ON THE WELL-POSEDNESS OF GALBRUN'S EQUATION

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ABSTRACT

The linearized Euler's equations constitute a standard model for sound propagation in moving fluids. Relying on a recently presented framework for Friedrichs' systems [1, 2], we establish existence, uniqueness, and continuous dependence on data of mild solutions to an initial-boundary-value problem for linearized Euler's equations. It is less known that linearized Euler's equations can be reduced to a vector wave equation in the Lagrangian displacement w. For homentropic background flows, the reduction attributed to H. Galbrun [3] reads

$$\rho_0 D_0\, \delta u(w) + \nabla(\rho_0 c_0\, \delta\hat{\rho}(w)) + \rho_0(\delta u(w)\cdot\nabla)u_0 - c_0(\nabla\rho_0)\,\delta\hat{\rho}(w) = \rho_0\,\delta\varphi \text{ in } \Omega \text{ for } t > 0, \quad (1)$$

where u_0, ρ_0, and c_0 denote the background fluid velocity, density, and speed of sound, respectively; $\delta\varphi$ denotes a source term; $D_0 = \partial_t + u_0\cdot\nabla$ the material derivative with respect to the background flow; $\delta u(w) = D_0 w - (w\cdot\nabla)u_0$ the acoustic velocity, and $\delta\hat{\rho}(w) = -c_0\rho_0^{-1}\nabla\cdot(\rho_0 w)$ the (scaled) acoustic density. We show that (sufficiently regular) solutions to an initial-boundary-value problem for Galbrun's equation (1), satisfy an energy estimate in the norm $\tau_0^{-2}\|\cdot\|^2 + \|\,\delta u(\cdot)\|^2 + \|\,\delta\hat{\rho}(\cdot)\|^2$, where $\tau_0 > 0$ has been introduced to homogenize the dimensions and where $\|\cdot\|$ denotes the $L^2(\Omega)$-norm. Moreover, we illustrate a procedure that generates solutions to Galbrun's equation (1) from solutions to linearized Euler's equations. If δu and $\delta\hat{\rho}$ solve an initial-boundary-value problem for linearized Euler's equations, we define w as the solution to the equation $D_0 w - (w\cdot\nabla)u_0 = \delta u$, supplemented with suitable initial and boundary conditions. If the so-called no resonance assumption, which is the key ingredient in the derivation of Galbrun's equation from linearized Euler's equations, is satisfied, and w is sufficiently regular, then w solves Galbrun's equation (1). In the special case that the background flow is everywhere tangential to $\partial\Omega$, the no resonance assumption can be enforced by selecting appropriate initial data for w.

REFERENCES

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