NONLINEAR STABILITY OF STEADY FLOW OF GIESEKUS VISCOELASTIC FLUID

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ABSTRACT

We analyse the stability of the steady flow of a viscoelastic fluid described by the Giesekus model subject to *finite* amplitude disturbances. We design a physically motivated Lyapunov functional

$$\mathcal{V} = \int_\Omega \frac{1}{2}\rho \left|\widetilde{\mathbf{v}}\right|^2 \mathrm{dv} + \int_\Omega \frac{1}{2}\mu \left[-\ln\det\left(\mathbb{I} + \widehat{\mathbb{B}_{\kappa_{p(t)}}}^{-1}\widetilde{\mathbb{B}_{\kappa_{p(t)}}}\right) + \mathrm{Tr}\left(\widehat{\mathbb{B}_{\kappa_{p(t)}}}^{-1}\widetilde{\mathbb{B}_{\kappa_{p(t)}}}\right)\right] \mathrm{dv},$$

to determine the asymptotic stability of the steady state flow. Here, $\widetilde{\mathbf{v}}$ denotes the perturbation from the steady velocity field, $\widehat{\mathbb{B}_{\kappa_{p(t)}}}$ denotes the steady extra stress tensor field and $\widetilde{\mathbb{B}_{\kappa_{p(t)}}}$ its perturbation. The constant density and shear modulus are denoted by ρ and μ, respectively.

Following estimate on the derivative of the Lyapunov functional is derived

$$\frac{\mathrm{d}\mathcal{V}}{\mathrm{d}t} \leq C_1(\nu, \nu_1, \mu) \int_\Omega \nabla\widetilde{\mathbf{v}} : \nabla\widetilde{\mathbf{v}} \,\mathrm{dv} + C_2(\nu, \nu_1, \mu) \int_\Omega \widetilde{\mathbb{B}_{\kappa_{p(t)}}} : \widetilde{\mathbb{B}_{\kappa_{p(t)}}} \,\mathrm{dv},$$

where the constants C_1, C_2 depend on the solvent viscosity ν, elastic viscosity ν_1, and shear modulus μ. We explicitly compute the constants C_1, C_2 for the Taylor–Couette flow to determine the regions of stability of the steady cylindrical flow.

The construction of the Lyapunov functional is based on the recently proposed method [1] and relies on the knowledge of the specific Helmholtz free energy and the entropy rate of the material. The proposed Lyapunov functional allows one to explicitly analyse the role of elasticity in the onset of instability which is a problem related to the so-called elastic turbulence [2].

REFERENCES

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[2] A. Groisman, and V. Steinberg, Elastic turbulence in a polymer solution flow, Nature 405, 53–55, 2000.