Mathematical Sciences and Application, Vol.11, Gakkotosho, pp.338-345, (1998).

[2] H.Kawarada and H.Suito, "Fuzzy Optimization Method", Computational Science for the 21th Century, John Wiley & Sons, pp.642-651, (1997).

[3] S.-L. Zhang, "GPBi-CG : Generalized product-type methods based on Bi-CG for solving nonsymmetric linear systems", SIAM Journal on Scientific Computing, 18-2, pp. 537-551, (1997).