GENETIC ALGORITHMS TO MINIMIZE A MULTI-VALUED LOGIC FUNCTION REPRESENTED BY ARITHMETICAL POLYNOMIAL FORMS

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ABSTRACT

One of actual applied problems of modern logic design on the base of multi-level elements is multi-valued logic (MVL) function minimisation. GA is more effective during general task solution. Therefore the author of this paper allow to use this algorithm for synthesis of some a priory minimal representation form of MVL functionsGeneral regularity is formulated by mathematical rules for computation of minimal form representation. In the case of incompletely defined MVL functions this class are essential extended by regulation of the domain of undeniable.

1. INTRODUCTION

In [1-3] it is demonstrated that development of creating digital networks technology on the basis of binary elements is physical limited by velocity of signal distribution in solid, area of interconnections in a crystal, dissipation power per unit of volume and other. Therefore intensive search of ways to solve these problems comes true within the framework of new technologies. One of them is to use multi-level elements, mathematical tools of multi-valued logic algebra, parallel and conveyor calculations.

Now, the essential progress in creating of circuit design on basis of on the multi-level logic elements is obtained [2-4]. So, the circuits of arithmetical adder and multiplier on the basis of multi-level logic elements were proposed in [5]. Several researches is dedicated to questions of multi-valued programmable logic array (MVPLA) [4, 6, 8].

The minimisation of the multiple-valued circuits is a urgent task of logic designing. The decision of this task is possible by of minimisation of logic function (multiple-valued logic (MVL) function), sold this circuit. Thus the algorithm of minimisation MVL-function depends on functionally complete basis in which the circuit for the given function will be realised.

A number of algorithms is known for minimisation MVL-function in MIN-MAX and MIN-TSUM basis's. The authors of [8-10] have received algorithm for minimisation MVL-function in basis Min-MAX for MPLA. In [11-14] the various algorithms for minimisation Reed-Muller expansion of MVL-function (MIN-TSUM basis) are offered. In [13-14] used genetics algorithms.

However, the authors [15-17] show, that for MVL-function in a various of cases, it is better to use arithmetic bases, instead of logic. Then MVL-function the form is represented as arithmetic polynom. Therefore new algorithms should be received for minimisation arithmetic polynomial form for MVL-function. The authors of [16] research minimisation task of MVL functions in class of arithmetic polynomial forms by means decision diagrams. Algorithm for minimisation incompletely specified MVL-function by Z.Zilic is offered. The authors [18] have received genetic algorithm of minimisation arithmetic polynomial form of MVL-function.

In the given work the results received in develop [18]. As against results stated in [18], in the present paper for minimisation arithmetic polynomial form such parameter as "polarity" arithmetic polynomial form in addition is offered to use.

2. ARITHMETICAL POLYNOMIAL FORM OF MVL REPRESENTATION

Definition 1. The MVL function on n variables is function $f(x_1,x_2,...,x_n)=f(X)$. The values and arguments of this function belong to set {0, 1,...,m-1}.

Table 1. Computing the values of $P(X)=(1/4)(4x_2+4x_1-10x_1x_2+4x_1x_2^2+4x_1^2x_2-2x_1^2x_2^2)$

$x_1\ x_2$	Arithmetical polynomial form	function values
0 0	**P(X)**=(1/4)(4·0+4·0-10·0·0+4·0·0+4·0·0-2·0·0)=(1/4)(0+0-0+0+0-0)	0
0 1	**P(X)**=(1/4)(4·0+4·1-10·0·1+4·0·1+4·0·1-2·0·1)=(1/4)(0+4-0+0+0-0)	1
0 2	**P(X)**=(1/4)(4·0+4·2-10·0·2+4·0·2+4·0·4-2·0·4)=(1/4)(0+8-0+0+0-0)	2
1 0	**P(X)**=(1/4)(4·1+4·0-10·1·0+4·1·0+4·1·0-2·1·0)=(1/4)(4+0-0+0+0-0)	1
1 1	**P(X)**=(1/4)(4·1+4·1-10·1·1+4·1·1+4·1·1-2·1·1)=(1/4)(4+4-10+4+4-2)	1
1 2	**P(X)**=(1/4)(4·1+4·2-10·1·2+4·1·4+4·1·2-2·1·4)=(1/4)(4+8-20+16+8-8)	2
2 0	**P(X)**=(1/4)(4·2+4·0-10·2·0+4·2·0+4·2·0-2·2·0)=(1/4)(8+0-0+0+0-0)	2
2 1	**P(X)**=(1/4)(4·2+4·1-10·2·1+4·2·1+4·4·1-2·4·1)=(1/4)(8+4-20+8+16-8)	2
2 2	**P(X)**=(1/4)(4·2+4·2-10·2·2+4·4·2+4·2·4-2·4·4)=(1/4)(8+8-40+32+32-32)	2

Definition 2. The truth table vector: $\mathbf{X}=[\,x^{(0)}\ x^{(1)}\ \ldots\ x^{(m^n-1)}]^T$ is the column of a truth table which corresponded the values of m-valued MVL function.

Any MVL function may by represented as arithmetical polynomial form [15].

Definition 3. The arithmetical polynomial form (P-form) of m-valued (m≥2) function with n variables is determined by the following equation:

$$P(X)=\sum_{i=0}^{m^n-1} p^{(i)}\ {}^{c_1}x_1^{i_1}\ {}^{c_2}x_2^{i_2}\ldots{}^{c_n}x_n^{i_n} \qquad (1)$$

were $p^{(i)}$ are the arithmetical form coefficients (it is a real numbers), and $(c_1c_2\ldots c_n)$ and $(i_1i_2\ldots i_n)$ are the m-ary expansions of c and i, respectively. The ${}^{c_j}x_j^{i_j}=(x_j^{\to c_j})^{i_j}=(x_j\oplus c_j)^{i_j}$, where $(x_j\oplus c_j)=\ =x_j\oplus c_j$ (over GF(m)).

This form is arithmetical expression. The variables of the P-form belong to set {0, 1 ,..., m-1}, but the coefficients $p^{(i)}$ are real numbers. Substituting the corresponded values of variables into (1) and executing necessary operations, we obtain the values of MVL function, which are belong to set {0, 1, ..., m-1}. So, the arithmetical polynomial form may be calculated for any m-valued MVL function (m is the number that belongs to set {0, 1, ..., m-1}) [15-18].

The calculation process of 3-valued MVL function (conjunction of 2 variables) is showed in table 1:

$$P(X)=(1/4)(4x_2+4x_1-10x_1x_2+4x_1x_2^2+4x_1^2x_2-2x_1^2x_2^2) \qquad (2)$$

Computing the P-form coefficients (1) is showed in papers [15, 18]. However, the arithmetic polynomial form based on (1), is synthesised only for completely specified functions.

But incompletely specified functions are used for a lot of number of applied problems. Therefore, the synthesis problem of incompletely specified MVL function representation form is timely.

Table 2. Arithmetical polynomial forms of MVL function $\mathbf{X}=[2\ *\ *\ *\ *\ 0\ *\ 2\ 2]^T$

Truth table vector	Coefficient vector of arithmetical polynomial form
$[2\ 0\ 0\ 0\ 0\ 0\ 0\ 2\ 2]^T$	$(1/4)[8\ -12\ 4\ -12\ 12\ -4\ 4\ 0\ 0]^T$
$[2\ 1\ 1\ 1\ 1\ 0\ 1\ 2\ 2]^T$	$(1/4)[8\ -6\ \ 2\ \ -6\ 10\ \ -6\ 2\ -2\ \ 2]^T$
$[2\ 1\ 1\ 0\ 2\ 0\ 0\ 2\ 2]^T$	$(1/4)[8\ -6\ 2\ -12\ 35\ -17\ 4\ -13\ 7]^T$
$[2\ 2\ 2\ 2\ 1\ 0\ 2\ 2\ 2]^T$	$(1/4)[8\ 0\ 0\ 0\ -8\ 0\ 0\ 4\ 0]^T$

3. THE MINIMAL ARITHMETICAL POLYNOMIAL FORM

An incompletely specified MVL function corresponds to set of the arithmetical polynomial forms (table 2). There are 7 nonzero coefficients in the first form, but there are only 3 nonzero coefficients in the last form. The using minimal arithmetical forms (with minimal number of nonzero coefficients)

is the most advantageous. Therefore, this minimal forms are usually applied for practical problems.

Author have researched the properties of minimal incompletely specified MVL function, represented by P-forms. The result of these analytical and experimental researches is following. Truthtable vectors of minimal arithmetical polynomial forms have inherent binary signs. These signs may be obtained on the basis of calculation rules for P-form coefficients and are conditioned DOT particulars [15]. There are two types of researched signs.

So, truth table column vector may be decomposed to several subvectors. The length of these subvectors is m^t ($t \le n-1$), and initial subvector elements is $x^{(ms)}$ ($0 \le s \le m^{n-2}$). The signs values for the first type depend on these subvectors (local sections) of truthtable vector. These values are determined by the expression [18]:

$$\Pi^1_{s/t}=1, \text{ if } \exists\, \mathbf{X}'=[x^{(ms)} \ldots x^{(ms+m^t)}]^T \text{ and } \mathbf{X}' \subset \mathbf{X} \text{ for } 1 < t \le n-1 \text{ и } 0 \le s \le m^{n-2}; \quad (3)$$

$\Pi^1_{s/t}=0$, otherwise.

As a result, vector of first type sign is formed: $\Pi^1=[\Pi^1_{0/1}\ \Pi^1_{0/2} \ldots \Pi^1_{s/t}]$.

The second type signs are showed periodical repeating of truth table column vector elements through interval m^t [18]. So, vector of second type sign is formed: $\Pi^2=[\Pi^2_{0/1}\ \Pi^2_{0/2} \ldots \Pi^2_{s/t}]$, where:

$$\Pi^2_{s/t}=1, \text{ if } \exists\, \mathbf{X}''=[x^{(ms)}\ x^{(ms+m^t)} \ldots x^{(ms+m^{t+1}-1)}]^T \text{ and } \mathbf{X}'' \subset \mathbf{X} \text{ for } 1 < t \le n-1 \text{ и } 0 \le s \le m^{n-2}; \quad (4)$$

$\Pi^2_{s/t}=0$, otherwise.

Change of polarity of arifmetical polynomial form permitted to minimise of nonzero coefficient in (1). For example, MVL-function X=[1 0 0 0 0 0 0 1 1]T corresponds an arithmetical polynomial form:

$$P(X) = (1/4)(4 - 6x_2 + 2x_2^2 - 6x_1 + 6x_1x_2 - 2x_1x_2^2 + 2x_1^2)$$

however, polarity of this form will change (c=(22)):

$$P(X) = (1/4)(-2x_1 + 2x_1^2).$$

4. GENETIC ALGORITHM TO MINIMIZE THE ARITMETICAL POLYNOMIAL FORM

Consider the main elements and stages of GA to minimise the incompletely specified MVL functions represented by arithmetical polynomial forms.

1. *Genes and chromosomes definition.*

The chromosome is formed from the undefined elements of truthtable vector. Therefore, every gene corresponds to the undefined value of m-valued function.

2.The forming of initial population.

Initial population contains the chromosomes with randomly formed genes. Size of the initial population is selected on the basis of experimental data (results of such experiments are discussed in 4-th section) .

3. *Fitness-function* provides the selection of fittest chromosomes in new population. In given case, chromosomes with maximum number of minimal arithmetical polynomial form signs have to be included in new population. Therefore, fitness-function is defined as :

$$F_{fitness}=\sum\Pi^1_{s/t}+\sum\Pi^2_{s/t} \quad (5)$$

where $\Pi^1_{s/t}$ and $\Pi^2_{s/t}$ are terminated by expressions (3) and (4).

So, determine the fitness-function is one of the main stages of GA design. For example, fitness-function in [22] is defined as the number of zero coefficients for arithmetical polynomial

form. However, particular feature of MVL function is following. Exchange of a function value leads to exchange of a lot of P-form coefficients.

For example, arithmetical polynomial form of 3-valued function with 2 variables ($\mathbf{X}$=[222210222]^T) has only 3 nonzero coefficients (tabl.2). Exchange only 2 values of this functions ($\mathbf{X}'$=[221210122]^T). P-form of last function already has 9 nonzero coefficients (tabl.2).

Consider defects of fitness-function, proposed in [18]. P-forms with minimal number of nonzero coefficients are selected in the new population by means selection procedures. The new population is formed on the basis of the truthtable vectors corresponding for these P-forms. However, new chromosomes, sometimes with worse properties, are generated on the basis of selected chromosomes by means of crossover and mutation procedures. Next defect of such determine the fitness-function is that it needs to compute the coefficient vector of P-form for each chromosome (truthtable vector).

The modification of fitness-function definition proposed in this work permits: (a) To remove computing the P-form coefficients from the algorithm; (b) To deliberate form truth table column vectors with maximum number of minimal arithmetical form signs. The fittest chromosomes is obtained in this case.

3. Crossover and mutation procedures.

The main idea of mutation procedure is chromosome transforming by means of genes randomly changing. The distinctive feature of mutation in proposed algorithm is different probability of exchanging genes. Describe this feature. Any gene is used at calculation of several minimal arithmetical form signs (3)-(4). The probability of gene exchange depends on the number of these nonzero signs. In other words, exchange genes with little number of nonzero signs is more priority. So, genes, which provide minimal arithmetical form zero signs, are exchanging during mutation procedure.

The maximum number of nonzero signs of both types must be obtained to synthesise the minimal arithmetical polynomial form . Practically, realisation of this condition is impossible by means only mutation operation. Therefore addition procedure for new chromosomes production is necessary. Such procedure in proposed algorithm is crossover.

The crossover procedure is exchange of chromosomes block genes. All genes in the such block are used for the calculation of specified minimal P-form sign. Therefore, the value of this sign does not exchange after crossover. However, the different variants of genes block combinations is searched. This approach permits to obtain the minimal arithmetical polynomial form during the evolutionary process.

Last stage of algorithm is a procedure of changing a polarity for vectors of arithmetical polynomial form factors, tinned for value vectors final population.

The authors carry out many experimental researches on the basis of this algorithm. The evaluates of algorithm efficiency are obtained.

5. EXPERIMENTAL RESULTS

The number of GA parameters has to be defined by empirical way for efficiency algorithm work. These parameters are initial and current population size, crossover and mutation speed, maximum of processing data dimension and way to determine the fitness-function.

Next criterion is used for evaluation of indicated parameters:

- number of populations, which are necessary to create the minimal arithmetical form synthesis;
- percent of really minimal arithmetical forms among all forms received at the of algorithm work execution.

Aims of experiments

1. Obtain comparative evaluation of method of fitness-function definition by proposed way (5) and way, described in [18].
2. Define maximum permissible dimension of initial m-valued function
3. Research influence of initial population size on execution result.

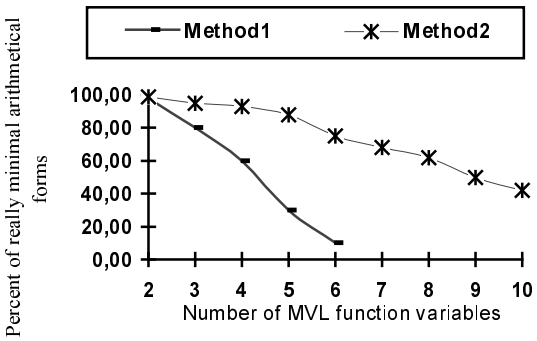

Fig.1. Dependence number of minimal P-form in result of minimisation processes for different algorithms.

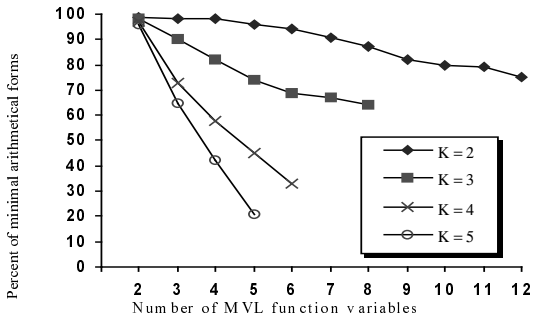

Fig.2. Efficiency of GA work at minimisation of MVL functions with different dimension.

4. Define dependence the main parameters of proposed algorithm on mutation and crossover speed.

The 3-valued MVL function with 2 variables is minimised in the experiments.

Describe obtained results.

1.The arithmetical polynomial forms are obtained for test set of incompletely specified 3-valued MVL functions (number of function variables is 2..6). This minimisation is executed by means algorithms with different fitness-functions (fig.1). The fitness-function defined as in [18] (method 1) and defined by expression (5) (method 2) are used. Further, the forms obtained by means algorithms with different fitness-functions, are compared on number of arithmetical polynomial form nonzero coefficients. The fig.6 shows, that parameters of P-form obtained by means method 2 are retrograded at increasing of variables number.

2. The result of MVL function depends on the initial data dimension. Experimental researches show that the number of minimal arithmetical polynomial forms, received at the algorithm execution, is decreased if dimension of processed MVL function is increased (fig.2).

3. Experimental researches show that the optimal size of initial and current populations belongs to the interval 10..15 (fig.3). In other words, population contain 10..15 chromosomes

4. The optimal mutation and crossover speeds for proposed algorithm are defined by empirical way. Fig. 4 shows the dependence the minimisation process speed on these parameters. Practically, the mutation and crossover speeds belonging to the interval [0,4..0,7] are optimal for discussed problem.

So, by experimental way it is obtained:

- the comparative evaluations of proposed algorithm and algorithm suggested in [18];
- the optimal parameters for proposed algorithm.

6. CONCLUSION

The genetic algorithm to minimise of incompletely specified MVL functions presented by arithmetical polynomial form is proposed in this paper. Distinctive features of this algorithm allows to:

1. Minimise the m-valued MVL function, presented by P-form (m is a prime).

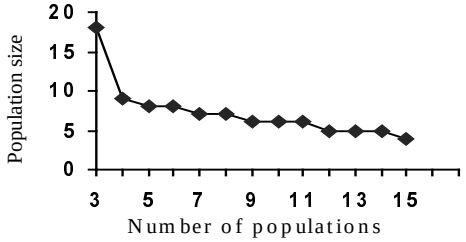

Fig.3. Dependence the number of GA populations on the population size.

1. Decrease of algorithm calculation complexity by means originally fitness-function definition.
2. Improve the algorithm characteristics (percent of obtained minimal arithmetical forms in result of minimisation processes and necessary number of populations) in comparison with algorithm [18]. However, authors so far do not essentially increase dimension of processing data.
   So, using the proposed algorithm permits to obtain minimal P-form for poorly specified MVL function (only 10-15% of truth table column vector components are defined).

   Main development direction of this approach is to increase of MVL variables number by means new decisions in the algorithm: using decomposition of equation system, employment the Gorner scheme for decrease of calculation complicity.

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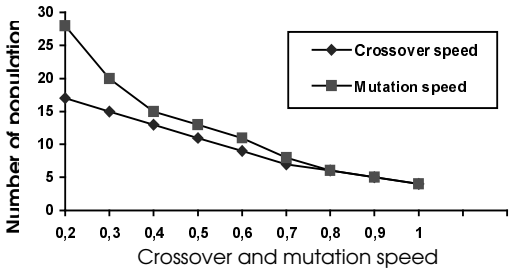

Fig. 4. Dependence the number of GA population on the mutation and crossover speeds.

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