FUNCTIONAL-TYPE A POSTERIORI ERROR ESTIMATES FOR BEM – A 2D-LAPLACIAN TOY-PROJECT

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ABSTRACT

In order to guarantee *efficiency* and *reliability* of a numerical solution, a posteriori error analysis is an important task when solving PDEs. In the case of BEM, many existing estimators (e.g. [5]) refer to error functionals with respect to the density function[1] ϕ_h on the boundary. For industrial purposes, one is also interested in the energy error with regard to the *global* reconstruction u_h on Ω. In this context, I present recent ideas from Prof. Dr. Sergey Repin which are subject to my master thesis. First implementation results of a majorant and two minorants within a 2D lowest-order BEM example[2] on a square will be displayed.

The main idea comes from a variational inequality[3] and the fact that BEM-solutions solve the equation exactly inside Ω. Therefore, special instances for majorants can be generated by finding "good" extensions of the boundary error function $e := g - \gamma_0 u_h$ to Ω. The minorants result from functional analysis arguments. Further, since e has zero mean boundary trace the sources [1] and [2] apply.

REFERENCES

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[3] S. Repin. A posteriori estimates for partial differential equations. Walter de Gruyter, Berlin, 2008.

[4] S. Repin and S. Kurz. Basic Introduction into the Boundary Element Method and Related Functional Error Estimates, 27.10.2017.

[5] Simple a posteriori error estimators for the h-version of the boundary element method. D. Praetorius and S. Ferraz-Leite, 2008.

[1] obtained by solving a boundary integral equation, e.g. Galerkin procedure.
[2] homogeneous Dirichlet-Laplacian
[3] e.g. in our example solutions are minimizers of the Dirichlet-Integral