HOMOGENISATION OF MAXWELL'S EQUATIONS REVISITED

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ABSTRACT

In this talk, I will introduce the homogenisation problem associated to the full time-dependent Maxwell equations. Highlighting earlier results and approaches to periodic homogenisation problems, I will proceed to discuss a certain non-periodic setting: Let $\Omega \subseteq \mathbb{R}^3$ open. A sequence $(a_n)_n$ in $L^\infty(\Omega)^{3\times 3}$ is said to be G-convergent to $a \in L^\infty(\Omega)^{3\times 3}$, if $a_n = a_n^*$, $a_n(x) \geq \alpha 1_{3\times 3}$ for some $\alpha > 0$ and a.e. $x \in \Omega$ and for all $f \in H^{-1}(\Omega)$ and $u_n \in H_0^1(\Omega)$ the equations

$$-\operatorname{div} a_n \operatorname{grad} u_n = f \quad (n \in \mathbb{N})$$

imply $u_n \to u$ weakly in $H_0^1(\Omega)$, where $u \in H_0^1(\Omega)$ satisfies

$$-\operatorname{div} a \operatorname{grad} u = f.$$

The main result will be the following: Let $\Omega \subseteq \mathbb{R}^3$ open, bounded with weak Lipschitz boundary. Let $(\varepsilon_n)_n$, $(\mu_n)_n$ be G-convergent sequences in $L^\infty(\Omega)^{3\times 3}$ with limits ε and μ. Then for all $(f, g) \in C_c^1(\mathbb{R}; L^2(\Omega)^6)$ and (E_n, H_n) being in certain (non-standard) Sobolev spaces ensuring existence and uniqueness of the Maxwell problem with E_n satisfying the electric boundary condition with the property

$$\begin{aligned} \partial_t \varepsilon_n E_n - \operatorname{curl} H_n &= f \\ \partial_t \mu_n H_n + \operatorname{curl} E_n &= g, \end{aligned}$$

we have that E_n and H_n converge weakly in $L^2_{\mathrm{loc}}(\mathbb{R} \times \Omega)$ to (E, H) with

$$\begin{aligned} \partial_t \varepsilon E - \operatorname{curl} H &= f \\ \partial_t \mu H + \operatorname{curl} E &= g. \end{aligned}$$

The results are related to findings in [1, 2], in particular to [1, Example 7.5].

REFERENCES

[1] Waurick, M. Nonlocal H-convergence, arXiv:1804.02026, 2018.

[2] Waurick, M. On the homogenization of partial integro-differential-algebraic equations Operators and Matrices, 10(2): 247-283, 2016