Numerical treatment of evolutionary systems (generalised Friedrichs systems)

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We consider the general formulation of linear evolutionary systems

$$\overline{(\partial_t M_0 + M_1 + A)}U = F,$$

in the sense of Picard's theory, where

- M_0, M_1 are bounded, linear operators on H,
- M_0 is self-adjoint,
- A is an unbounded, skew-selfadjoint operator on H,
- $\exists \rho_0 > 0,\, \gamma > 0\, \forall \rho \geq \rho_0,\, x \in H : \qquad \langle (\rho M_0 + M_1)x, x\rangle_\rho \geq \gamma \langle x, x\rangle_\rho.$

Then for each $\rho \geq \rho_0$ and $F \in H_\rho(\mathbb{R}, H)$ there is a unique solution $U \in H_\rho(\mathbb{R}, H)$ and it holds

$$|U|_\rho \leq \frac{1}{\gamma}|F|_\rho.$$

In our talk we will discuss the numerical treatment of such problems using finite elements in space and time. In particular we consider using discontinuous and continuous methods in time (dG- and cGP-methods) and continuous methods in space. We are especially interested in proving convergence orders with respect to the polynomial orders used for general classes of problems.

The discontinuous time-integration was already used in the two publications given below. The continuous approach yields unexpected problems that we want to illuminate too.

References

[1] S. Franz, S. Trostorff, and M. Waurick, Numerical methods for changing type systems, *IMA Journal of Numerical Analysis* (2018), https://doi.org/10.1093/imanum/dry007

[2] S. Franz and M. Waurick, Resolvent estimates and numerical implementation for the homogenisation of one-dimensional periodic mixed type problems, *ZAMM* (2018), https://doi.org/10.1002/zamm.201700329

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