MODELING EFFECTS OF ACOUSTIC VISCO–THERMAL BOUNDARY LAYERS AS A WENTZELL BOUNDARY CONDITION

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ABSTRACT

For certain devices, such as hearing aids or micro loudspeakers, sound propagation is significantly affected by the presence of viscous and thermal boundary layers appearing in the vicinity of solid boundaries. These effects can accurately be modeled in the linear regime by the linearized, compressible Navier–Stokes equations. However, the need for resolution of the very thin boundary layers typically makes numerical solutions of these equations computationally very expensive. A boundary-layer analysis of these equations yields explicit formulas for the acoustic velocity and the so-called excess density in the vicinity of a flat solid boundary. With these expressions as a basis, a boundary condition to the pressure Helmholtz equation can be devised that accurately takes visco-thermal boundary losses into account. The model is valid when the wavelength and the minimum radius of curvature of the wall are much larger than the boundary layer thickness, which are conditions that often are satisfied in practice. The boundary condition is of Wentzell type, that is, a condition that in addition to an impedance (or Robin) term also includes a term involving the surface Laplacian operator. Well-posedness of an associated boundary-value problem for the Helmholtz equation with this boundary condition can be established using a variation of standard techniques. In special geometries, the model reduces to well-known classical formulations, but it is applicable to most cases of acoustic calculations for which visco-thermal boundary losses need to be considered. The model has been assessed in the case of sound propagation through a compression driver, a kind of transducer that is commonly used to feed horn loudspeakers. The transmitted power spectrum through the device calculated numerically using our model agrees extremely well with computations using a hybrid model, where the full linearized, compressible Navier–Stokes equations are solved in the narrow regions of the device and the pressure Helmholtz equations elsewhere. However, our model needs two orders of magnitude less memory and computational time than the more complete model [1].

REFERENCES

[1] M. Berggren, A. Bernland, and D. Noreland. Acoustic boundary layers as boundary conditions. *J. Comput. Phys.*, 371:633–650, 2018. Also, arXiv: 1801.04177, .pdf.