Optimization Methods for Multidisciplinary Design in Aerospace Engineering Using Parallel Evolutionary Algorithms, Game Theory and Hierarchical Topology

Theoretical aspects and applications (1)

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Short Course on Integrated Multiphysics Simulation & Optimization, Laajavuori, March 13-14, 2009
Credit:

- K. Srinivas, E. Whitney, USYD
  Optimisation

- M. Sefrioui, Z. Johan, Courty, Dassault Aviation
  Hierarchical methods, UAV Design test cases, MDO Challenges

- M. Drela MIT
  Xfoil Software

TEKES for supporting this event in the context of the FiDiPro DESIGN Project
OUTLINE

1. OBJECTIVES

2. INTRODUCTION AND MOTIVATION

3. EVOLUTIONARY METHODS

4. MECHANICS OF GAs

5. MULTI-OBJECTIVE GAs AND GAME THEORY

6. DISTRIBUTED AND PARALLEL EAs

7. HIERARCHICAL EVOLUTIONARY ALGORITHMS (HEAs)

8. ASYNCHRONOUS PARALLEL EAs (HAPEAs)

9. UNCERTAINTY : PERFORMANCE VS STABILITY

10. THE DEVELOPMENT OF EVOLUTIONARY ALGORITHMS FOR DESIGN AND OPTIMISATION IN AERONAUTICS : EXAMPLE OF A UAV DESIGN

11. CONCLUSION
1. Main Objectives

These lectures describe theoretical and numerical aspects (part 1) with applications (part 2) of Evolutionary Design Methods in Aerospace Engineering.
2. Motivation: MASTERING COMPLEXITY, A COLLABORATIVE WORK....

- technological constraints
- economical constraints
- societal constraints
- integrated systems

- Targets (« doing better with less »)
  - Computational multidisciplinary tools
  - Decision maker algorithms for the design of industrial products
  - Time and cost reduction for system design and manufacturing

- Priorities
  - 1) Robustness (global solutions)
  - 2) Low cost efficiency (grid computing)
  - 3) Human interfaces
MDO simulations for civil aircraft (courtesy of Dassault Aviation)

**ENVIRONMENT**
- Acoustics
- Shapes

**AERODYNAMICS**
- Architecture
- Vibrations
- Shapes
- Flaps
- Anemometry

**FLIGHT DYNAMICS**
- Flight control
- Flight quality
- Thrust/drag control

**STRUCTURE**
- Landing gear
- Hydraulics
- Embedded Software

**ERGONOMY**
- Pilot information
- Visualizations

**EQUIPMENT**
- De-icing
- Inertial

Computational Physics and mathematics
MDO Challenge: Noise prediction and reduction (courtesy of Dassault Aviation)

Engine Noise:
- Fan, Compressor, Turbine, Combustion, Exhaust Jet

Airframe Noise:
- Landing Gear, Slats and Flaps, Wakes
NEW CONTEXT....

Multi Disciplinary

Search Space – Large
- Multimodal
- Non-Convex
- Discontinuous

Share knowledge:
- different cultures and technologies connected

Integration of software with interfaces and human factors

Trade off between Conflicting Requirements

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Multidisciplinary design problems involve search spaces that are multi-modal, non-convex or discontinuous.

Traditional methods use deterministic approach and rely heavily on the use of iterative trade-off studies between conflicting requirements.
Traditional optimization methods will fail to find the best answer in many complex engineering applications, (noise, complex or non differentiable objective functions); why? lack of robustness !!

The internal workings of validated in-house/ commercial solvers (Fluent, Cstar,…) are inaccessible from a modification point of view (black-boxes); why? Lack of flexibility for integration or modification!
3. EVOLUTIONARY ALGORITHMS (1)

Traditional Gradient Based methods for MDO might fail to find optimal solution if search space is:

- Large
- Multimodal
- Non-Convex
- Many Local Optimum
- Discontinuous

A real aircraft design optimization might exhibit one or several of these characteristics.
Multiple Goals  Minimise-Maximise

Search spaces for multiphysics solutions are complex

Optimal Solutions optimal Surface of UAV, μUAV

Payload Capacity  Structural Performance  Aerodynamic Performance

Multiple Goals

Minimise-Maximise

Payload Capacity

Aerodynamic Performance

Structural Performance

Optimal Solutions optimal Surface of UAV, μUAV

Traditional Gradient Based Techniques might fail or be trapped in local minima

Advanced Techniques are required

Advanced numerical techniques – Evolutionary Computing

Global minimum

local minimum

Time consuming process

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50’-60’: evolution could be used as an optimization tool for engineering problems

innovation: evolve a population of candidates to a given problem, using operators inspired by natural genetic variation and natural selection

approach: evolution inspired algorithms

60’: GAs invented by J. Holland, Univ. of Michigan

target: study the phenomena of adaptation as it occurs in nature

How? Developing ways how natural adaptation can be implemented into computer systems

result: GAs as an abstraction of biological evolution
Genetic Algorithms (GAs): notations (2)

- **chromosome**: a candidate solution to a problem, encoded as a bit string
- **genes**: single bit or short blocks that encode a particular element of a candidate solution
- **cross over**: exchanging material between the parent chromosomes
- **mutation**: flipping a bit at a randomly chosen locus
- **fitness**: criteria of function to minimize or maximize
- **example in CFD or CEM optimization problems:**
  - **population**: a set of airfoils; **chromosome**: an airfoil
  - **genes**: spline coefficient; **parents**: two airfoils
  - **offspring**: two children airfoils; **fitness**: drag or signature
  - **environment**: flow or wave (non) linear PDEs
Advanced Optimisation Tools:

**Evolutionary Optimisation**

- Good for all of the above
- Easy to parallelize
- Robust towards noise
- Explore larger search spaces
- Good for multi-objective problems

- Based on the Darwinian theory of evolution → populations of individuals evolve and reproduce by means of random mutation and crossover operators and compete in a set environment for survival of the fittest (selection).
GENETIC ALGORITHMS pioneered J. Holland in the 60’ with binary coding

![Diagram of Genetic Algorithm Cycle]

- Selection
- Crossover
- Mutation
- Fitness

1000110100101

- $P_c$
- $1 - P_c$
- $P_m$
GENETIC ALGORITHMS with 3 OPERATORS

- Selection (semi random, semi deterministic): survival of the fittest (Darwin principle)

- Cross over (random): \( P_c \) (binary coding)
  
  - Parents:
    - 100/01110
    - 001/00000

  - Offspring:
    - 10000000
    - 00101110

- Mutation (random): \( P_m \)

  - 10001110
  - 10001100
EVOLUTIONARY ALGORITHMS (3) : One Generation of the Algorithm...

Start Population

Select Parents

Recombine

Mutate

Evaluate

Failures

Join and Re-Rank

One Offspring

Successful

In Parallel

Offspring Population

Final Population
Genetic Algorithm: parameters

Population size: 30-100, problem dependent
Cross over rate: $P_c = 0.80-0.95$
Mutation rate: $P_m = 0.001 - 0.01$
Areas of applications with GAs

- **Optimization** and Machine learning (D. Golberg, 1989)
- Automatic programming (J. Koza, 1992)
- Economics (bidding strategies, economic markets)
- Immune systems (KrishnaKumar, 1998)
- Ecology (co evolution)
- Social systems (evolution of social behaviour in insect colonies, cooperation and communication of multi-agents systems)
- Complex adapted systems (Hidden order, J. Holland, 1997)
- …..
How are GAs different from traditional methods? (D. Goldberg, 1989)

- GAs work with a coding of the parameter set, not the parameters themselves
- GAs search from a population, not a single point
- GAs use payoff (objective function) information, not derivatives or other auxiliary knowledge
- GAs use probabilistic transition rules, not deterministic rules
- The central theme of research on GAs has been robustness
GAs Mechanisms: why they work?

- GAs are *indifferent to problems specifics* (no derivative needed to take a decision!)
- GAs use a coding of decision variables (DNA and adaptation of chromosomes)
- GAs process populations via evolutive generations
- GAs use randomized operators
- Theoretical foundations of GAs rely on a binary string representations of solutions and on the notion of schema
- The schema theorem (J. Holland): “*short, low order, above average schemata receive exponentially increasing trials in subsequent generation of a GAs*” (Michalewicz, 1992)
In this example: A chromosome or an individual are the control points \((y_i)\) that define the aerofoil shape.

\[
\begin{array}{cccccccc}
  y_1 & y_2 & y_3 & y_4 & y_5 & y_6 \\
\end{array}
+ \begin{array}{cccccccc}
  y_1' & y_2' & y_3' & y_4' & y_5' & y_6' & y_7' & y_8' & y_9' & y_{10}' \\
\end{array}
= \begin{array}{cccccccccccc}
  y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & y_1' & y_2' & y_3' & y_4' & y_5' & y_6' & y_7' & y_8' & y_9' & y_{10}' \\
\end{array}
\]

1000110100101

Candidate aerofoil
DRAWBACK OF EVOLUTIONARY ALGORITHMS

- Evolution process is time consuming/ high number of function evaluations is required.

- A typical MDO problem relies on CFD and FEA for aerodynamic and structural analysis.

- CFD and FEA software are time consuming!

*Gradient Based Methods or simple Evolutionary Algorithms are not efficient enough to capture global solutions for MO and MDO Problems—Therefore Advanced Techniques are required*
Aeronautical and aerospace design problems normally require a simultaneous optimisation of conflicting objectives and associated number of constraints.

They occur when two or more objectives that cannot be combined rationally. For example:

- Drag at two different values of lift.
- Drag and thickness.
- Pitching moment and maximum lift
- ……
Different Multi-Objective approaches

- Aggregated Objectives, main drawback is loss of information and a-priori choice of weights.

- Game Theory (von Neumann)
  - Game Strategies
    - Cooperative Games - Pareto
    - Competitive Games - Nash
    - Hierarchical Games - Stackelberg

- Vector Evaluated GA (VEGA) Schaffer,85
- Multi Objective Optimization with GAs K. Deb, 2001
MULTI-OBJECTIVE OPTIMISATION

Maximise/ Minimise

\[ f_i(x) \quad i = 1 \ldots N \]

Subjected to constraints

\[ g_i(x) = 0 \quad j = 1 \ldots N \]

\[ h_k(x) \leq 0 \quad k = 1 \ldots K \]

- \( f_i(x) \): Objective functions, output (e.g. cruise efficiency).

- \( x \): vector of design variables, inputs (e.g. aircraft/wing geometry)

- \( g(x) \): equality constraints and \( h(x) \): inequality constraints: (e.g. element von Mises stresses); in general these are nonlinear functions of the design variables.
MULTIPLE OBJECTIVE OPTIMIZATION

- Linear Combination of criteria (aggregation)

\[ C = \sum_{i=1}^{n} \omega_i \cdot c_i \]

**BUT**

- Dimensionless number
- Heavy bias from the choice of the weights

- VEGA (Vector-Evaluated GA) [Schaffer, 85]
  - bias on the extrema of each objective
GAME STRATEGIES

◦ Theoretical foundations: Von Neumann

◦ Applications to Economics and Politics: Von Neuman, Pareto, Nash, Von Stackelberg

◦ Decentralized optimization methods:
  Lions-Bensoussan-Temam in Rairo (1978, G. Marchuk, J.L. Lions, eds)

In this lecture: introduce and use Games strategies in Engineering for solving Multi Objective Optimization Problems
NOTATIONS

- For a game with 2 players, A and B
- For A
  - Objective function $f_A(x,y)$
  - A optimizes vector $x$
- For B
  - Objective function $f_B(x,y)$
  - B optimizes vector $y$

$\bar{A}$ = set of possible strategies for A
$\bar{B}$ = set of possible strategies for B
Pareto Dominance

- Pareto Optimality (minimization, 2 Players A and B).

is Pareto optimal if and only if:

\[(x^*, y^*)\]

\[\forall (x,y) \in \overline{A} \times \overline{B}, \begin{cases} f_A(x^*, y^*) \leq f_A(x, y) \\ f_B(x^*, y^*) \leq f_B(x, y) \end{cases}\]

- Pareto Dominance (for n players \((P_1, \ldots, P_n)\))

\[\square \text{Player } P_i \text{ has objective } f_i \text{ and controls } v_i\]

\[\square (v_1^*, \ldots, v_k^*, \ldots, v_n^*) \text{ dominates } (v_1, \ldots, v_k, \ldots, v_n) \text{ iff:}\]

\[\begin{cases} \forall i, f_i(x_1^*, \ldots, x_k^*, \ldots, x_n^*) \leq f_i(x_1, \ldots, x_k, \ldots, x_n) \\ \exists i, f_i(x_1^*, \ldots, x_k^*, \ldots, x_n^*) < f_i(x_1, \ldots, x_k, \ldots, x_n) \end{cases}\]
Pareto Front

- Pareto Optimality
  - A strategy \((v_1^*, ..., v_k^*, ..., v_n^*)\) is Pareto-optimal if it is not dominated.

- Pareto Front
  - Set of all NON-DOMINATED strategies
A set of solutions that are non-dominated w.r.t all others points in the search space, or that they dominate every other solution in the search space except fellow members of the Pareto optimal set.

EAs work on population based solutions …can find a optimal Pareto set in a single run

HAPMOEA: Captures Pareto Front, Nash and Stackelberg solutions
Nash Equilibrium

- Competitive symmetric games [Nash, 1951]
- For 2 Players A and B:

\[
f_A(\bar{x}^*, \bar{y}^*) = \inf_{x \in A} f_A(x, \bar{y}^*)
\]

\[
f_B(\bar{x}^*, \bar{y}^*) = \inf_{y \in B} f_B(\bar{x}^*, y)
\]

- For n Players:

\[
\forall i, \forall v_i, f_i(v_1^*, \ldots, v_{i-1}^*, v_i^*, v_{i+1}^*, \ldots, v_n^*) \leq f_i(v_1^*, \ldots, v_{i-1}^*, v_i^*, v_{i+1}^*, \ldots, v_n^*)
\]

« When no player can further improve his criterion, the system has reached a state of equilibrium named Nash equilibrium”
How to find a Nash Equilibrium?

- Let $D_A$ be the rational reaction set for A, and $D_B$ the rational reaction set for B.

\[
D_A = \{(x^*, y) \in \bar{A} \times \bar{B} \} \text{ such that } f_A(x^*, y) \leq f_A(x, y)
\]

\[
D_B = \{(x, y^*) \in \bar{A} \times \bar{B} \} \text{ such that } f_B(x, y^*) \leq f_B(x, y)
\]

- Which can be formulated:

\[
\begin{align*}
D_A &= \left\{ x, \frac{\partial f_A(x, y)}{\partial x} = 0 \right\} \\
D_B &= \left\{ y, \frac{\partial f_B(x, y)}{\partial y} = 0 \right\}
\end{align*}
\]

- Nash Equilibrium

A strategy pair $(x^*, y^*) \in D_A \cap D_B$ is a Nash Equilibrium!
Nash GAs

Player 1 = Population 1

Player 2 = Population 2

[Sefrioui & Periaux, 97] Optimizing $X \ Y$

Gen 0

$X_0 Y_r$  $X_1 Y_0$  $X_k Y_{k-1}$  $X_{k+1} Y_k$

Gen 1

Gen k

Gen k+1

$x_0 y_1$  $x_1 y_0$  $x_{k-1} y_k$  $x_k y_{k+1}$
Stackelberg Games

- Hierarchical strategies

- Stackelberg game, A leader
  - Stackelberg game with A leader and B follower:
    \[
    \min_{x \in D_A, y \in D_B} f_A(x, y)
    \]

- Stackelberg game, B leader
  - Stackelberg game with B leader and A follower:
    \[
    \min_{x \in D_A, y \in D_B} f_B(x, y)
    \]
Genetic Operators

[Genetic Operators]

Optimize

[Sefrioui & Periaux, 97]

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Example:

- Let us consider a game with 2 players A and B, with the following objective functions

\[ f_A = (x-1)^2 + (x-y)^2 \]
\[ f_B = (y-3)^2 + (x-y)^2 \]
Pareto : Analytic Resolution

- Let us consider the parametric function

\[ f_p(x,y) = \lambda \cdot ((x - 1)^2 + (x - y)^2) + (1 - \lambda) \cdot ((y - 3)^2 + (x - y)^2), \]

with \( 0 \leq \lambda \leq 1 \)

- The Pareto equilibria are the solution of

\[
\begin{align*}
\frac{\partial f_p(x,y)}{\partial x} &= 0 \\
\frac{\partial f_p(x,y)}{\partial y} &= 0 \\
\end{align*}
\]

\[
\begin{align*}
2\lambda x - 2\lambda + 2x - 2y &= 0 \\
-2\lambda y + 4y - 6 - 2x + 6\lambda &= 0 \\
\end{align*}
\]
Which yields

\[
\begin{align*}
    x &= \frac{\lambda^2 + \lambda - 3}{\lambda^2 - \lambda - 1} \\
    y &= \frac{3\lambda^2 - \lambda - 3}{\lambda^2 - \lambda - 1}
\end{align*}
\]
Stackelberg : Analytic

- Stackelberg, A leader
  - Minimize $f_A(x,y)$ on $D_B$.
  - $D_B$ is built by solving

\[
\frac{\partial f_B(x,y)}{\partial y} = 0
\]

\[
\frac{\partial f_B(x,y)}{\partial y} = 0 \iff 2(y - 3) - 2(x - y) = 0 \iff y = \frac{x + 3}{2}
\]

- $D_B$ is the line $y = \frac{x + 3}{2}$

- The problem consists now in solving

\[
\frac{\partial f_A(x,\frac{x + 3}{2})}{\partial x} = 0
\]
Stackelberg: Analytic (2)

\[ \frac{\partial f_A}{\partial x} \left( x, \frac{x + 3}{2} \right) = 0 \iff \frac{\partial ((x - 1)^2 + (x - \frac{x + 3}{2})^2)}{\partial x} = 0 \]

\[ \iff 2(x - 1) + \left( \frac{x}{2} - \frac{3}{2} \right) = 0 \iff x = \frac{7}{5} \]

- \( y \) is then:

\[ y = \frac{x + 3}{2} = \frac{7}{5} + \frac{3}{2} = \frac{22}{10} \]

- The first Stackelberg equilibrium \( S_A \) is the point:

\[ \left( \frac{7}{5}, \frac{22}{10} \right) = \left( 1.4, 2.2 \right) \]
Stackelberg : B leader

- Stackelberg, B leader and A follower
  - Minimize $f_B(x,y)$ on $D_A$. 
  - $D_A$ is built by solving
    \[
    \frac{\partial f_A(x,y)}{\partial x} = 0 \iff 2(x - 1) + 2(x - y) = 0 \iff y = 2x - 1
    \]
  - $D_A$ is the line $y = 2x - 1$

- The problem consists now in solving
  \[
  \frac{\partial f_B}{\partial y} \left( \frac{y + 1}{2}, y \right) = 0
  \]
Stackelberg: B leader (2)

\[
\frac{\partial f_B}{\partial y} \left( \frac{y + 1}{2}, y \right) = 0 \iff \frac{\partial}{\partial y} \left( (y - 3)^2 + \left( \frac{y + 1}{2} - y \right)^2 \right) = 0
\]

\[
\iff 2(y - 3) - \left( \frac{1 - y}{2} \right) = 0 \iff y = \frac{13}{5}
\]

\square \ x \ is \ then:

\[
x = \frac{y + 1}{2} = \frac{13}{5} + \frac{1}{2} = \frac{18}{10}
\]

\square \ The \ second \ Stackelberg \ equilibrium \ S_B \ is \ the \ point:

\[
\left( \begin{array}{c}
\frac{18}{10} \\
\frac{13}{5}
\end{array} \right) = \left( \begin{array}{c}
1.8 \\
2.6
\end{array} \right)
\]
The Nash Equilibrium is the intersection of the two rational reaction sets $D_A$ and $D_B$. Finding the Nash Equilibrium consists in solving:

\[
\begin{align*}
    y &= 2x - 1 \\
    y &= \frac{x + 3}{2}
\end{align*}
\]

\[
\begin{align*}
    y &= 2x - 1 \\
    y &= \frac{x + 3}{2}
\end{align*}
\]

\[
\begin{align*}
    y &= 2x - 1 \\
    3y &= 7
\end{align*}
\]

\[
\begin{align*}
    x &= \frac{5}{3} \\
    y &= \frac{7}{3}
\end{align*}
\]

The Nash Equilibrium $E_N$ is the point

\[
\begin{pmatrix}
    5 \\
    3 \\
    7 \\
    3
\end{pmatrix} = \begin{pmatrix}
    1.66 \\
    2.33
\end{pmatrix}
\]
Pareto Equilibrium
Optimization results with GAs

- Try to optimize the function $f_A$ and $f_B$ with the optimization tools presented earlier
  - With a Pareto/ GA game
  - With a Nash/ GA game
  - With a Stackelberg/ GA game
Nash GA : convergence
- $f_A$ converges towards 0.896 and $f_B$ towards 0.88
- Both those are the values on the objective plane!
- And we can check that
  \[ f_A\left(\frac{5}{3},\frac{7}{3}\right) = 0.896 \quad \text{and} \quad f_B\left(\frac{5}{3},\frac{7}{3}\right) = 0.88 \]
- So the Nash GA finds the theoretic Nash Equilibrium

**Specifics**
- 2 populations, each of size 30
- $P_c = 0.95 \quad P_m = 0.01$
- Exchange frequency: every generation
- $(x,y)$ in $[-5,5] \times [-5,5]$
Pareto GA

\[ f_A = 0.895891 \]
\[ f_B = 0.880220 \]
Stackelberg GA : convergence

![Graph showing Stackelberg-GA convergence with fitness on the y-axis and number of generations on the x-axis. Two lines represent Stackelberg, A leader and Stackelberg, B leader.]
In both cases (with either A or B leaders), the algorithms converges towards 0.8. But in the objective plane.

In the plane (x,y), we can see that the first game converges towards (1.4,2.2) and that the second game converges towards (1.8,2.6)
Converged game solutions for GAs vs analytical approaches
Pareto Equilibrium

Equilibres Théoriques

Equilibre de Pareto Théorique
Equilibre de Nash Théorique
Stackelberg A leader
Stackelberg B leader

$D_A : y = 2x - 1$

$D_B : y = (x+3)/2$
Two-objective Inverse Design in Aerodynamics

The optimization problem:

Optimization problem: reconstructing two different pressure distributions

Problem: find all the profiles existing between the low-drag profile and the high lift profile

\[ M_a = 0.2 \quad \alpha = 10.8^\circ \]
\[ \min f_1 = \int_{\Gamma_1} (p(w) - p_{sub})^2 \, ds \]

\[ M_a = 0.77 \quad \alpha = 1^\circ \]
\[ \min f_2 = \int_{\Gamma_1} (p(w) - p_{trans})^2 \, ds \]
MULTI-OBJECTIVE DESIGN with gradient method

Pareto-front, Stackelberg points, Nash equilibrium (Parameterization with Hicks-Henne functions)
MULTI-OBJECTIVE DESIGN: Pareto solution set

Parameterization with Hicks-Henne functions

Pareto Front of Airfoil (parameterization is Hicks-Henne function)
Comparison of pareto-fronts computed by different parameterization

Comparison of pareto-fronts computed by GAs and Deterministic method

MULTI-OBJECTIVE DESIGN: Comparisons
6. Parallel GAs mechanisms

- PGAs: a particular instance of GAs
  - sub-population (H. Muhlenbein, 1989)
  - network of interconnected sub-populations (*Island Model*)
  - smaller sub-populations versus a single large one

![Diagram showing a network of interconnected sub-populations with labels for '1 node = 1 sub-population' and 'Neighbours']
Parallel GAs mechanisms: a road map to robustness! (M. Sefrioui & JP, 1996)

- **Isolation and Migration**
  - sub-populations evolve independently for a given period of time (epoch)
  - after each epoch, migration between sub-populations before isolation resumes
  - promising solutions shared by sub-populations via their neighbours
Test-case: Rastrigin Function

\[ f = 10 \cdot 20 + \sum_{i=1}^{20} \left( x_i^2 - 10 \cdot \cos(2\pi \cdot x_i) \right) \]

In dimension 2, the surface is:
Convergence for Sequential and Parallel GA
Rastrigin Function

Parallel and Sequential GA convergence

Number of Generations

- Interactions of the 3 layers: solutions go up and down the layers.
- The best ones keep going up until they are completely refined.
- No need for great precision during exploration.
- Time-consuming solvers used only for the most promising solutions.
- Think of it as a kind of optimisation and population based *multi grid*.
HIERARCHICAL TOPOLOGY - MULTIPLE MODELS

Start migration:

- **Layer 1:** Receive (1/3 population) best solutions from layer 2 reevaluate using type 1 integrated analysis.

- **Layer 2:** Receive (1/3 population) random solutions from layer 1 and best from layer 3 reevaluate them using type 2 integrated analysis.

- **Layer 3:** Receive (1/3 population) random solutions from layer 2 reevaluates them using type 3 integrated analysis.
10. Discontinuous Pareto Front industrial test case:
Two Objective UAV Airfoil Section Design (Eurogen 2003)

- Design of a single element aerofoil for a low-cost UAV application.
- Two subsonic design points considered for optimisation
  - Loitering flight
  - Rapid-transit flight.
Design Variables: Bounding Envelope of the Aerofoil Search Space

Two Bezier curves representation:

Four control points on the mean line.

Six control points on the thickness distribution.

10 Design variables for the aerofoil

Constraints:
- Thickness > 12% x/c
- Pitching moment > -0.065
Fitness Functions and Design Constraints

Specifications: Z. Johan, Dassault Aviation

Min $f_1$ (Cd transit)  Mach=0.60 and Re= 14.0x10**6, Cm>-0.065
Min $f_2$ (Cd loiter) Mach=0.15 and Re= 3.5x10**6

- Constraints are applied by equally penalizing both fitness values via a penalty method.

- Aerofoil generated outside the thickness bounds of 10% to 15% are rejected immediately, before analysis.
Solver

- XFOIL software written by Drela.
- It comprises a higher order panel method with coupled integral boundary layer.
- We have allowed free transition points for the boundary layer.
- Locally sonic flow will be prevented by checking:

  The value of $C_p$: $C_{pi} < C_p$ then the candidate is rejected immediately
Hierarchical Asynchronous Parallel EA (HAPEA)

Exploitation
Population size = 20

Intermediate
Population size = 20

Exploration
Population size = 10

Model 1
Grid= 119 panels

Model 2
Grid=99 panels

Model 3
Grid= 79 panels
Discontinuous Pareto Front for Aerofoil

This case was run for 5300 function evaluations of the head node, and took approximately four hours on a single 1.0 GHz processor.
Classical aerodynamic shapes have been evolved,
Optimum Airfoils for Cruise and Loiter

Evolved a conventional low-drag pressure distribution and overall form

Classical 'rooftop' type pressure distribution upper surface Almost constant favorable pressure gradient lower surface

Objective 1: Optimal Aerofoil – Cruise $CP$ Distribution.

Objective 2: Optimal Aerofoil – Loiter $CP$ Distribution.
Marked favorable gradient on the lower surface in both flow regimes

Conventional Pressure distribution,

Pronounced S-shaped camber distribution.

Cruise CP Distribution

Loiter CP Distribution.

**Why asynchronous?**

Converged PDE solutions to MO and MDO -> variable time to complete

Time to solve non-linear PDE -> depends upon geometry

Ignore any concept of a *generation*

Solution can be generated in and out of order

Processors – Can be of different speeds
- Added at random
Parallelization Strategy

Classification of our model (S. Armfield, USYD):

- The algorithm: classified as a hierarchical Hybrid pMOEA model [Cantu Paz], uses a Master slave PMOEA but incorporates the concept of isolation and migration through hierarchical topology binary tree structure where each level executes different MOEAs/parameters (heterogeneous)

- The distribution of objective function evaluations over the slave processors is where each slave performs k objective function evaluations.

Parallel Processing system characteristics:

- Cluster of maximum 18 PCs with Heterogeneous CPUs, RAMs, caches, memory access times, storage capabilities and communication attributes.

Inter-processor communication:

- Using the Parallel Virtual Machine (PVM)
Asynchronous Evaluation (1)

- Ignores the concept of generation-based solution.
- Fitness functions are computed asynchronously.
- Only one candidate solution is generated at a time, and only one individual is incorporated at a time rather than an entire population at every generation as is traditional EAs.
- Solutions can be generated and returned out of order.
Asynchronous Evaluation (2)

- No need for synchronicity → no possible wait-time bottleneck.
- No need for the different processors to be of similar speed.
- Processors can be added or deleted dynamically during the execution.
- There is no practical upper limit on the number of processors we can use.
- All desktop computers in an organization are fair game.
Results So Far...

- HAPEA technique is approximately three times faster than other similar EA methods.

<table>
<thead>
<tr>
<th></th>
<th>Evaluations</th>
<th>CPU Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional EA</td>
<td>2311 ± 224</td>
<td>152m ± 20m</td>
</tr>
<tr>
<td>New Technique</td>
<td>504 ± 490 (-78%)</td>
<td>48m ± 24m</td>
</tr>
</tbody>
</table>

- A test bench for single and multi objective problems has been developed and tested successfully.

- We have successfully coupled the optimisation code to different compressible CFD codes and also to some aircraft design codes:
  - **CFD**: HDASS, MSES, XFOIL, FLO22, Nsc2ke
  - **Aircraft Design**: Flight Optimisation Software (FLOPS), ADS (In house)
9. Robust design : TAGUCHI METHOD (Uncertainty)

Robust Design method, also called the Taguchi Method (uncertainty), pioneered by Genichi Taguchi in 1978, improves a quality of engineering productivity. An optimisation problem could be defined as:

$$\text{Max or Min } f = f(x_1,\ldots,x_n,x_{n+1},\ldots,x_m)$$

Where $x_1,\ldots,x_n$ represent design parameters and $x_{n+1},\ldots,x_m$ represent uncertainty parameters that are in fine step size.

Taguchi optimization method minimizes the variability of the performance under uncertain operating conditions. Define two different objectives associated to the function to optimise: mean value and variance.

**MEAN**

$$f = \frac{1}{K} \sum_{j=1}^{K} f_j$$

**VARIANCE**

$$\delta f = \frac{1}{K-1} \left( \sum_{j=1}^{K} |f_j - \bar{f}| \right)$$
**UNCERTAINTY**

**MEAN**

\[
f = \frac{1}{K} \sum_{j=1}^{K} f_j
\]

**VARIANCE**

\[
\delta f = \frac{1}{K-1} \left( \sum_{j=1}^{K} \left| f_j - f \right| \right)
\]

---

**Uncertainty design technique**

**Single-objective design optimisation**

\[ f = \min(C_D) \text{ at } M_t \]

---

**Uncertainty based Single-criteria design optimisation**

\[ f_1 = \min(C_D) \text{ and } f_2 = \min(\delta C_D) \]

\[ M_\varepsilon \in [M_s - \varepsilon, M_s, M_s + \varepsilon] \]
UNCERTAINTY BASED MULTI DISCIPLINARY DESIGN OPTIMISATION OF J-UCAV

- Fitness functions are

\[ \text{fitness} (f_1) = \min \left( \frac{1}{L/D} \right) \]

\[ \text{fitness} (f_2) = \min \left( \frac{L}{D} \right) \]

where \( \frac{L}{D} = \sqrt{\frac{1}{K} \sum_{i=1}^{K} \left( \frac{L}{D_i} \right) \frac{M_i^3}{M_s^2}} \) and \( \delta \frac{L}{D} = \frac{1}{(K-1) \sum_{i=1}^{K} \left( \frac{L}{D_i} \right)} \left( \frac{M_i^2}{M_s^2} - \frac{L}{D} \right)^2 \)

\[ f_3 = \min \left( \overline{RCS_{\theta,\phi}} \right) = \frac{1}{2} \left[ \left( \overline{RCS_{\theta,\phi}} + \delta \overline{RCS_{\theta,\phi}} \right) + \left( \overline{RCS_{\theta,\phi}} + \delta \overline{RCS_{\theta,\phi}} \right) \right] \]

where \( \theta = [0^\circ : 3^\circ : 360^\circ] \) and \( \phi = [0^\circ : 0^\circ : 0^\circ] \) (Monostatic)

where incident angles \( \theta = 135^\circ, \phi = 90^\circ \) at \( \theta = [0^\circ : 3^\circ : 360^\circ], \phi = [0^\circ : 0^\circ : 0^\circ] \) (Bistatic)

- Variability of flight conditions and radar frequencies

\( M_\alpha \in [0.75, 0.775, 0.80, 0.825, 0.85] \)

\( \alpha_\alpha \in [4.662, 3.968, 3.275^\circ, 2.581, 1.887] \)

\( ATI_\alpha \in [30062, 25093, ATI_s = 20125 \text{ ft}, 15156, 10187] \)

\( F_\alpha \in [1.0, 1.25, F_s = 1.5 \text{ GHz}, 1.75, 2.0] \)
RESULT: \textit{Pareto set planforms and aerofoil sections}

\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
Models & AR & \(b\) & \(\lambda_{c_1}\) & \(\lambda_{c_2}\) & \(\Lambda_{2-\text{c}_1}\) & \(\Lambda_{\text{c}_1-\text{c}_2}\) & \(\Lambda_{\text{c}_2-\text{T}}\) \\
\hline
Baseline & 4.45 & 18.9 & 19.7 & 19.7 & 55\degree & 29\degree & 29\degree \\
\hline
ParetoM1 (BO1) & 6.02 & 22.20 & 18.1 & 18.1 & 58.0\degree & 31.2\degree & 30.9\degree \\
\hline
ParetoM8 & 4.30 & 18.32 & 22.4 & 22.4 & 56.3\degree & 30.0\degree & 29.2\degree \\
\hline
ParetoM10 (BO2-BO3) & 3.46 & 16.47 & 29.0 & 27.0 & 57.26\degree & 27.2\degree & 26.7\degree \\
\hline
\end{tabular}
13. CONCLUSION (1)

- This lecture has described the basic concepts of EAs, and a short review of different approaches and industrial needs for MDO presented.

- Details of Evolutionary Algorithms and their specific applications to aeronautical design problems discussed.

- The lecture provided specific details on a particular EA used in this research named HAPEA.

- It is noticed that there are different methods, architectures and applications of optimisation and multidisciplinary design optimisation methods for aeronautical problems.

- However, still further research for alternative methods are still required to address the industrial and academic challenges and needs of aeronautic industry.

- EAs is an alternative option to satisfy some of these needs, as they can be easily coupled, particularly adaptable, easily parallelised, require no gradient of the objective function(s), have been used for multi-objective optimisation and successfully applied to different aeronautical design problems.

- Nonetheless, EAs have seen little application at an industrial level due to the computational expense involved in this process and the fact that they require a larger number of function evaluations, compared to traditional deterministic techniques.

- The continuing research has focused on development and applications of canonical evolution algorithms for their application to aeronautical design problems. It is worth to have a single framework that allows:
  - Solving single and multi-objective problems that can be deceptive, discontinuous, multi-modal.
  - Incorporation of different game strategies-Pareto, Nash, Stackelberg
  - Implementation of multi-fidelity approaches
  - Taking into account uncertainties
  - Parallel Computations
  - Asynchronous evaluations
Conclusion (2): KEY CONCEPTS

- **systemic** technology like the one required by UAVs will increase in the future (see Part 3).

- In order to obtain true optimised-global solution we need to think multidisciplinary.

- **Evolutionary Algorithms** are techniques to consider as it provides fruitful and optimal results.

- **Simple** EAs are not sufficient: the complex task of MO and MDO in aeronautics required **advanced** EAs.
REFERENCES


7. D.S. Lee, Uncertainty Based Multi Objective and Multidisciplinary Design Optimization in Aerospace Engineering, PhD, University of Sydney, NSW, Australia, 2008.