Principles and Methodologies of Multi-Criterion Problem Solving

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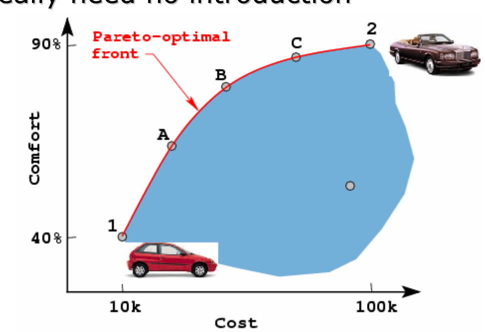
Overview

- Multi-Criterion Optimization
- Evolutionary Multi-Criterion Optimization (EMO)
- Advantages of EMO
- Applications of EMO
  - Decision-making
  - *Innovization*: Innovation through EMO
  - Aiding in other problem-solving tasks
- EMO and Decision Making
- Conclusions

EMO Books (Since 2001)

Multi-Objective Optimization

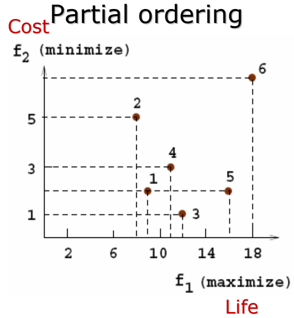
- Really need no introduction



Which Solutions are Optimal?

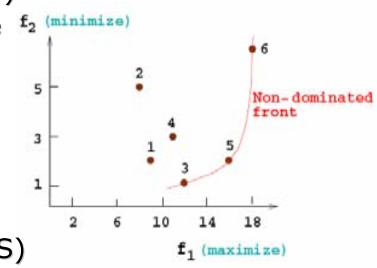
- Relates to the concept of domination
- $x^{(1)}$ dominates $x^{(2)}$, if
  - $x^{(1)}$ is no worse than $x^{(2)}$ in all objectives
  - $x^{(1)}$ is strictly better than $x^{(2)}$ in at least one objective
- Examples:
  - 3 dominates 2
  - 3 does not dominate 5

Partial ordering



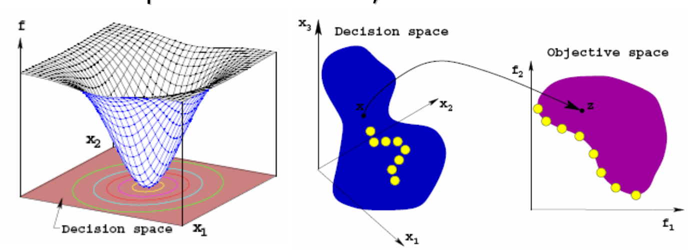
Pareto-Optimal Solutions

- P'=Non-dominated(P)
  - Solutions which are not dominated by any member of the set P
- O(N log N) algorithms exist
- Pareto-Optimal set = Non-dominated(S)
- A number of solutions are optimal

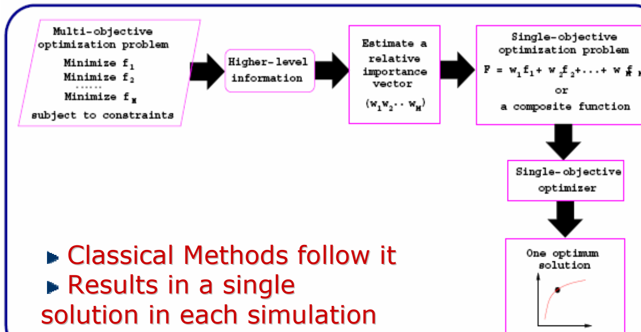


Differences with Single-Objective Optimization

- One optimum versus multiple optima
- Requires search and decision-making
- Two spaces of interest, instead of one



Preference-Based Methods



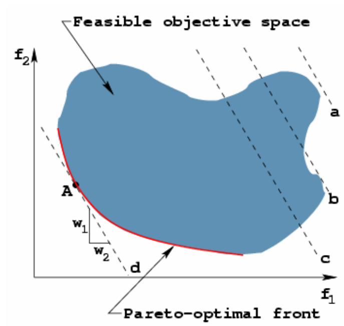
- Classical Methods follow it
- Results in a single solution in each simulation

Classical Approach: Weighted Sum Method

- Construct a weighted sum of objectives and optimize

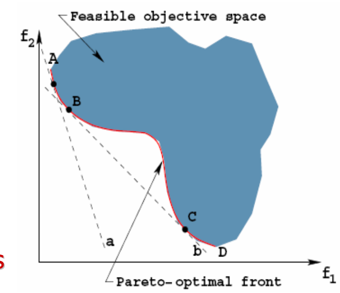
$$F(x) = \sum_{i=1}^{M} w_i f_i(x)$$

- User supplies weight vector *w*



Difficulties with Weighted-Sum Method

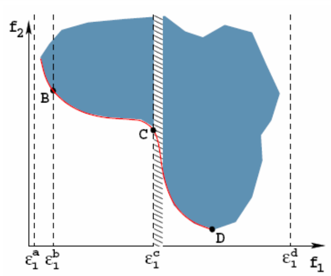
- Need to know **w**
- Non-uniformity in Pareto-optimal solutions
- Inability to find some Pareto-optimal solutions (those in non-convex region)
- However, a solution of this approach is always Pareto-optimal



ε-Constraint Method

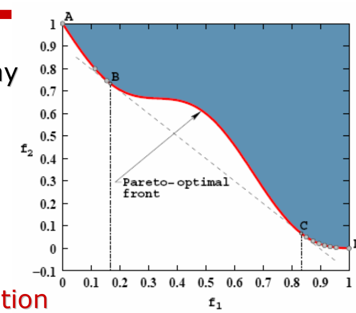
- Constrain all but one objective
- Need to know relevant **ε** vectors
- Non-uniformity in Pareto-optimal solutions
- However, any Pareto-optimal solution can be found with this approach

$$\begin{array}{ll} \text{Minimize} & f_\mu(\mathbf{x}), \\ \text{subject to} & f_m(\mathbf{x}) \le \epsilon_m, \; m \ne \mu; \end{array}$$



Difficulties with Most Classical Approaches

- Need to run a single-objective optimizer many times
- Expect a lot of problem knowledge
- Even then, good distribution is not guaranteed
- Multi-objective optimization as an application of single-objective optimization



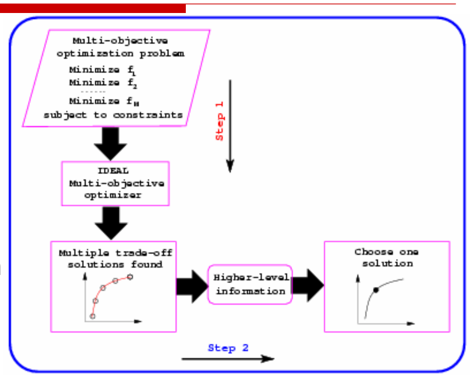
Ideal Multi-Objective Optimization

Step 1 :

Find a set of Pareto-optimal solutions

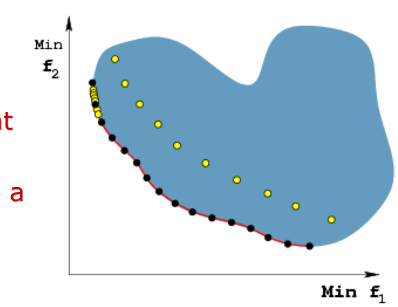
Step 2 :

Choose one from the set



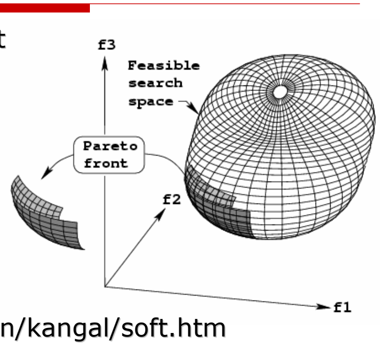
Two Goals in Ideal Multi-Objective Optimization

- Converge to the Pareto-optimal front
- Maintain as diverse a distribution as possible



Elitist Non-dominated Sorting Genetic Algorithm (NSGA-II)

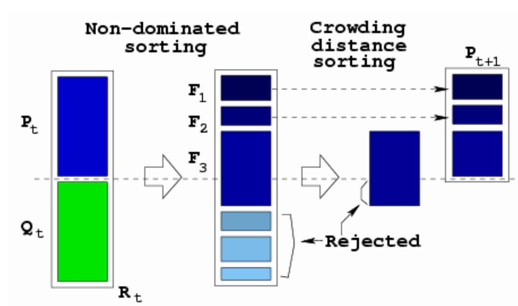
- NSGA-II can extract Pareto-optimal frontier
- And find a well-distributed set of solutions
- Adopted by iSIGHT and ModeFrontier
- Code downloadable http://www.iitk.ac.in/kangal/soft.htm



IEEE TEC paper awarded 'Fast Breaking Paper in Engg. by ISI Web of Sc.

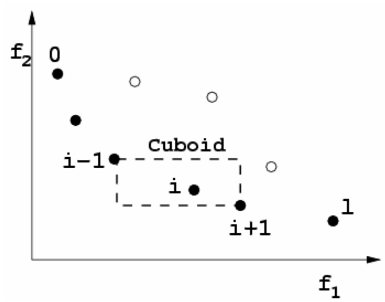
NSGA-II Procedure

- Elites are preserved
- Non-dominated solutions are emphasized



NSGA-II (cont.)

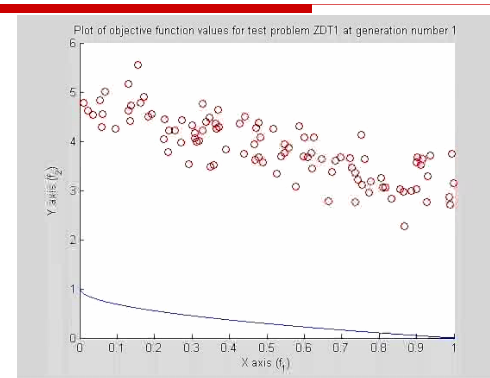
- Diversity is preserved



Overall Complexity
O($N \log^{M-1} N$)

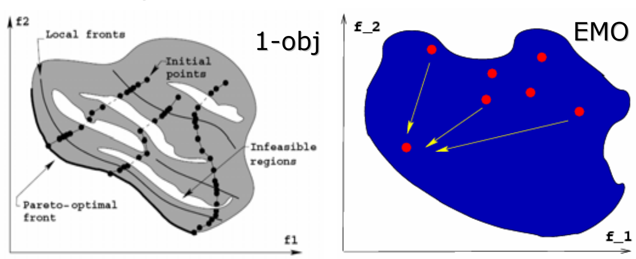
Improve diversity by
- k-mean clustering
- Euclidean distance measure
- Other techniques

Simulation on ZDT1



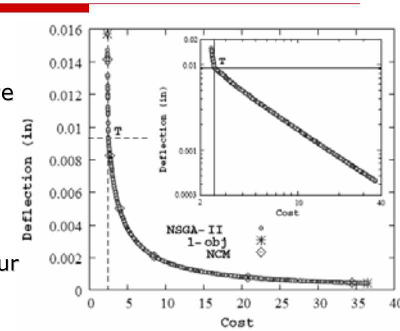
Multiple Single-Objective Optimization Versus EMO

- Repetitive use may be computationally expensive
- EMO is parallel and overall faster and reliable



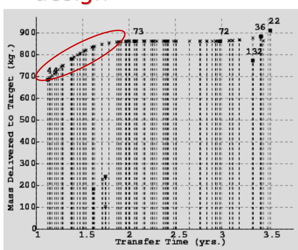
Performing Reliable and Confident EMO Simulations

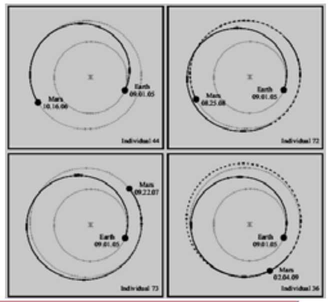
- EMO is a numerical method
- Verify by single-objective optimizations
- Verify by smaller-objective optimizations
- Cluster the frontier
  - Check to see if they are KKT points (see our CEC-2007 paper)



EMO Applications:
1. *Better Decision-Making*

- Identify different trade-off solutions for choosing one (Better and more confident decision-making)
- Inter-planetary trajectory design

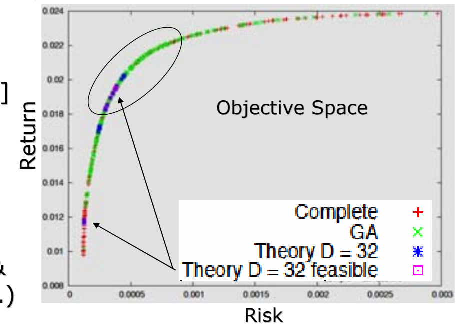




Portfolio Optimization

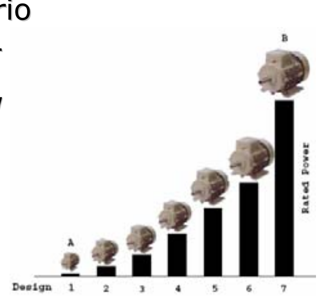
$$\min\{\mathbf{x}^T\Sigma\mathbf{x}\}$$
$$\max\{\mu^T\mathbf{x}\}$$
$$s.t.\quad \mathbf{1}^T\mathbf{x}=1$$
$$x_i = 0 \text{ or } x_i \in [\alpha,\omega] \quad \alpha \geq 0$$
and Card(x)=D

- Minimize risk, maximize return
- Complete: Assuming any x
- GA: x=0 or [α,w] and fixed D=32
- Theory D=32: Complete with D=32
- Theory D=32 feasible: D=32 & feasible (9 solns.)



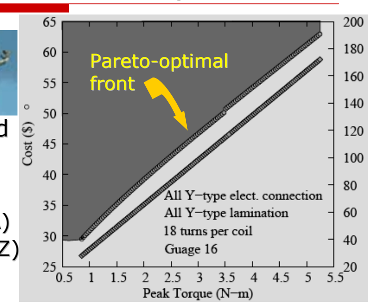
2. *Innovization*:
Discovery of Innovative design principles through optimization

- Understand important design principles in a routine design scenario
- Example: Electric motor design with varying ratings, say 1 to 10 kW
  - Each will vary in size and power
  - Armature size, number of turns etc.
- How do solutions vary?
  - Any common principles!



Brushless DC Permanent Magnet Motor Design for Cost and Peak Torque

- Six variables (all discrete), three constraints
- Non-convex, disconnected P-O front

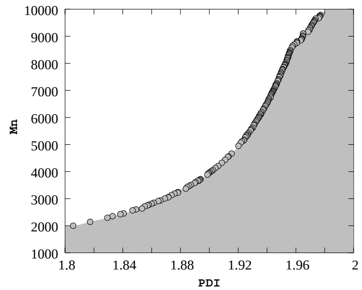


- *Innovizations*:
  - Connection: Y (betn. Y & Δ)
  - Lamination Type: Y (X, Y, Z)
  - 1 out of 16 wire guages
  - 18 turns per coil (10,80)
  - *More peak torque by adding linearly more laminations*

Recipe for design

Epoxy Polymerization

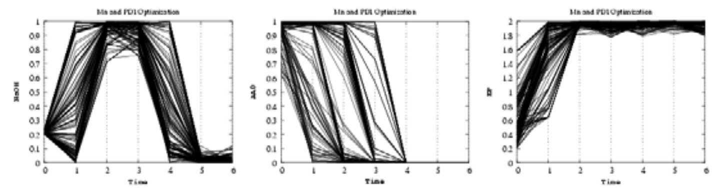
- Three ingredients added hourly
- 54 ODEs solved for a 7-hour simulation
- Maximize chain length (Mn)
- Minimize polydispersity index (PDI)
- Total 3x7 or 21 variables
- (Deb et al., 2004)



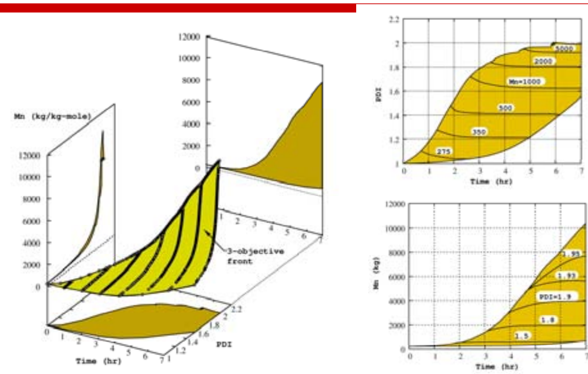
A non-convex frontier

Epoxy Polymerization (cont.)

- Some patterns emerge among obtained solutions
- Chemical significance unveiled



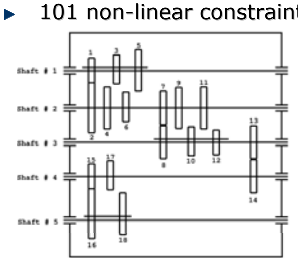
Innovized Principles: *An Optimal Operating Chart*

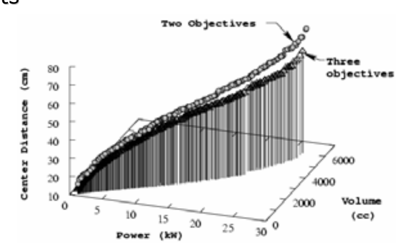


Gear-box Design

- A multi-spindle gear-box design (Deb and Jain, 2003)
- 28 variables (integer, discrete, real-valued)
- 101 non-linear constraints
- Important insights obtained

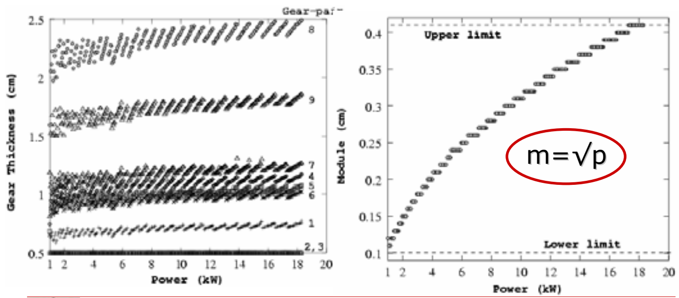
(larger module for more power)





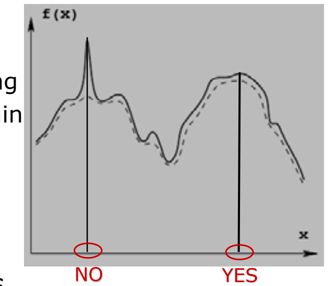
Innovized Principles

- Module varies proportional to square-root of power
- Keep other 27 variables more or less the same



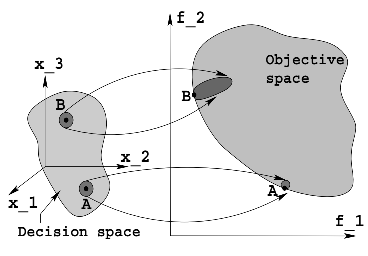
Robust Optimization
Handling uncertainties

- Parameters are uncertain and sensitive to implementation
  - Tolerances in manufacturing
  - Material properties uncertain
  - Loading is uncertain
- Evaluation uncertainty
- Who wants a sensitive optimum solution?
- Single-objective robust EAs exist



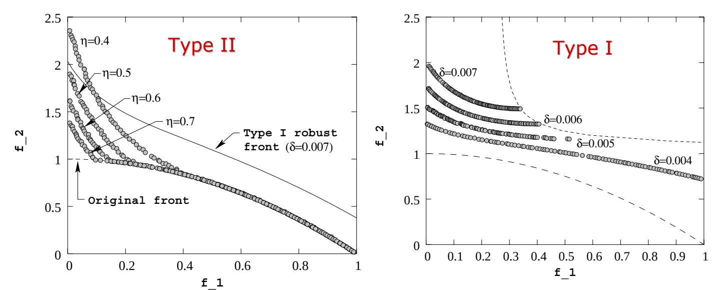
Multi-Objective Robust Solutions

- Not all Pareto-optimal points may be robust
- A is robust, but B is not
- Decision-makers will be interested in knowing robust part of the front



Robust Frontiers for Two Objectives

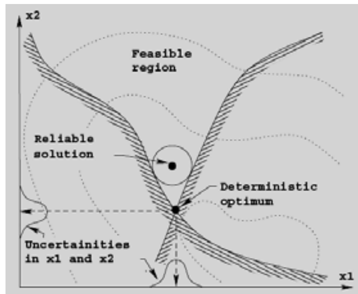
- Identify robust region
- Allows a control on desired robustness



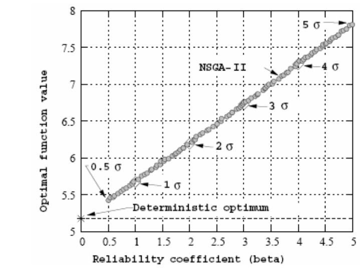
Reliability-Based Optimization:
Making designs safe against failures

- Deterministic optimum is not usually reliable
- Reliable solution is an interior point
- Chance constraints with a given reliability

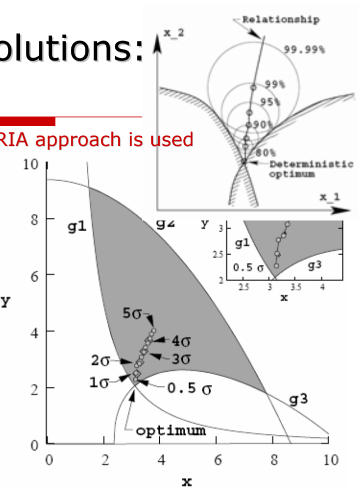
Minimize $\mu_f + k\sigma_f$
Subject to $Pr(g_j(x) \geq 0) \geq \beta_j$
β_j is user-supplied



Multiple Reliability Solutions:
Get a better insight



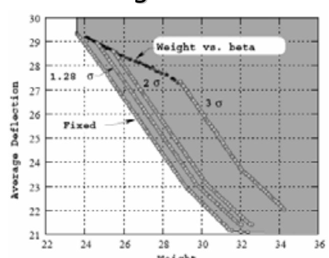
RIA approach is used

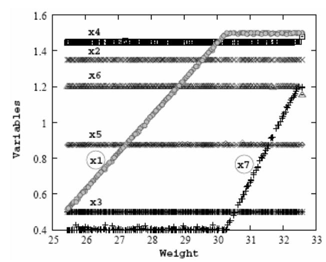


Maximize $x + y$,
Subject to $g_1(x, y) \equiv \frac{1}{20}x^2y - 1 \geq 0$,
$g_2(x, y) \equiv \frac{1}{30}(x + y - 5)^2 + \frac{1}{120}(x - y - 12)^2 - 1 \geq 0$,
$g_3(x, y) \equiv \frac{80}{x^2+8y+5} - 1 \geq 0$,
$0 \leq x, y \leq 10$.

Multi-Objective Reliability-Based Optimization

- Reliable fronts show rate of movement
- What remains unchanged and what gets changed!





Handling Many Objectives

- Identify redundant objectives, if exist
- EMO+PCA in iterations

10-objective DTLZ5 problem

Iter.1

Iter. 1 : PCA-1 (58.83 % variance)	f_7 f_{10}
PCA-2 (28.26 % variance)	f_1
PCA-3 (06.53 % variance)	f_8
PCA-4 (03.27 % variance)	f_8

	f_1	f_7	f_8	f_{10}
f_1	+	-	+	-
f_7	-	+	+	-
f_8	+	+	+	-
f_{10}	-	-	-	+

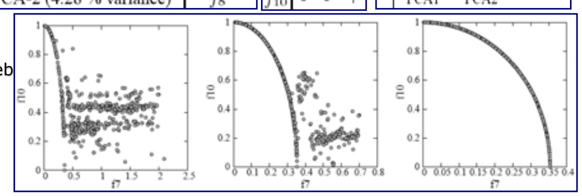
Iter.2

Iter. 2 : PCA-1 (94.58 % variance)	f_7 f_{10}
PCA-2 (4.28 % variance)	f_8

	f_7	f_8	f_{10}
f_7	+	+	-
f_8	+	+	-
f_{10}	-	-	+

	e1:0.9458	e2:0.0428	
f_7	+0.543	-0.275	c7=0.5253
f_8	+0.457	+0.672	c8=0.4610
	PCA1	PCA2	

Saxena and Deb (CEC-2006, EMO-2007, CEC-2007)

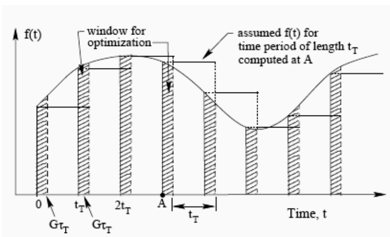


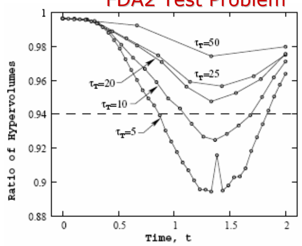
Algorithm Flexibility

Dynamic (On-line) Optimization

- Assume a statis in problem for a time step
- Find minimum time step by a off-line study for a pre-defined performance

FDA2 Test Problem

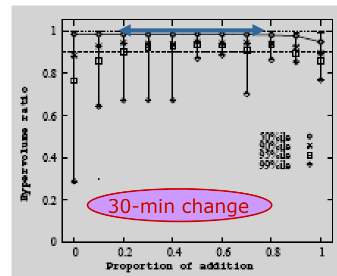


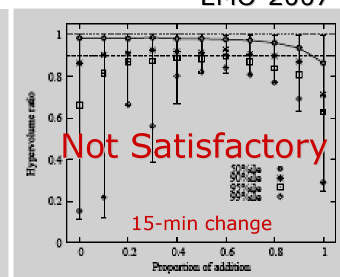


Dynamic Multi-Objective Hydro-Thermal Power Scheduling

- Addition of random or mutated points at changes
- 30-min change found satisfactory

EMO-2007

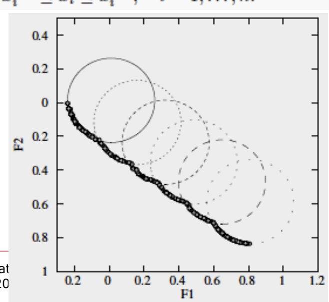




Multi-Objective Bilevel Programming (Deb and Sinha, 2009)

- Upper level solution is feasible only if it is a lower level PO solution
- Often appears in engineering problems to deal with stability, equilibrium etc.
- NSGA-II with a local search

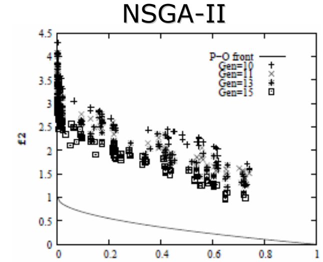
$$\begin{aligned} \min_{(\mathbf{x}_u,\mathbf{x}_l)} \quad & \mathbf{F}(\mathbf{x}) = (F_1(\mathbf{x}),\ldots,F_M(\mathbf{x})), \\ \text{s.t.} \quad & \mathbf{x}_l \in \operatorname{argmin}_{(\mathbf{x}_l)} \left\{ \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ldots,f_m(\mathbf{x})) \middle| \mathbf{g}(\mathbf{x}) \ge \mathbf{0}, \mathbf{h}(\mathbf{x}) = \mathbf{0} \right\}, \\ & \mathbf{G}(\mathbf{x}) \ge \mathbf{0}, \mathbf{H}(\mathbf{x}) = \mathbf{0}, \\ & x_i^{(L)} \le x_i \le x_i^{(U)}, \quad i = 1,\ldots,n. \end{aligned}$$



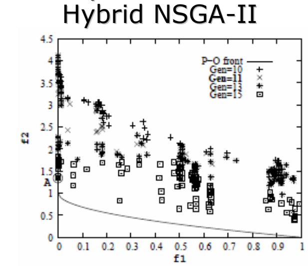
Hybrid EMO (Karthik, Deb & Miettinen, 2008)

- Improve EMO's convergence properties
- Introduce local search based on achievement scalarization function occasionally

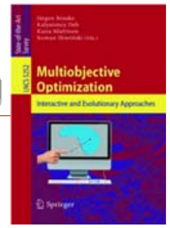
NSGA-II



Hybrid NSGA-II



EMO and Decision-Making



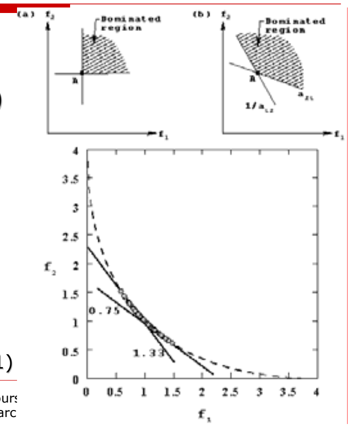
- Need to choose a single solution!
- How to choose one solution (MCDM)
- First EMO, then MCDM
- EMO+MCDM all along
  - Use where multiple, repetitive applications are sought
  - Use where, instead of a point, a trade-off region is sought
  - Use for finding points with specific properties (nadir point, knee point, etc.)
  - Use for robust, reliable or other fronts
- Use EMO for an idea of the front, then decision-making (I-MODE)

MDO Short Course at Laajavuori (13 March 2009) 41

Finding a Partial Pareto Frontier

- Using a DM's preference (not a solution but a region)
- Guided domination principle: Biased niching approach
- Weighted domination approach

(Branke et al., 2001; Deb, 2001)

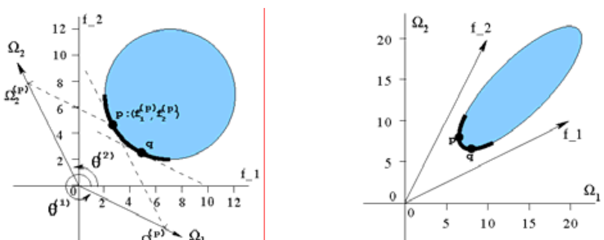


MDO Short Cours (13 Marc

Distributed Computing of Pareto-Optimal Set

Deb, Zope & Jain (EMO-2003)

- Guided domination concept to search different parts of Pareto-optimal region
- Distributed computing of different parts



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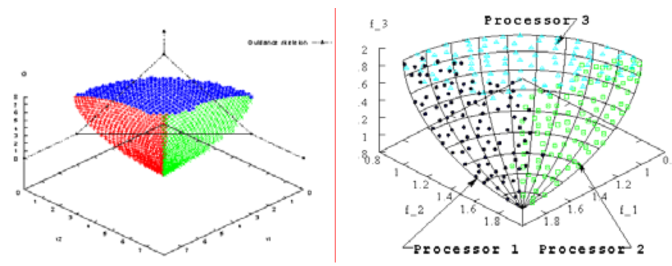
Distributed computing: A Three-Objective Problem

- Spatial computing, not temporal



Theory

NSGA-II Simulations

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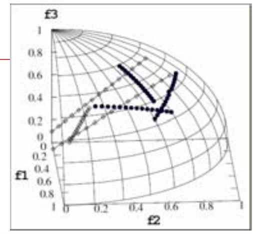
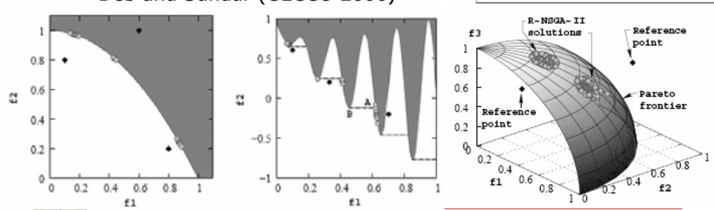
Making Decisions: Current Focus

Deb and Kumar (GECCO-2007)



- Ranking based on closeness to each reference point or a reference direction

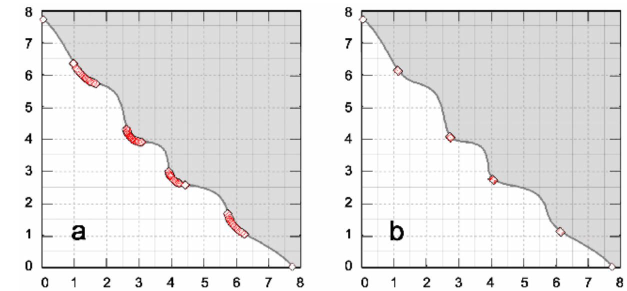
Deb and Sundar (GECCO 2006)



'Light Beam' Approach in CEC-07

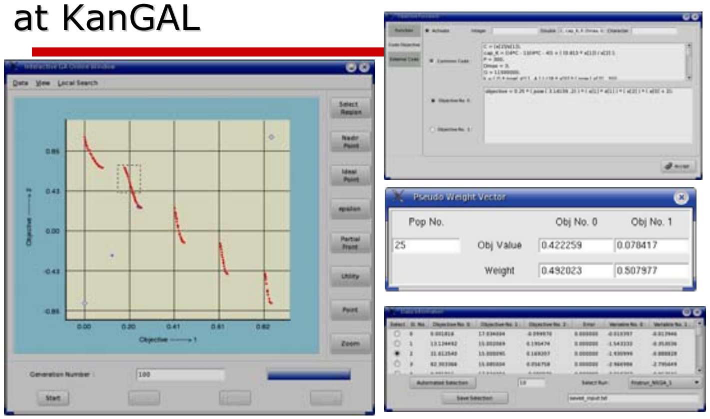
Finding Knee Solutions (Branke et al., 2004)

- Find only the knee or near-knee solutions



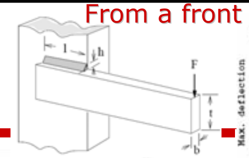
I-MODE Software Developed at KanGAL

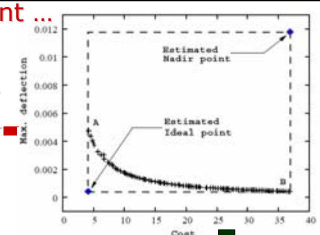
Deb and Chaudhuri, EMO-07

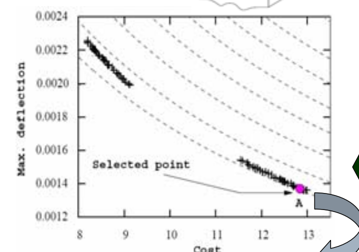


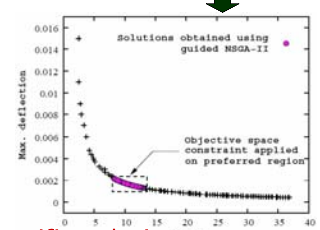
Example:

From a front ...









Decision Variables				Objective Values	
h (in)	l (in)	t (in)	b (in)	Cost	Max. Deflection (in)
0.917	1.009	9.856	1.672	12.838	0.001371

... to a specific solution

Conclusions

- EMO is a fast-growing field of research and application
  - Exciting for field of Evolutionary Computing
- Practical applications and challenges surfacing
- EMO+MCDM, EMO+Math optimization
- Commercial softwares available
  - ModeFrontier, iSIGHT
- Computer codes freely downloadable
- Many conference and journal publication opportunities

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Regular EMO Activities

http://www.emo09.org

- A dedicated two-yearly conference (EMO):
  EMO-01 (Zurich),
  EMO-03 (Faro),
  EMO-05 (Guanajuato),
  EMO-07 (Sendai)
- Next one in Nantes, France (EMO-09)
- Other major EA conferences (EMO tracks)
- Special issues of journals
- 150+ PhD theses so far since 1993

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