EVOLUTION AND OPTIMAL CONTROL OF RATE-INDEPENDENT SYSTEMS VIA VANISHING VISCOSITY

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ABSTRACT

In order to describe the state of an object that is subject to an external loading ℓ during an observation period $[0,T]$, we want to find a curve $z:[0,T]\to\mathcal{Z}$ solving the following evolution inclusion:

$$0\in\partial\mathcal{R}(\dot{z}(t))+D_z\mathcal{I}(\ell(t),z(t)) \text{ for almost all } t\in[0,T],\quad z(0)=z_0.$$

Here, $\mathcal{R}$ is the dissipation potential, $\partial\mathcal{R}$ denotes its convex subdifferential, $\mathcal{I}$ is the potential energy, and z_0 in the Banach space $\mathcal{Z}$ a given initial state. The dissipation potential measures the loss of energy due to interior friction that occurs as the object changes its state. In some applications (dry friction, plasticity, fracture), the dissipative force does not depend on the velocity of the process. These processes are called rate-independent and have to be described by a positively 1-homogeneous dissipation potential which is often modelized by a norm. In our case, we assume the existence of two more Banach spaces such that $\mathcal{Z}\overset{\text{compact}}{\hookrightarrow}\mathcal{V}\hookrightarrow\mathcal{X}$ (think of, e.g., $H_0^1(\Omega)\hookrightarrow L^2(\Omega)\hookrightarrow L^1(\Omega)$) and set $\mathcal{R}(z):=\|z\|_{\mathcal{X}}$. However, since $\mathcal{I}$ is non-convex in z, we cannot expect continuous solutions, even if ℓ is smooth. Therefore, we approximate the rate-independent system by a sequence of viscous systems with vanishing viscosity $\varepsilon>0$. More precisley, we replace $\mathcal{R}$ by $\mathcal{R}_\varepsilon(z):=\|z\|_{\mathcal{X}}+\frac{\varepsilon}{2}\|z\|_{\mathcal{V}}^2$ and show existence of absolutely continuous solutions z_ε to the resulting systems via a time discretization scheme. Then, we reparameterize z_ε in such a way that we can prove the existence of a limiting solution for $\varepsilon\to0$ in the sense of parameterized BV solutions. The aim is to solve an optimal control problem of the rate-independent system in the parameterized picture, where ℓ is the control variable.

REFERENCES

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