A time dependent Stokes interface problem: well-posedness and space-time FEM discretization

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Overview

1. The Problem
   - Strong Formulation
   - Weak Formulation
   - Well-posedness
   - Regularity and Pressure
2. Discretization
   - in Space
   - in Space-Time
3. Numerical Results

The Problem

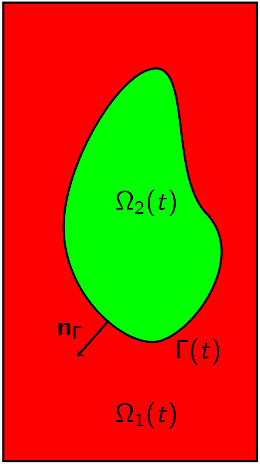
Strong formulation

$$\begin{cases} \rho_i(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}) = \operatorname{div}\boldsymbol{\sigma}_i + \mathbf{g} \\ \operatorname{div}\mathbf{u} = 0 \end{cases} \text{ in } \Omega_i(t),$$

$$\mathbf{u}(0) = \mathbf{u}_0 \text{ in } \Omega_i(0)$$

$$\begin{aligned} [\boldsymbol{\sigma}\mathbf{n}_\Gamma] &= -\tau\kappa\mathbf{n}_\Gamma \quad \text{on } \Gamma(t), \\ [\mathbf{u}] &= 0 \quad \text{on } \Gamma(t), \\ V_\Gamma &= \mathbf{u}\cdot\mathbf{n}_\Gamma \quad \text{on } \Gamma(t) \end{aligned}$$

with the Newtonian stress tensor $\boldsymbol{\sigma}_i = -p\mathbf{I} + \mu_i(\nabla\mathbf{u} + (\nabla\mathbf{u})^T)$ and a normal velocity of the interface given by V_Γ.



The Problem

Strong formulation

$$\begin{cases} \rho_i\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) - \mu_i \Delta \mathbf{u} + \nabla p = \mathbf{g} \\ \operatorname{div} \mathbf{u} = 0 \end{cases} \text{ in } \Omega_i(t),$$

$$\mathbf{u}(0) = \mathbf{u}_0 \ \text{ in } \Omega_i(0)$$

$$[p\mathbf{n}_\Gamma + \left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right)\mathbf{n}_\Gamma] = -\tau\kappa\mathbf{n}_\Gamma \quad \text{on} \quad \Gamma(t),$$

$$[\mathbf{u}] = 0 \quad \text{on} \quad \Gamma(t),$$

$$V_\Gamma = \mathbf{u}\cdot\mathbf{n}_\Gamma \quad \text{on} \quad \Gamma(t)$$

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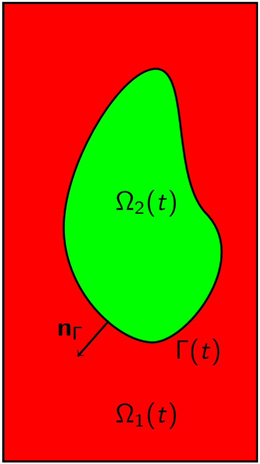
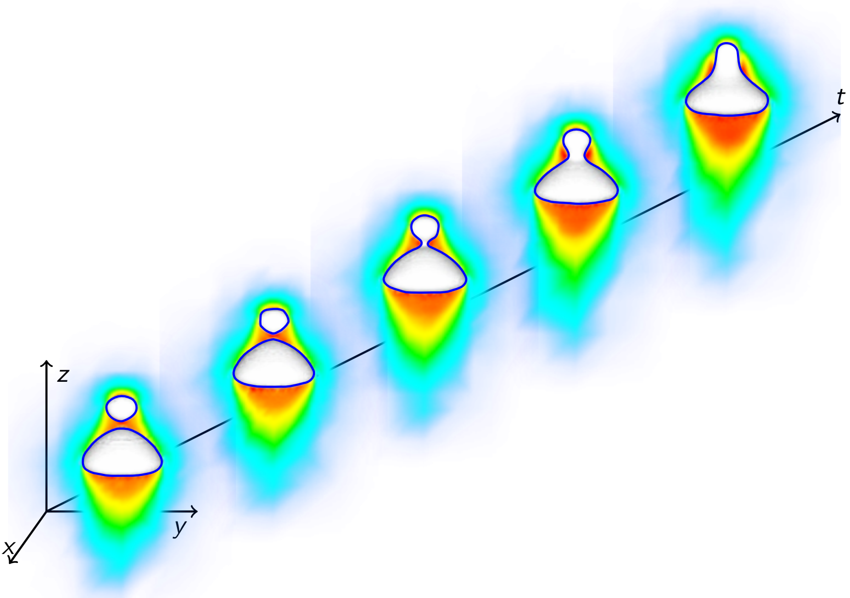


A Typical Problem



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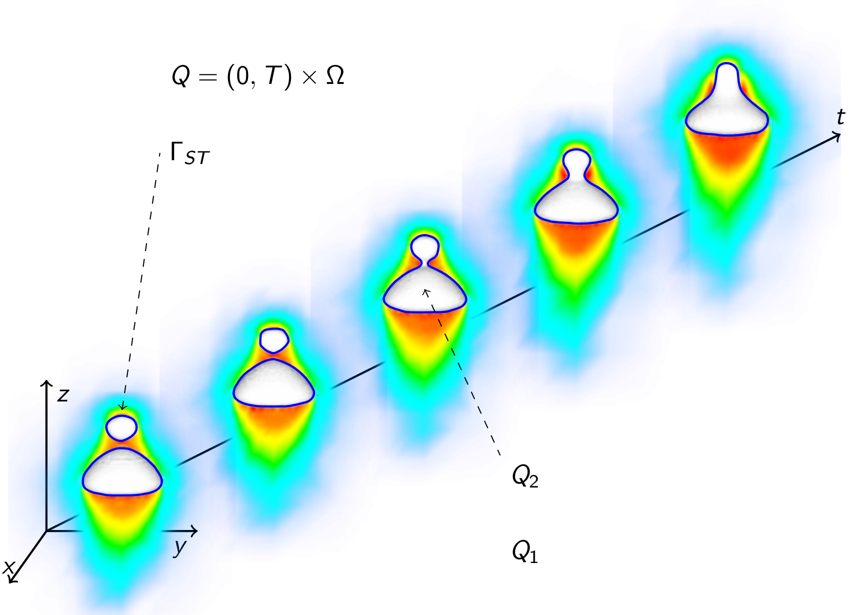
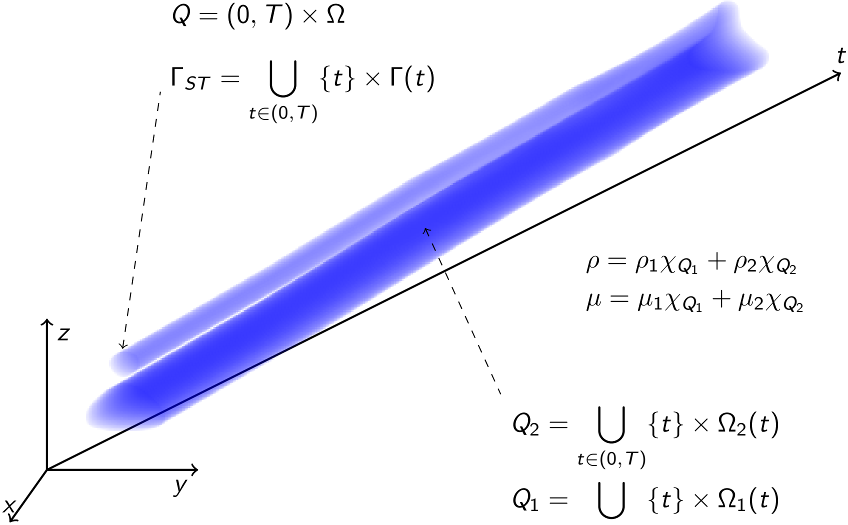


Space-Time Domains

$$Q = (0, T) \times \Omega$$

$$\Gamma_{ST} = \bigcup_{t \in (0,T)} \{t\} \times \Gamma(t)$$



$$\rho = \rho_1 \chi_{Q_1} + \rho_2 \chi_{Q_2}$$

$$\mu = \mu_1 \chi_{Q_1} + \mu_2 \chi_{Q_2}$$

$$Q_2 = \bigcup_{t \in (0,T)} \{t\} \times \Omega_2(t)$$

$$Q_1 = \bigcup_{t \in (0,T)} \{t\} \times \Omega_1(t)$$

Weak Formulation

Space-Time formulation for velocity and pressure

Find **u** such that

$$\rho(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u}) - \Delta_\mu \mathbf{u} + \nabla p = \mathbf{f} \quad \text{in } L^2(0,T;H^{-1}(\Omega))^d,$$

$$\operatorname{div} \mathbf{u} = 0 \quad \text{in } L^2(Q),$$

$$\mathbf{u}(0) = \mathbf{u}_0 \quad \text{in } L^2(\Omega)^d,$$

where

$$\langle \Delta_\mu \mathbf{u}, \mathbf{v} \rangle_\Omega := \langle \mu\big(\nabla \mathbf{u} + (\nabla \mathbf{u})^T\big), \nabla \mathbf{v} + (\nabla \mathbf{v})^T \rangle_\Omega.$$

Physical coefficients are constant in Q_i

$$\frac{\partial \rho}{\partial t} + (\mathbf{u} \cdot \nabla)\rho = 0, \frac{\partial \mu}{\partial t} + (\mathbf{u} \cdot \nabla)\mu = 0 \quad \text{in } H^{-1}(Q)$$

$$\rho(0) = \rho_0, \mu(0) = \mu_0.$$

Weak Formulation

Linearized Space-Time formulation for velocity and pressure

Find $\mathbf{u}$ such that

$$\rho\underbrace{(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{w}\cdot\nabla)\mathbf{u})}_{=\dot{\mathbf{u}}} - \Delta_\mu \mathbf{u} + \nabla p = \mathbf{f} \quad \text{in } L^2(0,T;H^{-1}(\Omega))^d,$$

$$\operatorname{div}\mathbf{u} = 0 \quad \text{in } L^2(Q),$$

$$\mathbf{u}(0) = \mathbf{u}_0 \quad \text{in } L^2(\Omega)^d,$$

where $\mathbf{w} \in L^\infty(Q)^d$, $\operatorname{div}\mathbf{w} = 0$.

Physical coefficients are constant in Q_i

$$\dot{\rho} = \frac{\partial \rho}{\partial t} + (\mathbf{w}\cdot\nabla)\rho = 0, \dot{\mu} = \frac{\partial \mu}{\partial t} + (\mathbf{w}\cdot\nabla)\mu = 0 \quad \text{in } H^{-1}(Q)$$

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Divergence Free Formulation

Spaces of Divergence free functions

Standard spaces:

$$\mathcal{V} := \{\, \mathbf{v} \in H_0^1(\Omega)^d \mid \operatorname{div} \mathbf{v} = 0 \,\}, \quad X := L^2(I; \mathcal{V}).$$

Solution spaces of Bochner-type:

$$\begin{aligned} W &:= \{\mathbf{v} \in X | \rho\dot{\mathbf{v}} \in X'\}, \quad \|\mathbf{v}\|_W^2 = \|\mathbf{v}\|_X^2 + \|\rho\dot{\mathbf{v}}\|_{X'}^2, \\ V &:= \overline{H^1(I; \mathcal{V})}^{\|\cdot\|_W} \subset W. \end{aligned}$$

Note: W and V do not have the standard Tensor-product structure if ρ is time dependent. Is $W = V$?

Weak divergence free formulation

Find $\mathbf{u} \in V$ such that

$$\int_0^T \langle \rho\dot{\mathbf{u}}, \mathbf{v}\rangle_\Omega + \int_0^T a(t; \mathbf{u}(t), \mathbf{v}(t)) = \int_0^T \langle \mathbf{f}, \mathbf{v}\rangle_\Omega, \quad \text{for all } \mathbf{v} \in X,$$

where $a(t; \mathbf{u}(t); \mathbf{v}(t)) = \langle \Delta_{\mu(t)} \mathbf{u}(t), \mathbf{v}(t)\rangle_\Omega$.

Well-posedness

Conditions on the bilinear form a

A bilinear form a is called uniformly elliptic and uniformly continuous if (resp.)

$$\exists\,\gamma > 0: \quad a(t;\mathbf{v},\mathbf{v}) \geq \gamma \|\mathbf{v}\|^2_{H^1(\Omega)^d} \quad \text{for all} \;\; \mathbf{v} \in \mathcal{V},\; t \in I,$$
$$\exists\,\Gamma > 0: \quad a(t;\mathbf{u},\mathbf{v}) \leq \Gamma \|\mathbf{u}\|_{H^1(\Omega)^d} \|\mathbf{v}\|_{H^1(\Omega)^d} \quad \text{for all} \;\; \mathbf{u},\mathbf{v} \in \mathcal{V},\; t \in I.$$

Main Result

Let $a(t;\cdot,\cdot)$ be a uniformly elliptic, uniformly continuous, bilinear form on $\mathcal{V} \times \mathcal{V}$.
For every $\mathbf{f} \in X'$ there exists a unique $\mathbf{u} \in V$ with $u(0) = 0$ such that

$$b(\mathbf{u},\mathbf{v}) := \langle \rho\dot{\mathbf{u}},\mathbf{v}\rangle_Q + \int_0^T a(t;\mathbf{u}(t),\mathbf{v}(t))\,dt = \langle \mathbf{f},\mathbf{v}\rangle_Q \quad \text{for all} \;\; \mathbf{v} \in X. \qquad \text{(P)}$$

Furthermore, the map $\mathbf{f} \mapsto \mathbf{u}$ is continuous.

Proof of Well-posedness

Problem

For every $\mathbf{f} \in X'$ there exists a unique $\mathbf{u} \in V$ with $\mathbf{u}(0) = 0$ such that

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Is this a standard problem?

The well-posedness of similar problems is typically proven by using either a Galerkin argument or by applying Banach-Nečas-Babuška Theory. Here this is not possible is a direct way. This is due to the fact that V and W do not have a tensor product structure. A combination of these two needs to be used.
BNB Theorem: necessary conditions on the continuous bilinear form b:

$$\inf_{0 \neq \mathbf{u} \in V,\ \mathbf{u}(0)=0} \ \sup_{0 \neq \mathbf{v} \in X} \frac{b(\mathbf{u}, \mathbf{v})}{\|\mathbf{u}\|_V \|\mathbf{v}\|_X} \geq c_s, \tag{BNB 1}$$

If $b(\mathbf{u}, \mathbf{v}) = 0$ holds for all $\mathbf{u} \in V$, $\mathbf{u}(0) = 0$, then $\mathbf{v} = 0$. (BNB 2)

Proof of Well-posedness

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Structure of the Proof

The proof of the main result is obtained in several steps.

(1) Well-posedness of (P) is first proven for a symmetric, time-*independent* bilinear form a. This is done using a Galerkin argument (use $\mathbf{v} = \frac{\partial \mathbf{u}}{\partial t}$).

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(2) Using the BNB Theorem, well-posedness of (P) can be proven for a symmetric, time-*dependent* bilinear form a. (1) is used to verify (BNB 2).

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(2) Using the BNB Theorem, well-posedness of (P) can be proven for a symmetric, time-*dependent* bilinear form a. (1) is used to verify (BNB 2).

(3) Inductively adding small (in terms of γ) anti-symmetric perturbations to the bilinear form a from (2), on can verify that (P) holds for any time-*dependent* bilinear form.

Regularity and Pressure

Regularity

If $\mathbf{f} \in (\overline{X}^{L^2})' \cap L^2(I; H^{-1}(\Omega)^d) \supset L^2(Q)^d$ and $\mathbf{w} \in C^2(Q)^d$, then the solution $\mathbf{u}$ of

$$\langle \rho\dot{\mathbf{u}}, \mathbf{v} \rangle_Q + \int_0^T a(t; \mathbf{u}(t), \mathbf{v}(t))\, dt = \langle \mathbf{f}, \mathbf{v} \rangle_Q \quad \text{for all} \quad \mathbf{v} \in X, \qquad \text{(P)}$$

has the additional regularity $\mathbf{u} \in H^1(I; L^2(\Omega)^d)$.

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Pressure

If the solution $\mathbf{u}$ of (P) satisfies $\mathbf{u} \in H^1(I; L^2(\Omega)^d)$, then there exists a unique p such that $(\mathbf{u}, p) \in H^1(Q) \times L^2(I; L^2(\Omega)/\mathbb{R})$ is the unique solution of

$$\langle \rho \frac{\partial \mathbf{u}}{\partial t}, \mathbf{v} \rangle_Q + \int_0^T \tilde{a}(t; \mathbf{u}(t), \mathbf{v}(t))\, dt + \langle \nabla p, \mathbf{v} \rangle_Q = \langle \mathbf{f}, \mathbf{v} \rangle_Q \text{ for all } \mathbf{v} \in L^2(I; H_0^1(\Omega)^d),$$

$$\langle \operatorname{div} \mathbf{u}, q \rangle_Q = 0 \quad \text{for all} \quad q \in L^2(I; L^2(\Omega)/\mathbb{R}))$$

where $\tilde{a}(t; \mathbf{u}(t), \mathbf{v}(t)) = \langle \rho(t) \mathbf{w}(t) \cdot \nabla \mathbf{u}(t), \mathbf{v}(t) \rangle_\Omega + a(t; \mathbf{u}(t), \mathbf{v}(t))$.

Discretization

Continuous Variational Formulation

Find $(\mathbf{u}, p) \in H^1(Q) \times L^2(I; L^2(\Omega)/\mathbb{R})$ (with $\mathbf{u}(0) = 0$) such that

$$\langle \rho \frac{\partial \mathbf{u}}{\partial t}, \mathbf{v} \rangle_Q + \int_0^T \tilde{a}(t; \mathbf{u}(t), \mathbf{v}(t))\, dt + \langle \nabla p, \mathbf{v} \rangle_Q = \langle \mathbf{f}, \mathbf{v} \rangle_Q \text{ for all } \mathbf{v} \in L^2(I; H_0^1(\Omega)^d),$$

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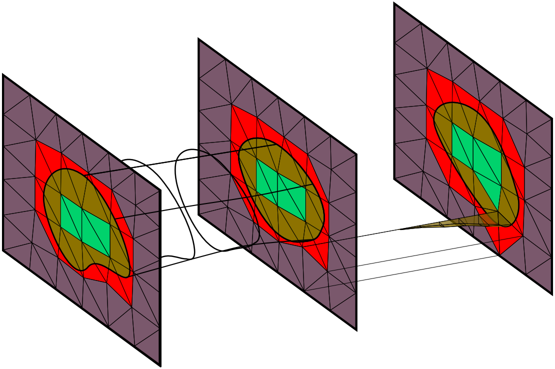
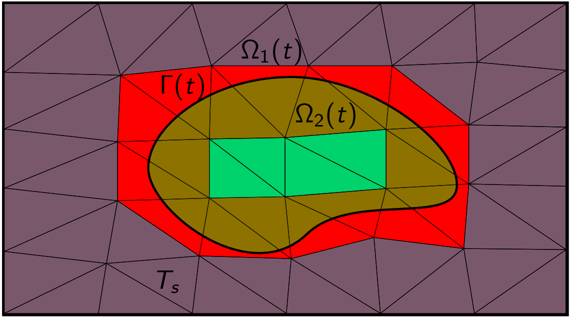


Discretization in Space

Finite Elements in Space - XFEM

$$U_h = \{\mathbf{v} \in C_0(\Omega)^d | \forall T_s \in \mathcal{T}_h : \mathbf{v}|_{T_s} \in \mathcal{P}_{k+1}(T_s)^d\}$$
$$P_h(t) = \{q \in C(\Omega_1(t) \cup \Omega_2(t))/\mathbb{R} | \forall T_s \in \mathcal{T}_h, \forall i = 1,2 : q|_{T_s \cap \Omega_i} \in \mathcal{P}_k(T_s \cap \Omega_i)\}$$



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Semi-Discrete Variational Formulation

Find $(\mathbf{u}_h, p_h) \in H^1(I; U_h) \times L^2(I; L^2(\Omega)/\mathbb{R})$ (with $\mathbf{u}(0) = 0$) with $\forall t \in I : p_h(t) \in P_h(t)$ such that

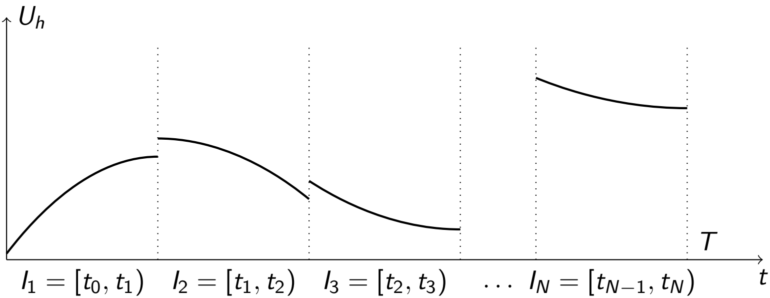
$$\langle \rho \frac{\partial \mathbf{u}_h}{\partial t}, \mathbf{v} \rangle_Q + \int_0^T \tilde{a}(t; \mathbf{u}_h(t), \mathbf{v}(t))\, dt + \langle \nabla p_h, \mathbf{v} \rangle_Q = \langle \mathbf{f}, \mathbf{v} \rangle_Q \quad \forall \mathbf{v} \in L^2(I; U_h),$$
$$\langle \operatorname{div} \mathbf{u}_h, q \rangle_Q = 0 \quad \forall q \in L^2(Q), q(t) \in P_h(t).$$

Discretization in Space-Time

Finite Elements - ST-XFEM

$$U_{N,h} = \left\{ \mathbf{v} : [0, T) \to C_0(\Omega)^d \middle| \forall T_s \in \mathcal{T}_h, 1 \leq n \leq N : \mathbf{v}|_{I_n \times T_s} \in \mathcal{P}_\ell(I_n, \mathcal{P}_{k+1}(T_s)^d) \right\}$$

$$P_{N,h} = \left\{ q : [0, T) \to C(Q_1 \cup Q_2) \middle| \forall T_s \in \mathcal{T}_h, 1 \leq n \leq N, \forall i = 1, 2 : q|_{(I_n \times T_s) \cap Q_i} \in \mathcal{P}_\ell(I_n, \mathcal{P}_k(T_s))|_{Q_i} \right\}$$



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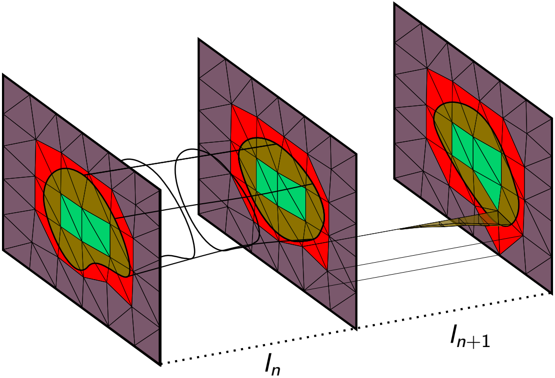

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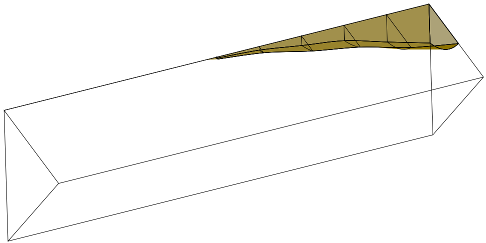

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Discrete Variational Formulation

Find $(\mathbf{u}, p) \in U_{N,h} \times P_{N,h}$ (with $\mathbf{u}(0) = 0$) such that

$$\langle \rho D_t \mathbf{u}_h, \mathbf{v}\rangle_Q + \int_0^T \tilde{a}(t; \mathbf{u}_h(t), \mathbf{v}(t))\, dt + \langle \nabla p_h, \mathbf{v}\rangle_Q = \langle \mathbf{f}, \mathbf{v}\rangle_Q \quad \forall \mathbf{v} \in U_{N,h},$$

$$\langle \operatorname{div} \mathbf{u}_h, q\rangle_Q = 0 \quad \forall q \in P_{N,h}.$$

Discretization in Space-Time

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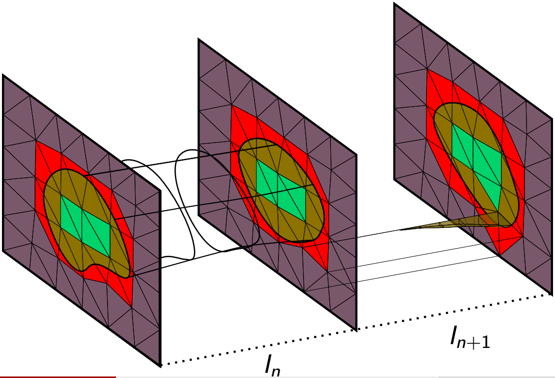


Discretization in Space-Time

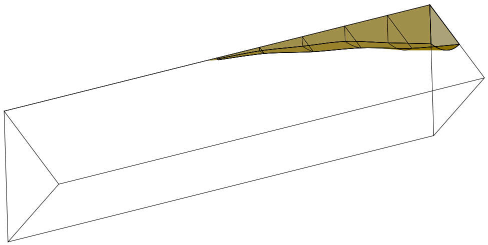
Discrete Variational Formulation

Find $(\mathbf{u}, p) \in U_{N,h} \times P_{N,h}$ (with $\mathbf{u}(0) = 0$) such that

$$\langle \rho D_t \mathbf{u}_h, \mathbf{v} \rangle_Q + \int_Q \mu(t,x) \nabla_S \mathbf{u} : \nabla_S \mathbf{v} \, dxdt + \langle \nabla p_h, \mathbf{v} \rangle_Q = \langle \mathbf{f}, \mathbf{v} \rangle_Q \quad \forall \mathbf{v} \in U_{N,h},$$

$$\langle \mathrm{div}\, \mathbf{u}_h, q \rangle_Q = 0 \quad \forall q \in P_{N,h}.$$

where $\nabla_S \mathbf{u} = \nabla \mathbf{u} + (\nabla \mathbf{u})^T$.

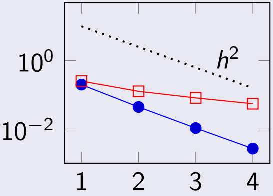


Numerical Results - $\mathcal{P}_2$-$\mathcal{P}_1^X$ in Space/$\mathcal{P}_1$ in Time

Convergence of velocity

$S \backslash T$	16	32	64	128
4	0.19753	0.19552	0.19510	0.19507
8	0.04699	0.04390	0.04332	0.04318
16	0.01736	0.01154	0.01064	0.01047
32	0.01326	0.00525	0.00306	0.00267

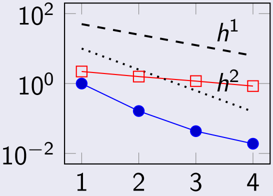
Error in $L^2(I; H^1(\Omega)^3)$-norm.



Convergence of pressure

$S \backslash T$	16	32	64	128
4	0.99726	0.96497	0.97083	0.98616
8	0.17501	0.16322	0.16244	0.16315
16	0.10682	0.04883	0.04331	0.04355
32	0.17825	0.07309	0.02646	0.01895

Error in $L^2(I; L^2(\Omega))$-norm.



Conclusions and Outlook

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- Space-time quadrature for discontinuous coefficients,
- Optimal convergence results for velocity (numerically).

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Thank you!