MATHEMATICAL ELASTICITY – WHEN CALCULUS OF VARIATIONS MEETS MECHANICS

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ABSTRACT

Modern approaches to elasticity are based on the assumption that the first Piola-Kirchhoff stress tensor possesses a potential called stored energy density, W, which depends on the deformation gradient. Such materials are then called hyperelastic. If we additionally assume that external forces applied on a body are conservative, equilibrium equations of elasticity are formally Euler-Lagrange equations for minimizers of the elastic-energy functional. Existence of minimizers can be ensured if W is polyconvex, for instance. Polyconvexity also allows for physically realistic behavior of W, i.e., orientation-preservation of deformations and that $W(F) \to +\infty$ if $\det F \to 0$. On the other hand, it is far not a necessary condition to state a well-posed minimization problem. If we give up some important physical assumptions, we can work with (Morrey) quasiconvexity of W which is then a necessary and sufficient condition for weak lower semicontinuity of the energy functional. We will indicate difficulties which prevent us from extending quasiconvexity to physically acceptable energies.

Moreover, many materials cannot obey polyconvex or quasiconvex stored energy density. A prominent example are e.g. shape-memory alloys. A possible solution, often found in literature, is to assume that the stored energy density depends also on the second deformation gradient and is convex in it. We show the existence of minimizers under weaker assumptions, namely, we make the energy density depend on gradients of nonlinear minors of the deformation gradients. Additionally, we outline some interesting properties of minimizers and a few applications to modeling of shape memory materials and plasticity.

This talk is based on a joint work with B. Benešová and A. Schlömerkemper (both from Würzburg).

REFERENCES

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