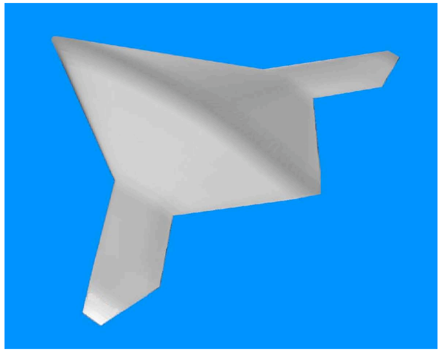
Optimization Methods for Multidisciplinary Design in Aerospace Engineering Using Parallel Evolutionary Algorithms, Game Theory and Hierarchical Topology
Theoretical aspects and applications (1)



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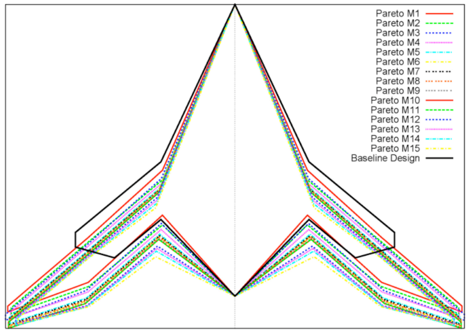

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Credit:

- K.Srinivas, E. Whitney, USYD
  Optimisation
- M. Sefrioui , Z. Johan, Courty, Dassault Aviation
  Hierarchical methods, UAV Design test cases, MDO Challenges
- M. Drela MIT
  Xfoil Software

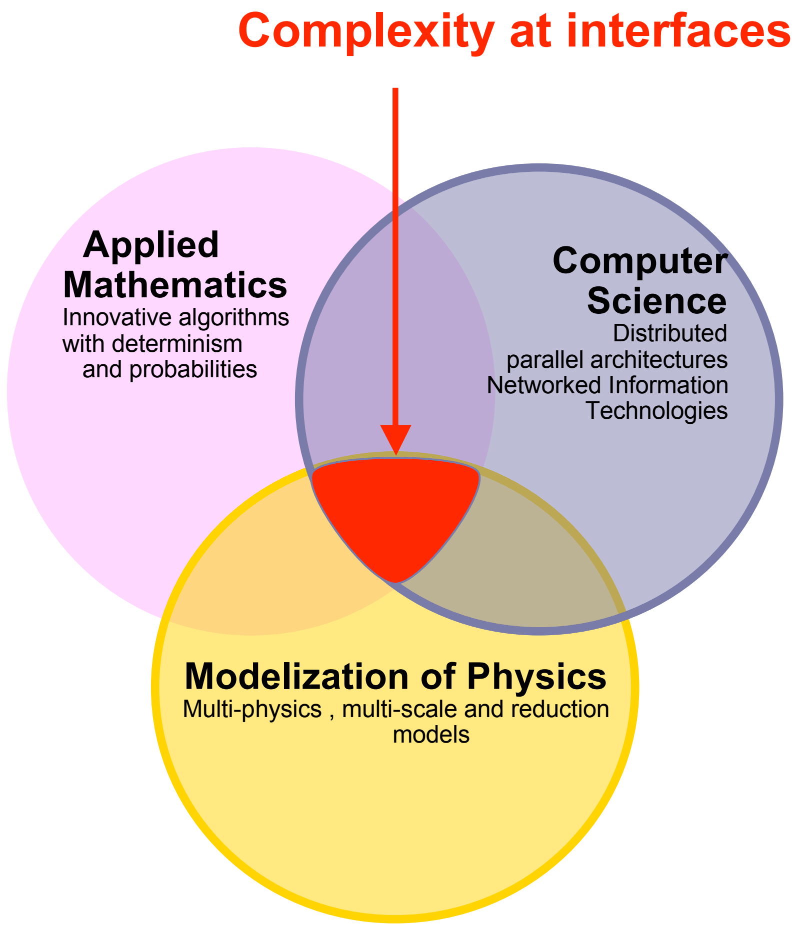
TEKES for supporting this event in the context of the FiDiPro DESIGN Project

OUTLINE

1. Main Objectives

These lectures describe theoretical and numerical aspects (part 1) with applications (part 2) of Evolutionary Design Methods in Aerospace Engineering

2. Motivation: MASTERING COMPLEXITY, A COLLABORATIVE WORK….

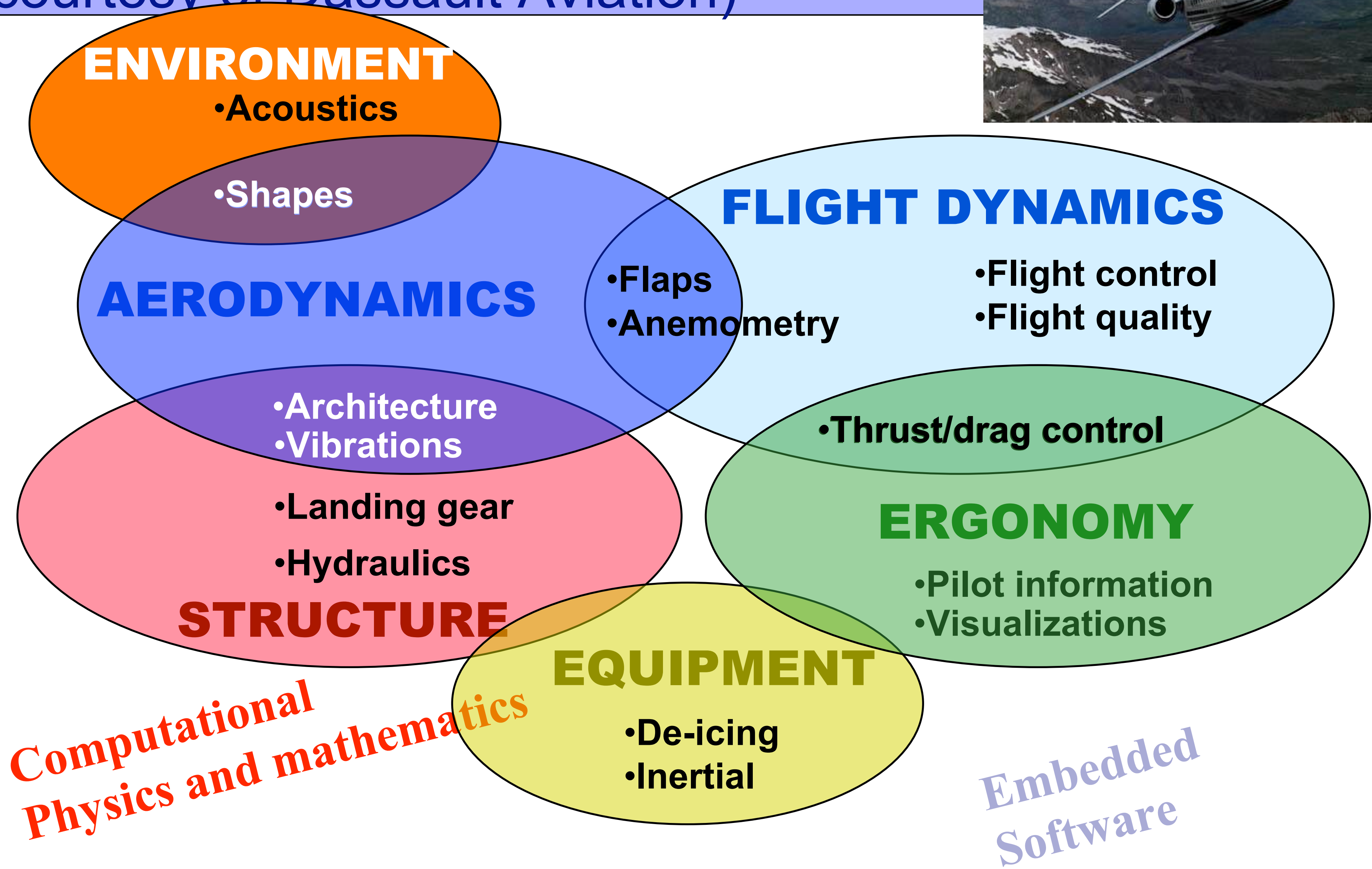


- technological constraints
- economical constraints
- societal constraints
- integrated systems

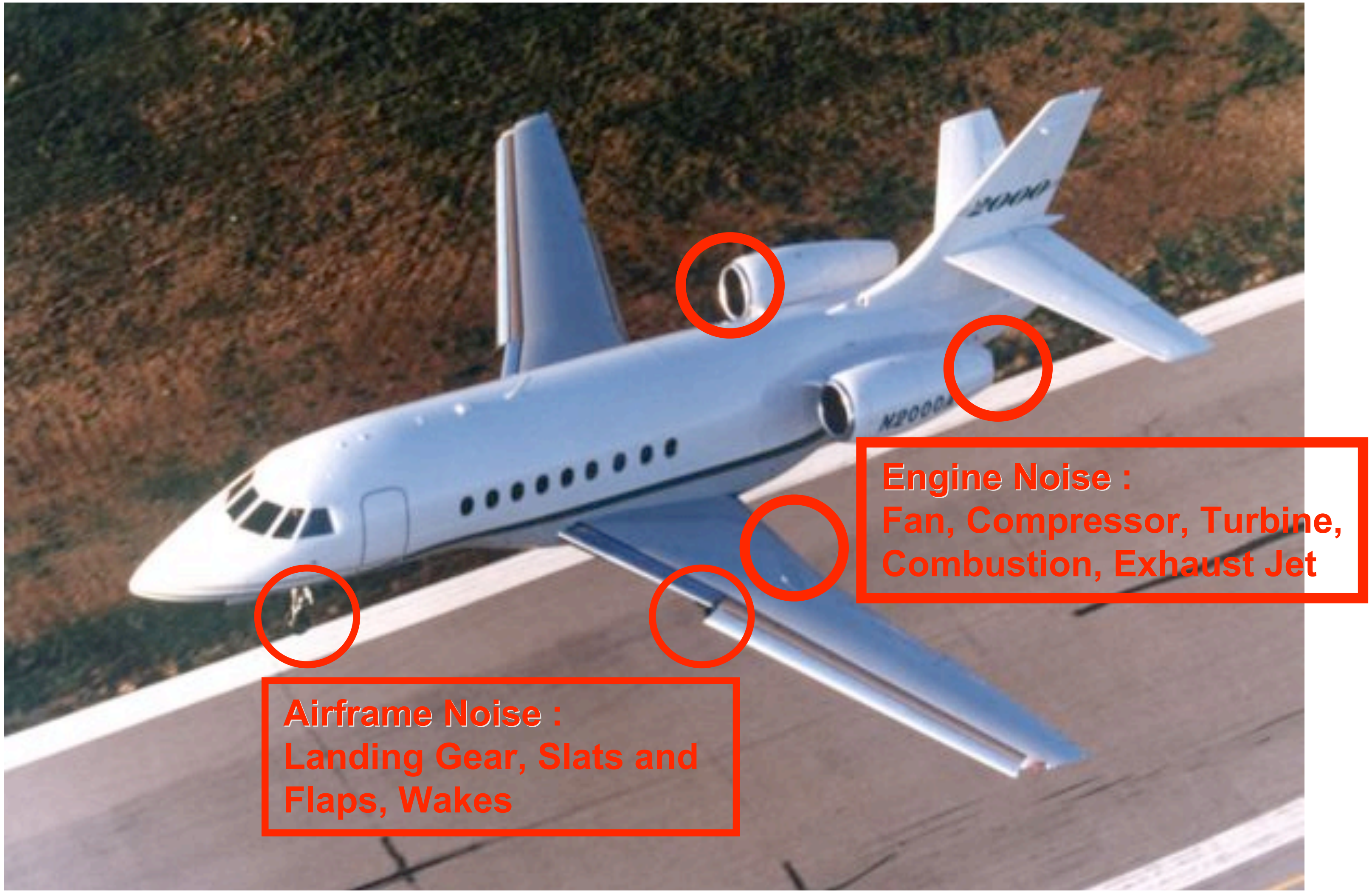
- Targets (« doing better with less »)
  - Computational multidiscilinary tools
  - Decision maker algorithmsfor the design of industrial products
  - Time and cost reduction for **system design** and **manufacturing**

- Priorities
  - 1) Robustness (global solutions)
  - 2) Low cost efficiency (grid computing)
  - 3) Human interfaces

MDO simulations for civil aircraft (courtesy of Dassault Aviation)



MDO Challenge : Noise prediction and reduction (courtesy of Dassault Aviation)



NEW CONTEXT….

Multi Disciplinary

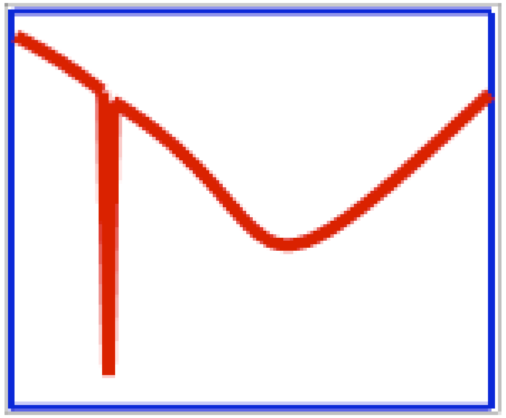
Search Space – Large
Multimodal
Non-Convex
Discontinuous

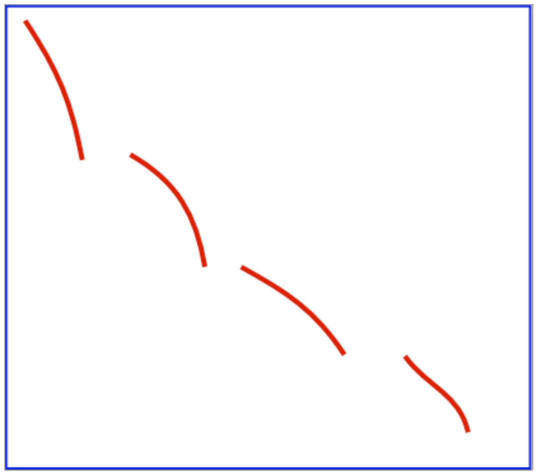
Share knowledge:
different cultures and
technologies connected

Integration of software
with interfaces and
human factors

Trade off between Conflicting Requirements

PROBLEMS IN AERODYNAMIC OPTIMISATION (1)





- Multidisciplinary design problems involve search spaces that are multi-modal, non-convex or discontinuous

- Traditional methods use deterministic approach and rely heavily on the use of iterative trade-off studies between conflicting requirements.

PROBLEMS IN AERODYNAMIC OPTIMISATION (2)

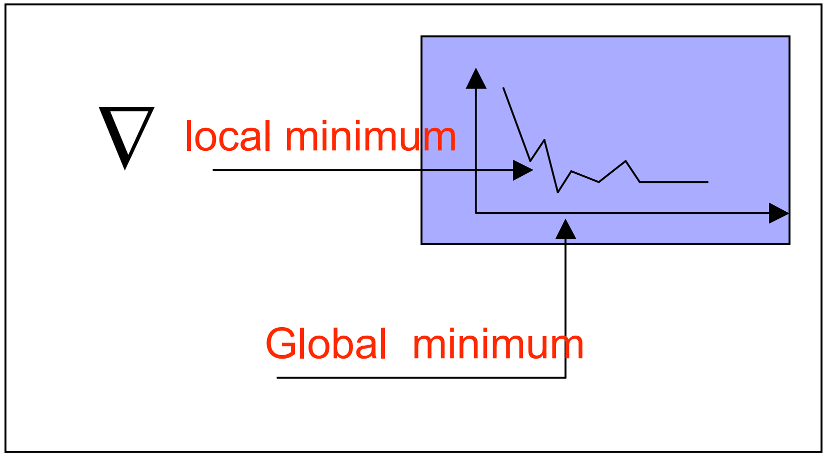
- Traditional optimization methods will fail to find the best answer in many complex engineering applications, (noise, complex or non differentiable objective functions); why? lack of robustness !!

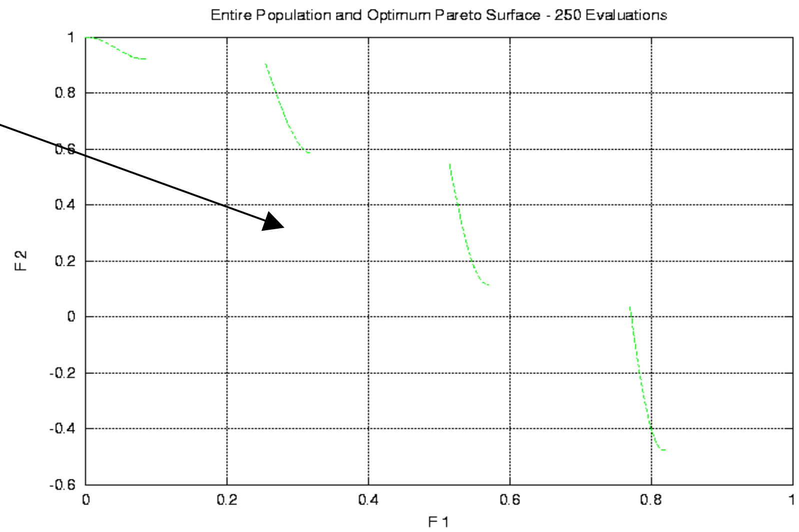
- The internal workings of validated in-house/ commercial solvers (Fluent, Cstar,…) are inaccessible from a modification point of view (black-boxes); why? Lack of flexibility for integration or modification!

3. EVOLUTIONARY ALGORITHMS (1)

Traditional Gradient Based methods for MDO might fail to find optimal solution if search space is:

- Large
- Multimodal
- Non-Convex
- Many Local Optimum
- Discontinuous

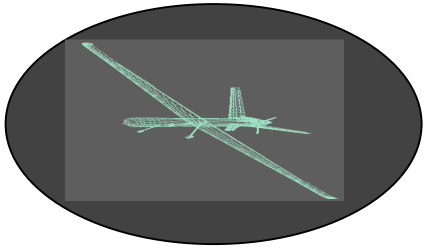


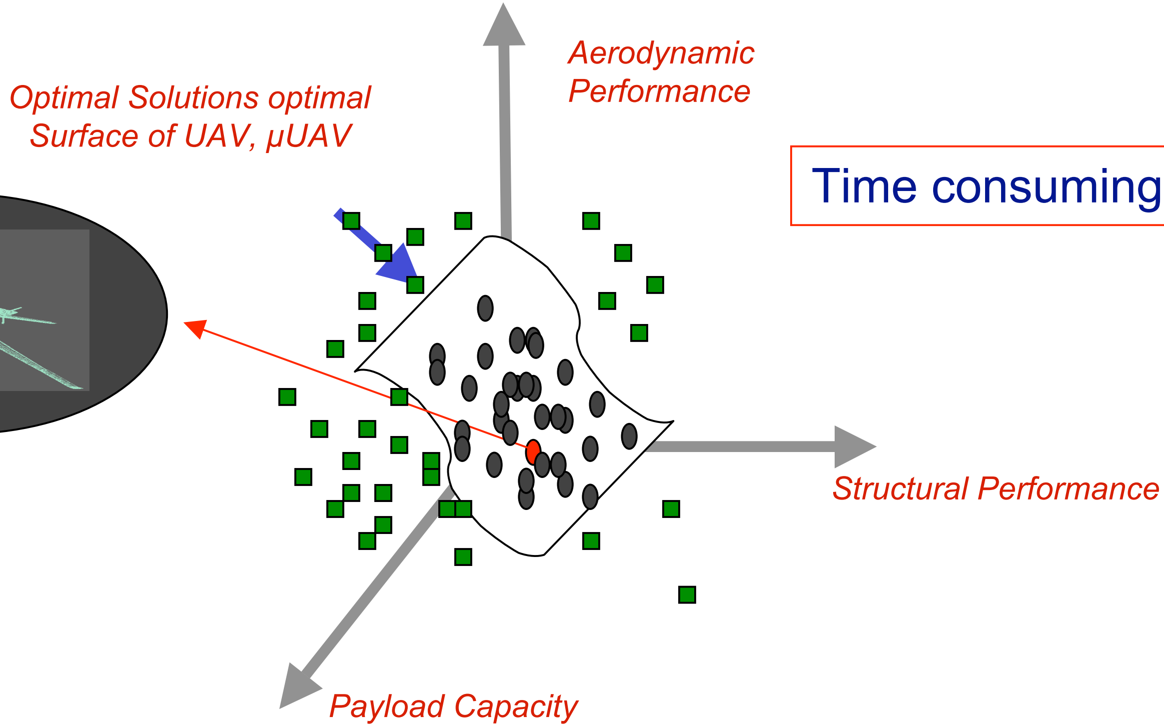


A real aircraft design optimization might exhibit one or several of these characteristics

Multiple Goals Minimise-Maximise

Search spaces for multiphysics solutions are complex

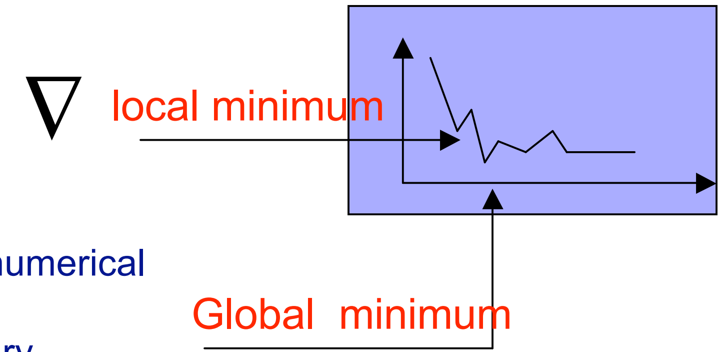




Time consuming process

Traditional Gradient Based Techniques might fail or be trapped in local minima

Advanced Techniques are required



Advanced numerical techniques –Evolutionary Computing

A brief history of GAs, M. Mitchell, 1996

- *50'-60'*: evolution could be used as an optimization tool for engineering problems
- *innovation*: evolve a population of candidates to a given problem, using operators inspired by natural genetic variation and natural selection
- *approach*: evolution inspired algorithms
- *60'*: GAs invented by J. Holland, Univ. of Michigan
- *target:* study the phenomena of adaptation as it occurs in nature
- *How* ? Developing ways how natural adaptation can be implemented into computer systems
- *result*: GAs as an abstraction of biological evolution

Genetic Algorithms (GAs): notations (2)

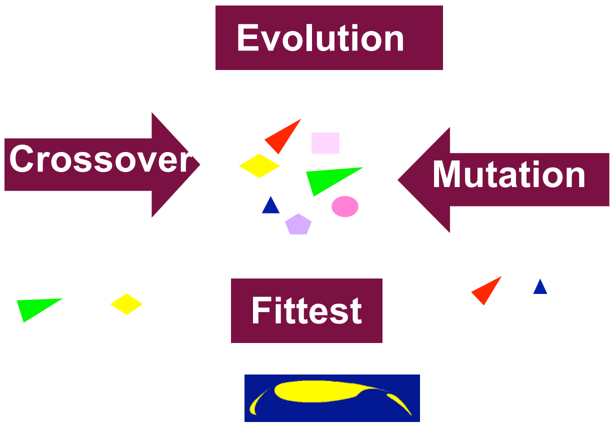
- chromosome: a candidate solution to a problem, encoded as a bit string
- genes: single bit or short blocks that encode a particular element of a candidate solution
- cross over: exchanging material between the parent chromosomes
- mutation: flipping a bit at a randomly chosen locus
- fitness: criteria of function to minimize or maximize
- example in CFD or CEM optimization problems:
  - *population*: a set of airfoils; *chromosome*: an airfoil
  - *genes*: spline coefficient; *parents*: two airfoils
  - *offspring*: two children airfoils; *fitness* : drag or signature
  - *environment*: flow or wave (non) linear PDEs

EVOLUTIONARY ALGORITHMS (GAs, ESs, EAs, MAs,…)

Advanced Optimisation Tools:

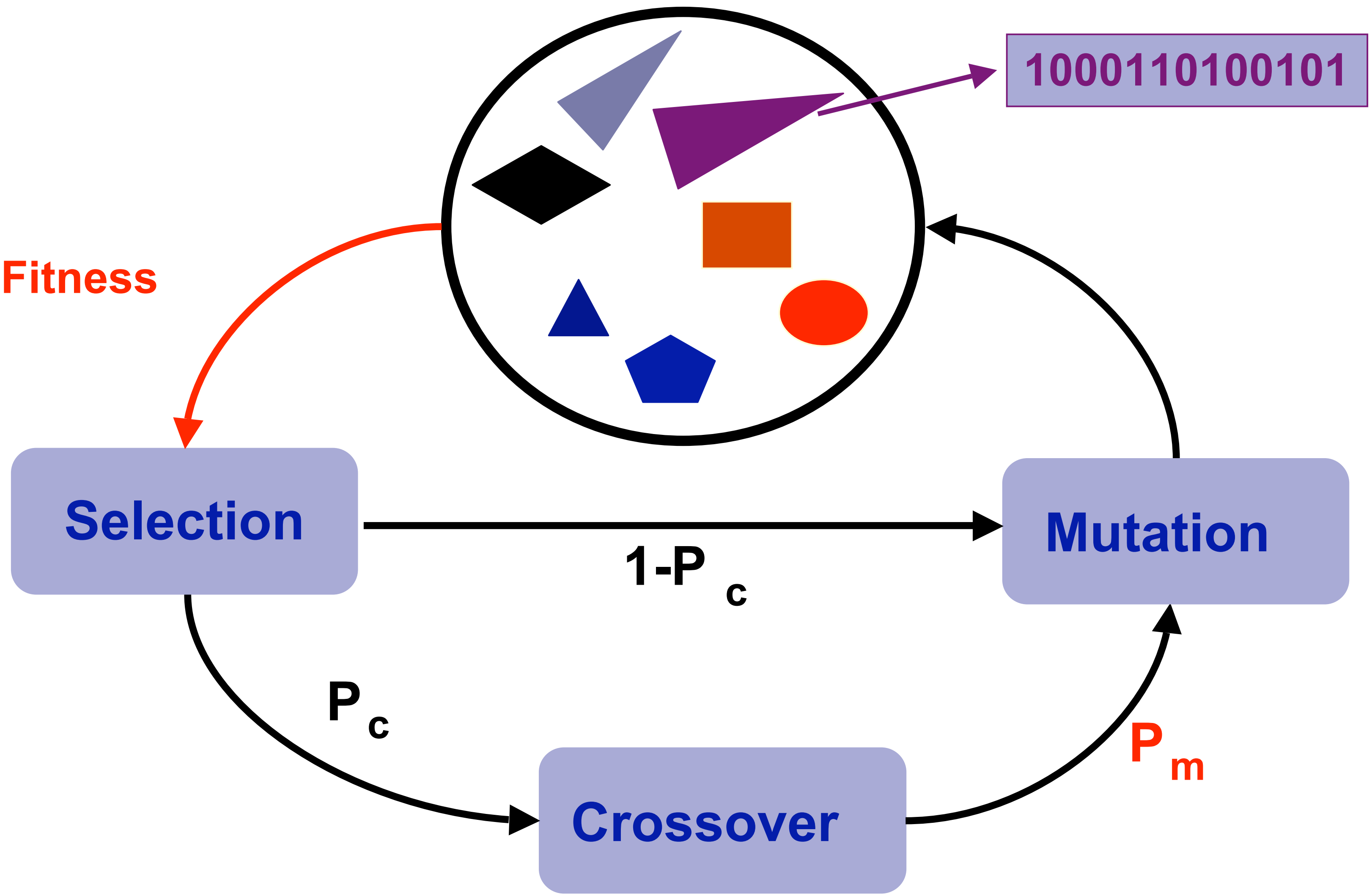
Evolutionary Optimisation

- Good for all of the above
- Easy to parallelize
- Robust towards noise
- Explore larger search spaces
- Good for multi-objective problems



- Based on the Darwinian theory of evolution → populations of individuals evolve and reproduce by means of random mutation and crossover operators and compete in a set environment for survival of the fittest (selection).

GENETIC ALGORITHMS pioneered J. Holland in the 60' with binary coding



GENETIC ALGORITHMS with 3 OPERATORS

- Selection (semi random, semi deterministic) : survival of the fittest (Darwin principle)

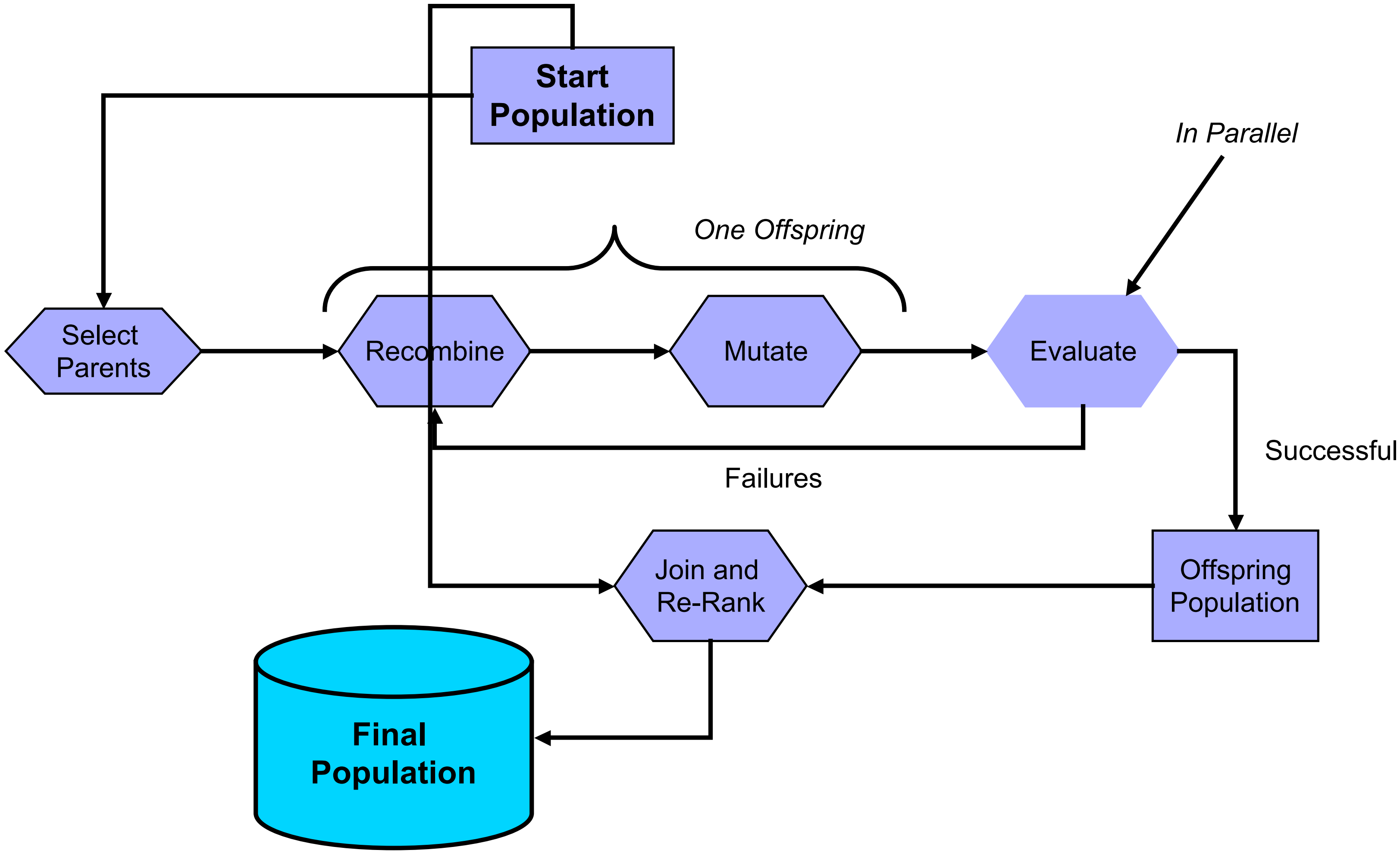
- Cross over (random): Pc (binary coding)

Parents	**Offspring**
100/01110	*100*00000
001/00000	**001***01110*

- Mutation (random): Pm

10001110 → 10001100

EVOLUTIONARY ALGORITHMS (3) : One Generation of the Algorithm...



Genetic Algorithm: parameters

Population size: 30-100 , problem dependent

Cross over rate: Pc= 0.80-0.95

Mutation rate: Pm= 0.001- 0.01

Areas of applications with GAs

- *Optimization* and Machine learning(D. Golberg, 1989)
- Automatic programming (J. Koza, 1992)
- Economics (bidding strategies, economic markets)
- Immune systems (KrishnaKumar,1998)
- Ecology (co evolution)
- Social systems (evolution of social behaviour in insect colonies, cooperation and communication of multi-agents systems)
- Complex adapted systems (Hidden order, J.Holland, 1997)
- …..

How are GAs different from traditional methods ? (D. Goldberg, 1989)

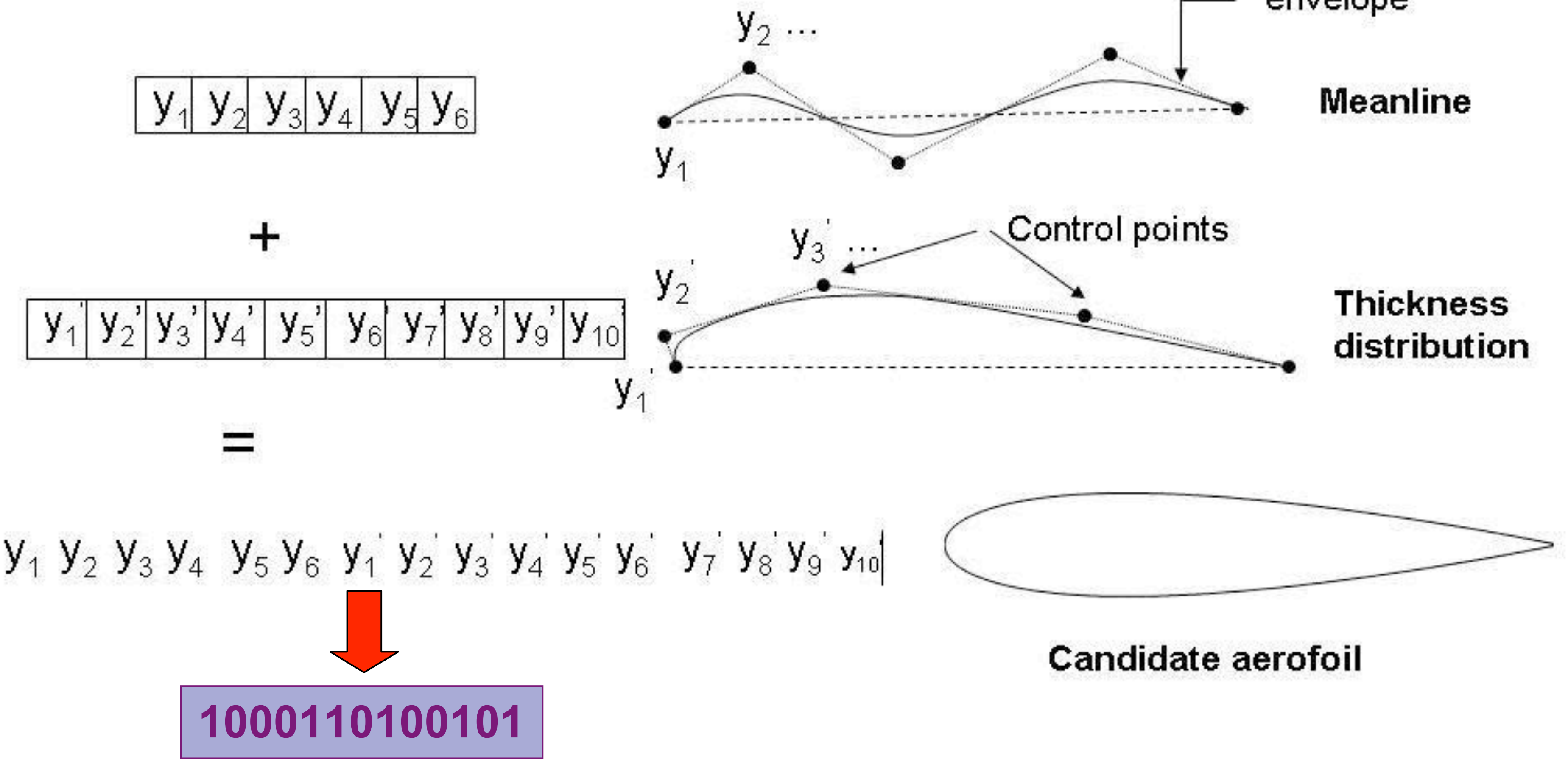
- GAs work with a coding of the parameter set, not the parameters themselves
- GAs search from a population, not a single point
- GAs use payoff (objective function) information, not derivatives or other auxiliary knowledge
- GAs use probalistic transition rules, not deterministic rules
- The central theme of research on GAs has been robustness

GAs Mechanisms: why they work ?

- GAs are *indifferent to problems specifics* (no derivative needed to take a decision!)
- GAs use a coding of decision variables (DNA and adaptation of chromosomes)
- GAs process populations via evolutive generations
- GAs use randomized operators
- Theoretical foundations of GAs rely on a binary string representations of solutions and on the notion of schema
- The schema theorem (J. Holland): *“short, low order, above average schemeta receive exponentially increasing trials in subsequent generation of a GAs”* (Michalewicz, 1992)

EVOLUTIONARY ALGORITHMS : EXAMPLE OF A CHROMOSOME OR INDIVIDUAL

In this example: A chromosome or an individual are the control points (y_i) that define the aerofoil shape



DRAWBACK OF EVOLUTIONARY ALGORITHMS

- Evolution process is time consuming/ high number of function evaluations is required.
  - A typical MDO problem relies on CFD and FEA for aerodynamic and structural analysis.
    - CFD and FEA software are time consuming !

Gradient Based Methods or simple Evolutionary Algorithms are not efficient enough to capture global solutions for MO and MDO Problems- Therefore Advanced Techniques are required

4. MULTI-OBJECTIVE OPTIMISATION (1)

- Aeronautical and aerospace design problems normally require a simultaneous optimisation of conflicting objectives and associated number of constraints.

- They occur when two or more objectives that cannot be combined rationally. For example:

  - Drag at two different values of lift.
  - Drag and thickness.
  - Pitching moment and maximum lift
  - ……

MULTI-OBJECTIVE OPTIMISATION

Different Multi-Objective approaches

- Aggregated Objectives, main drawback is loss of information and a-priori choice of weights.

- Game Theory (von Neumann)
  - Game Strategies
    - -Cooperative Games - Pareto
    - -Competitive Games - Nash
    - -Hierarchical Games - Stackelberg

- Vector Evaluated GA (VEGA) Schaffer,85
- Multi Objective Optimization with GAs K. Deb , 2001

MULTI-OBJECTIVE OPTIMISATION

Maximise/ Minimise	$f_i(x) \quad i = 1 \ldots N$
Subjected to constraints	$g_i(x) = 0 \quad j = 1 \ldots N$ $h_k(x) \leq 0 \quad k = 1 \ldots K$

- $f_i(x)$ → Objective functions, output (e.g. cruise efficiency).
- *x:* vector of design variables, inputs (e.g. aircraft/wing geometry)
- *g(x)* equality constraints and *h(x)* inequality constraints: (e.g. element von Mises stresses); in general these are nonlinear functions of the design variables.

MULTIPLE OBJECTIVE OPTIMIZATION

- Linear Combination of criteria (aggregation)

$$C = \sum_{i=1}^{n} \omega_i \cdot c_i$$

BUT

- Dimensionless number
- Heavy bias from the choice of the weights

- VEGA (Vector-Evaluated GA) **[Schaffer, 85]**
  - bias on the extrema of each objective

GAME STRATEGIES

- Theoretical foundations: Von Neumann

- Applications to Economics and Politics: Von Neuman, Pareto, Nash, Von Stackelberg

- Decentralized optimization methods:

Lions-Bensoussan-Temam in Rairo (1978, G. Marchuk, J.L. Lions, eds)

In this lecture: introduce and use Games strategies in Engineering for solving Multi Objective Optimization Problems

NOTATIONS

- For a game with 2 players, A and B
- For A
  - Objective function $f_A(x,y)$
  - A optimizes vector x
- For B
  - Objective function $f_B(x,y)$
  - B optimizes vector y

$$\overline{A} = \text{set of possible strategies for A}$$

$$\overline{B} = \text{set of possible strategies for B}$$

Pareto Dominance

- Pareto Optimality (minimization, 2 Players A and B). (x^*, y^*) is Pareto optimal if and only if:

$$\forall (x,y) \in \overline{A} \times \overline{B}, \begin{cases} f_A(x^*, y^*) \leq f_A(x,y) \\ f_B(x^*, y^*) \leq f_B(x,y) \end{cases}$$

- Pareto Dominance (for n players (P_1,…,P_n)
  - Player P_i has objective f_i and controls v_i
  - (v_1*,..,v_k*,..,v_n*) dominates (v_1,..,v_k,..,v_n) iff:

$$\begin{cases} \forall i, f_i(x_1^*, \ldots, x_k^*, \ldots, x_n^*) \leq f_i(x_1, \ldots, x_k, \ldots, x_n) \\ \exists i, f_i(x_1^*, \ldots, x_k^*, \ldots, x_n^*) < f_i(x_1, \ldots, x_k, \ldots, x_n) \end{cases}$$

Pareto Front

- Pareto Optimality
  - a strategy $(v_1^*,..,v_k^*,..,v_n^*)$ is Pareto-optimal if it is not dominated

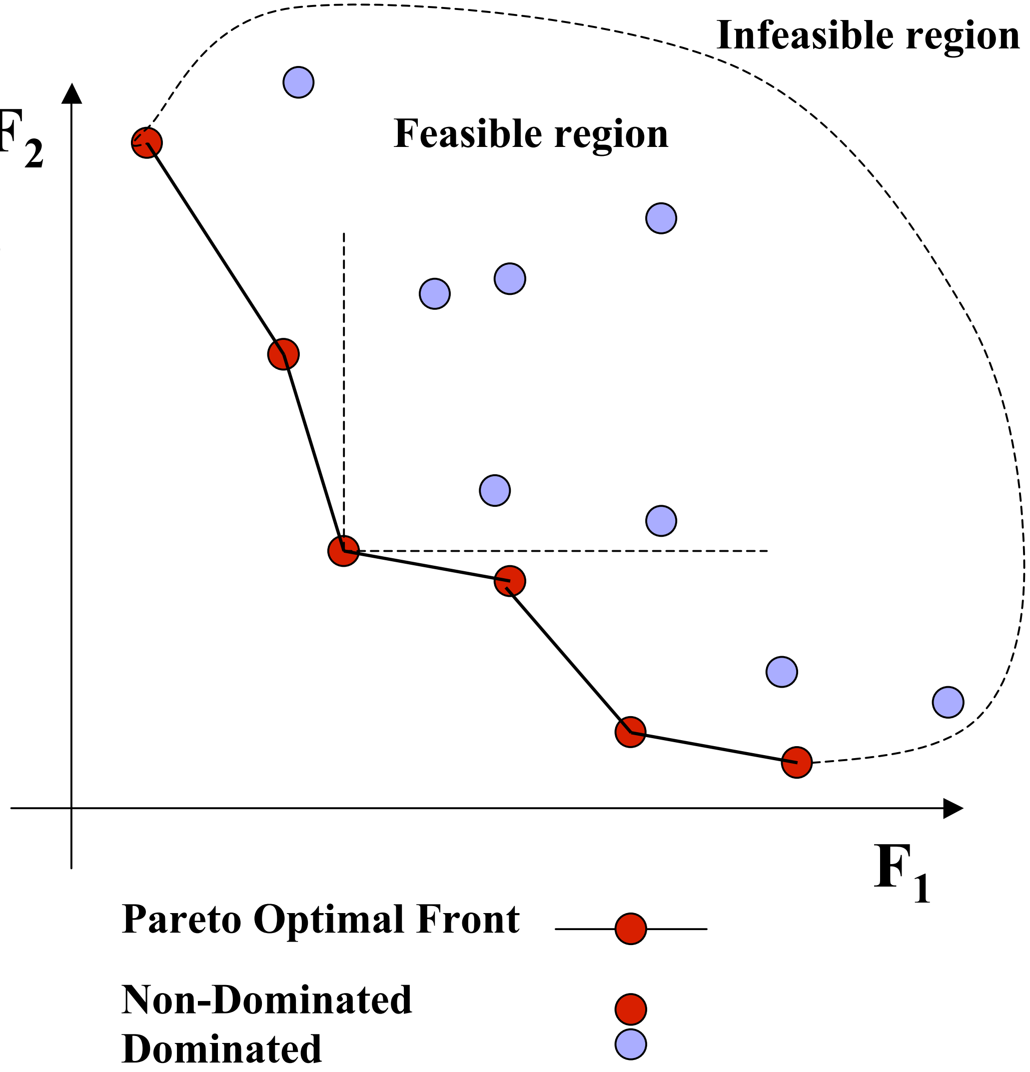
- Pareto Front
  - Set of all NON-DOMINATED strategies

PARETO OPTIMAL SET : DEFINITION

- A set of solutions that are non-dominated w.r.t all others points in the search space, or that they dominate every other solution in the search space except fellow members of the Pareto optimal set.

- EAs work on population based solutions …can find a optimal Pareto set in a single run

- HAPMOEA: Captures Pareto Front, Nash and Stackelberg solutions



Nash Equilibrium

- Competitive symmetric games [Nash, 1951]
- For 2 Players A and B:

$$f_A(\vec{x}^*, \vec{y}^*) = \inf_{x \in \bar{A}} f_A(x, \vec{y}^*)$$

$$f_B(\vec{x}^*, \vec{y}^*) = \inf_{y \in \bar{B}} f_B(\vec{x}^*, y)$$

- For n Players :

$$\forall i, \forall v_i, f_i(\vec{v}_1^{\,*}, \ldots, \vec{v}_{i-1}^{\,*}, \vec{v}_i^{\,*}, \vec{v}_{i+1}^{\,*}, \ldots, \vec{v}_n^{\,*}) \leq f_i(\vec{v}_1^{\,*}, \ldots, \vec{v}_{i-1}^{\,*}, \vec{v}_i, \vec{v}_{i+1}^{\,*}, \ldots, \vec{v}_n^{\,*})$$

« When no player can further improve his criterion, the system has reached a state of equilibrium named Nash equilibrium”

How to find a Nash Equilibrium ?

- Let D_A be the rational reaction set for A, and D_B the rational reaction set for B.

$$\begin{cases} D_A = \left\{(x^*,y) \in \bar{A} \times \bar{B}\right\} \text{ such that } f_A(x^*,y) \le f_A(x,y) \\ D_B = \left\{(x,y^*) \in \bar{A} \times \bar{B}\right\} \text{ such that } f_B(x,y^*) \le f_B(x,y) \end{cases}$$

- Which can be formulated:

$$\begin{cases} D_A = \left\{x, \dfrac{\partial f_A(x,y)}{\partial x} = 0\right\} \\ D_B = \left\{y, \dfrac{\partial f_B(x,y)}{\partial y} = 0\right\} \end{cases}$$

- Nash Equilibrium

A strategy pair $(x^*,y^*) \in D_A \cap D_B$ is a Nash Equilibrium !

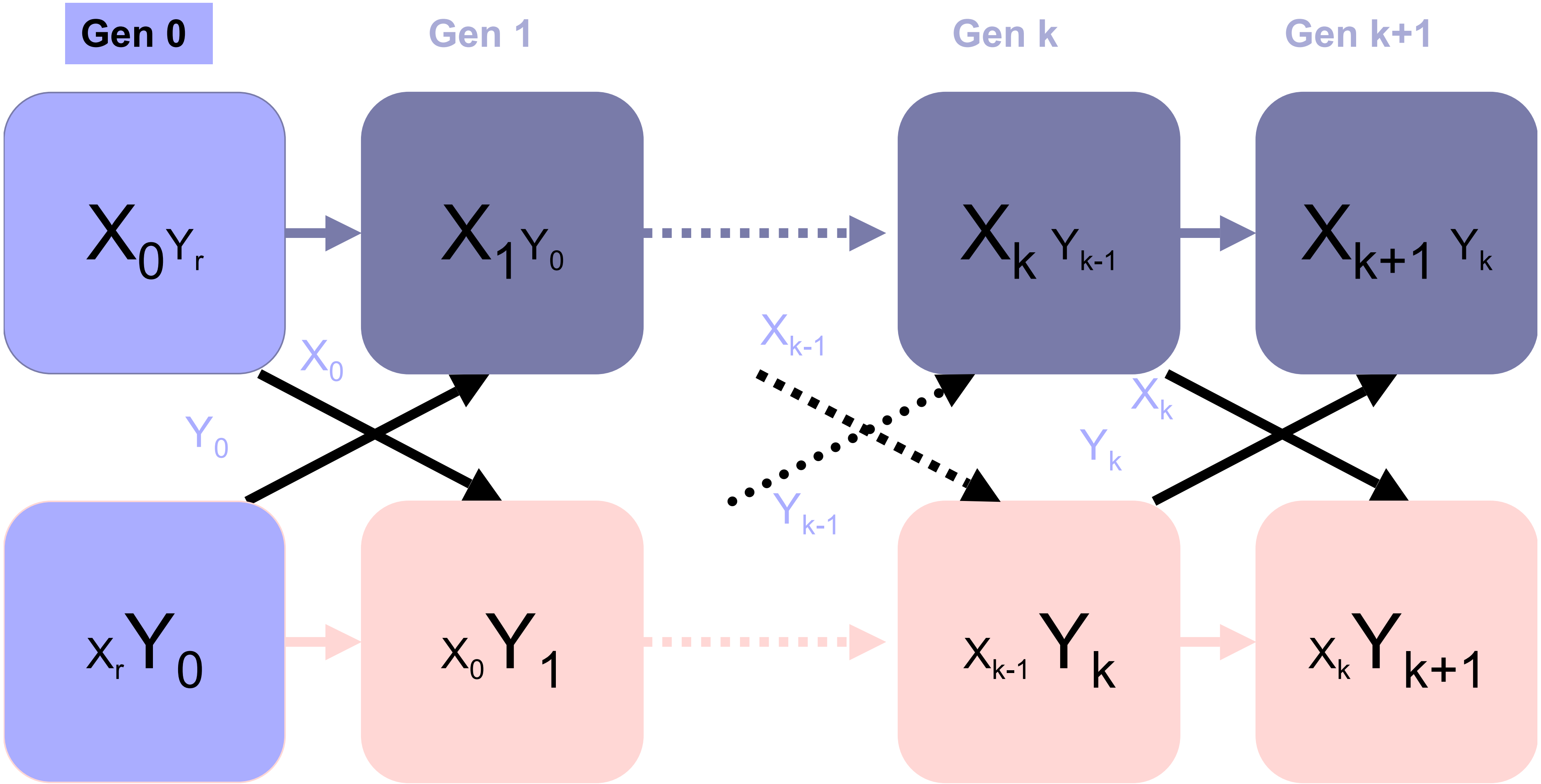
Nash GAs

[Sefrioui & Periaux, 97]

Optimizing **X Y**

Player 1 **Player 2**

Player 1 = Population 1



Player 2 = Population 2

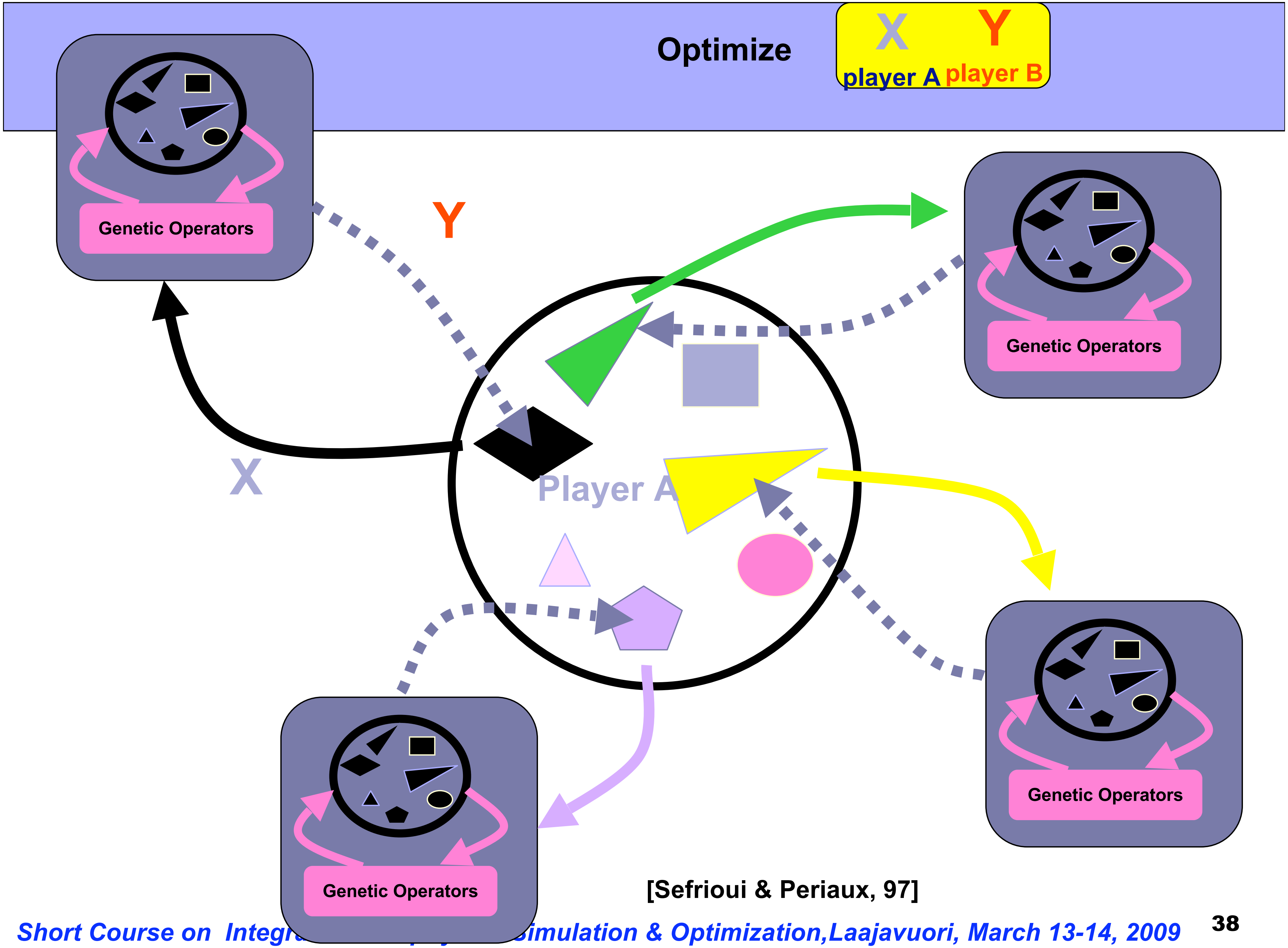
Stackelberg Games

- Hierarchical strategies

- Stackelberg game, A leader
  - Stackelberg game with A leader and B follower :
    minimize $f_A(x,y)$ with y in D_B

- Stackelberg game, B leader
  - Stackelberg game with B leader and A follower :

$$\min_{x \in D_A, y \in \bar{B}} f_B(x,y)$$

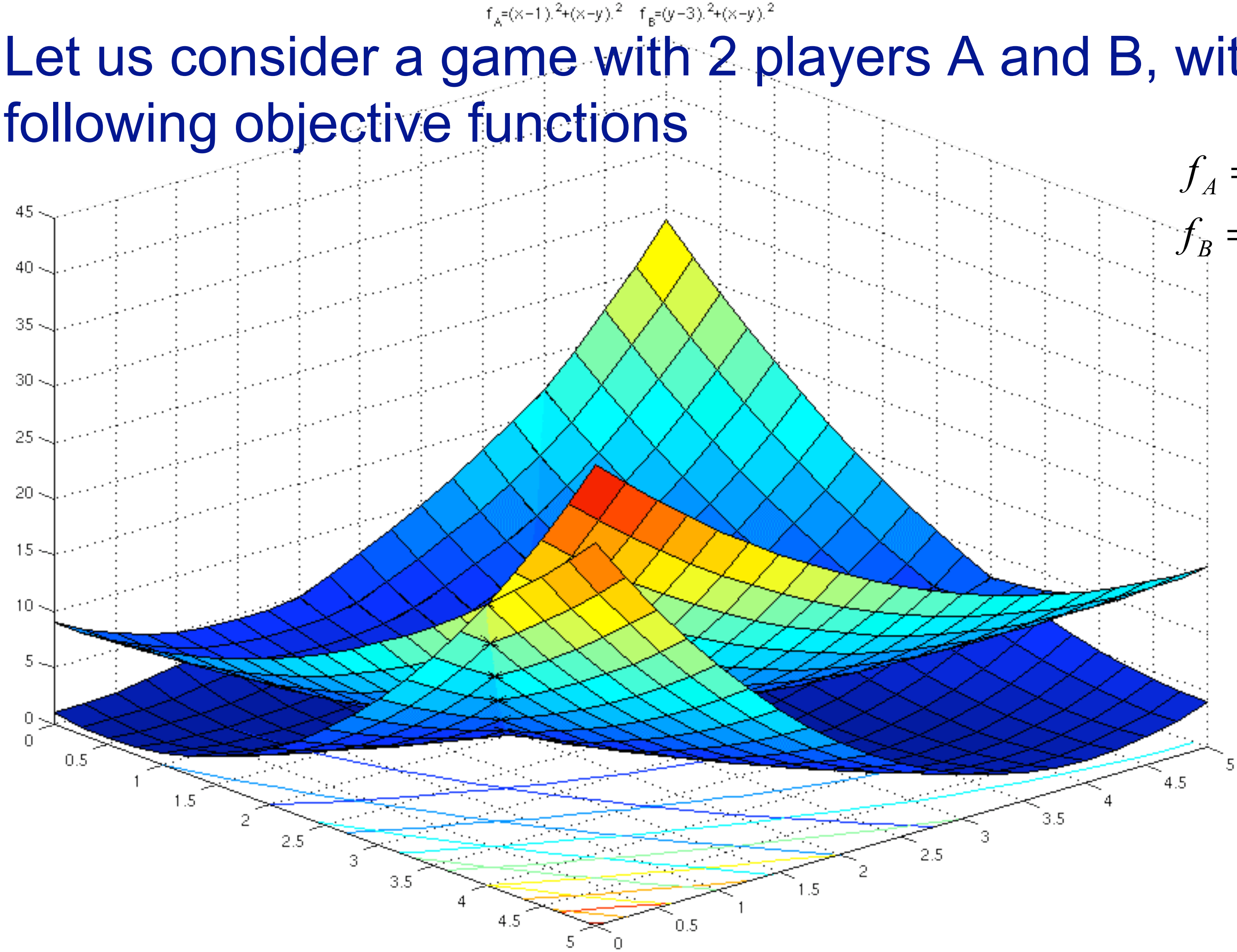


[Sefrioui & Periaux, 97]

Example:

- Let us consider a game with 2 players A and B, with the following objective functions

$$f_A = (x-1)^2 + (x-y)^2$$
$$f_B = (y-3)^2 + (x-y)^2$$



Pareto : Analytic Resolution

- Let us consider the parametric function

$$f_p(x,y) = \lambda \cdot ((x-1)^2 + (x-y)^2) + (1-\lambda) \cdot ((y-3)^2 + (x-y)^2)$$

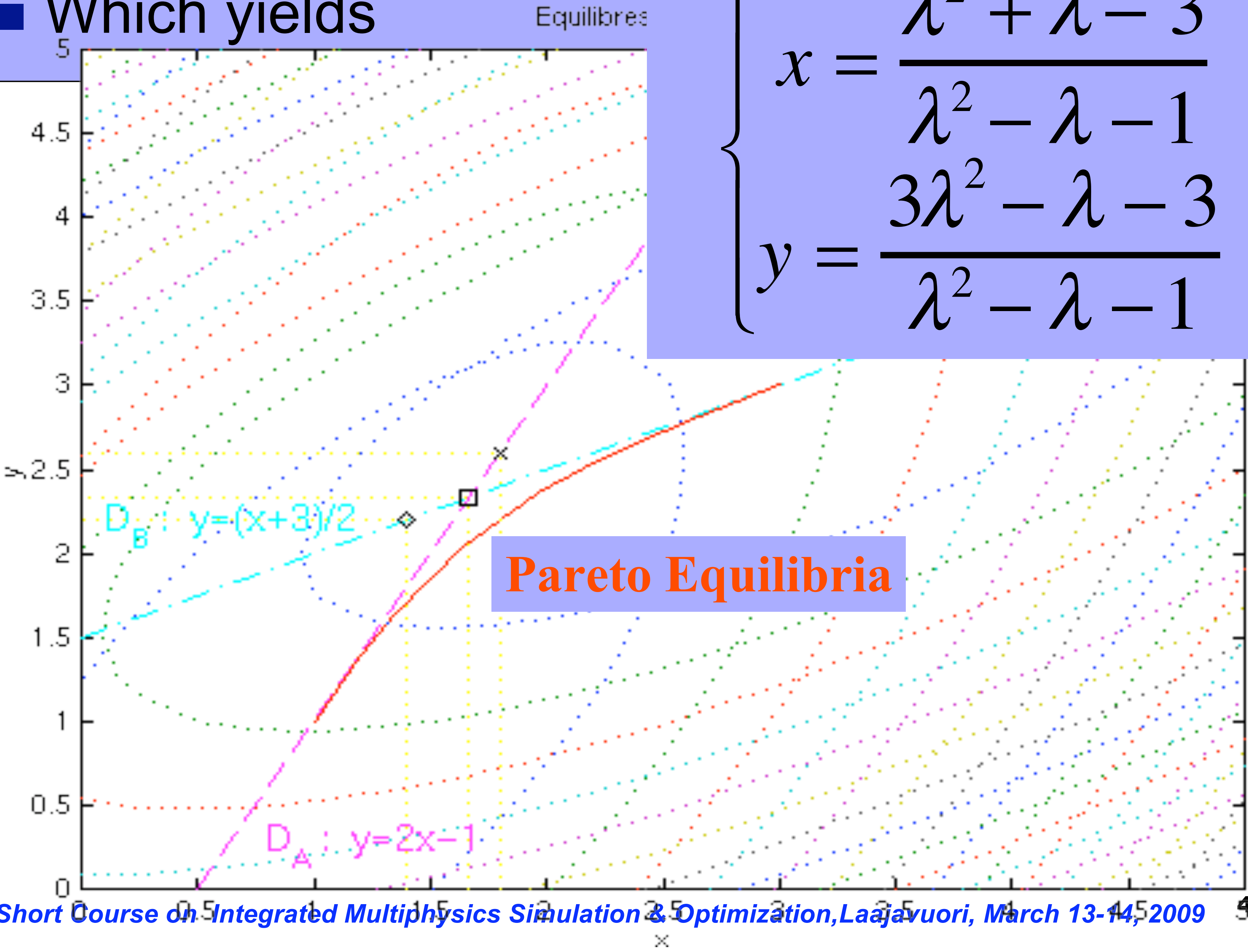
$$\text{with } 0 \le \lambda \le 1$$

- The Pareto equilibria are the solution of

$$\begin{cases} \dfrac{\partial f_p(x,y)}{\partial x} = 0 \\ \dfrac{\partial f_p(x,y)}{\partial y} = 0 \end{cases} \Rightarrow \begin{cases} 2\lambda x - 2\lambda + 2x - 2y = 0 \\ -2\lambda y + 4y - 6 - 2x + 6\lambda = 0 \end{cases}$$

- Which yields

$$\begin{cases} x = \dfrac{\lambda^2 + \lambda - 3}{\lambda^2 - \lambda - 1} \\ y = \dfrac{3\lambda^2 - \lambda - 3}{\lambda^2 - \lambda - 1} \end{cases}$$



Stackelberg : Analytic

- Stackelberg, A leader
  - Minimize $f_A(x,y)$ on D_B.
  - D_B is built by solving

$$\frac{\partial f_B(x,y)}{\partial y} = 0$$

$$\frac{\partial f_B(x,y)}{\partial y} = 0 \Leftrightarrow 2(y-3) - 2(x-y) = 0 \Leftrightarrow y = \frac{x+3}{2}$$

  - D_B is the line $y = \dfrac{x+3}{2}$
  - The problem consists now in solving

$$\frac{\partial f_A\left(x, \frac{x+3}{2}\right)}{\partial x} = 0$$

Stackelberg : Analytic (2)

$$\frac{\partial f_A\left(x, \frac{x+3}{2}\right)}{\partial x} = 0 \Leftrightarrow \frac{\partial((x-1)^2 + (x - \frac{x+3}{2})^2)}{\partial x} = 0$$

$$\Leftrightarrow 2(x-1) + (\frac{x}{2} - \frac{3}{2}) = 0 \Leftrightarrow x = \frac{7}{5}$$

- y is then: $y = \frac{x+3}{2} = \frac{\frac{7}{5}+3}{2} = \frac{22}{10}$

- The first Stackelberg equilibrium S_A is the point : $\begin{pmatrix} \frac{7}{5} \\ \frac{22}{10} \end{pmatrix} = \begin{pmatrix} 1.4 \\ 2.2 \end{pmatrix}$

Stackelberg : B leader

- Stackelberg, B leader and A follower
  - Minimize $f_B(x,y)$ on D_A.
  - D_A is built by solving $\frac{\partial f_A(x,y)}{\partial x} = 0$

$$\frac{\partial f_A(x,y)}{\partial x} = 0 \Leftrightarrow 2(x-1) + 2(x-y) = 0 \Leftrightarrow y = 2x-1$$

  - D_A is the line $y = 2x-1$
  - The problem consists now in solving

$$\frac{\partial f_B\left(\frac{y+1}{2}, y\right)}{\partial y} = 0$$

Stackelberg : B leader (2)

$$\frac{\partial f_B\left(\frac{y+1}{2},y\right)}{\partial y}=0 \Leftrightarrow \frac{\partial((y-3)^2+(\frac{y+1}{2}-y)^2)}{\partial y}=0$$

$$\Leftrightarrow 2(y-3)-(\frac{1-y}{2})=0 \Leftrightarrow y=\frac{13}{5}$$

- x is then: $x=\frac{y+1}{2}=\frac{\frac{13}{5}+1}{2}=\frac{18}{10}$
- The second Stackelberg equilibrium S_B is the point : $\begin{pmatrix}\frac{18}{10}\\ \frac{13}{5}\end{pmatrix}=\begin{pmatrix}1.8\\ 2.6\end{pmatrix}$

Nash : Analytic

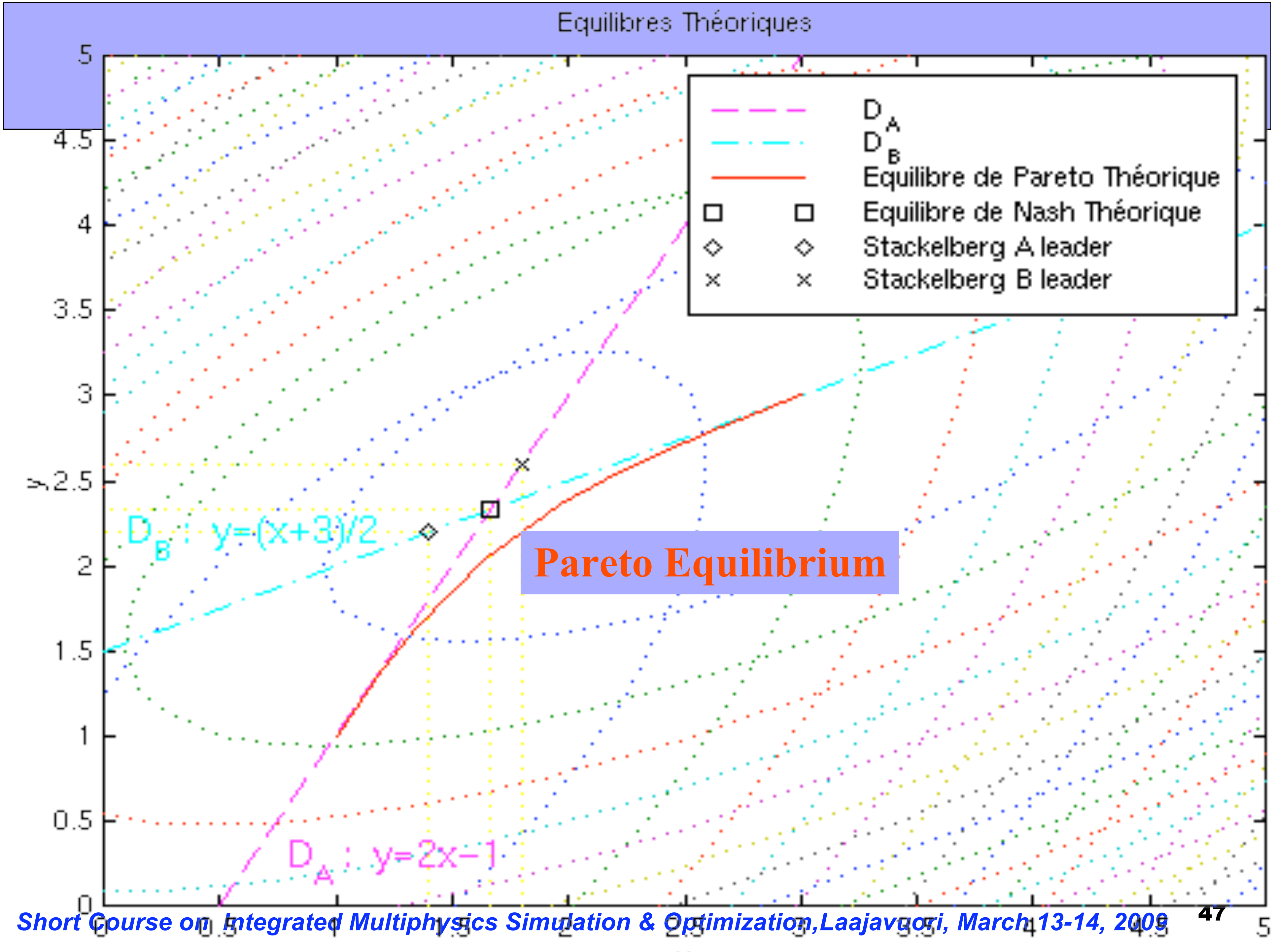
- The Nash Equilibrium is the intersection of the two rational reaction sets D_A and D_B. Finding the Nash Equilibrium consists in solving:

$$\begin{cases} y = 2x - 1 \\ y = \dfrac{x+3}{2} \end{cases}$$

$$\begin{cases} y = 2x - 1 \\ y = \dfrac{x+3}{2} \end{cases} \Leftrightarrow \begin{cases} y = 2x - 1 \\ 3y = 7 \end{cases} \Leftrightarrow \begin{cases} x = \dfrac{5}{3} \\ y = \dfrac{7}{3} \end{cases}$$

  - The Nash Equilibrium E_N is the point

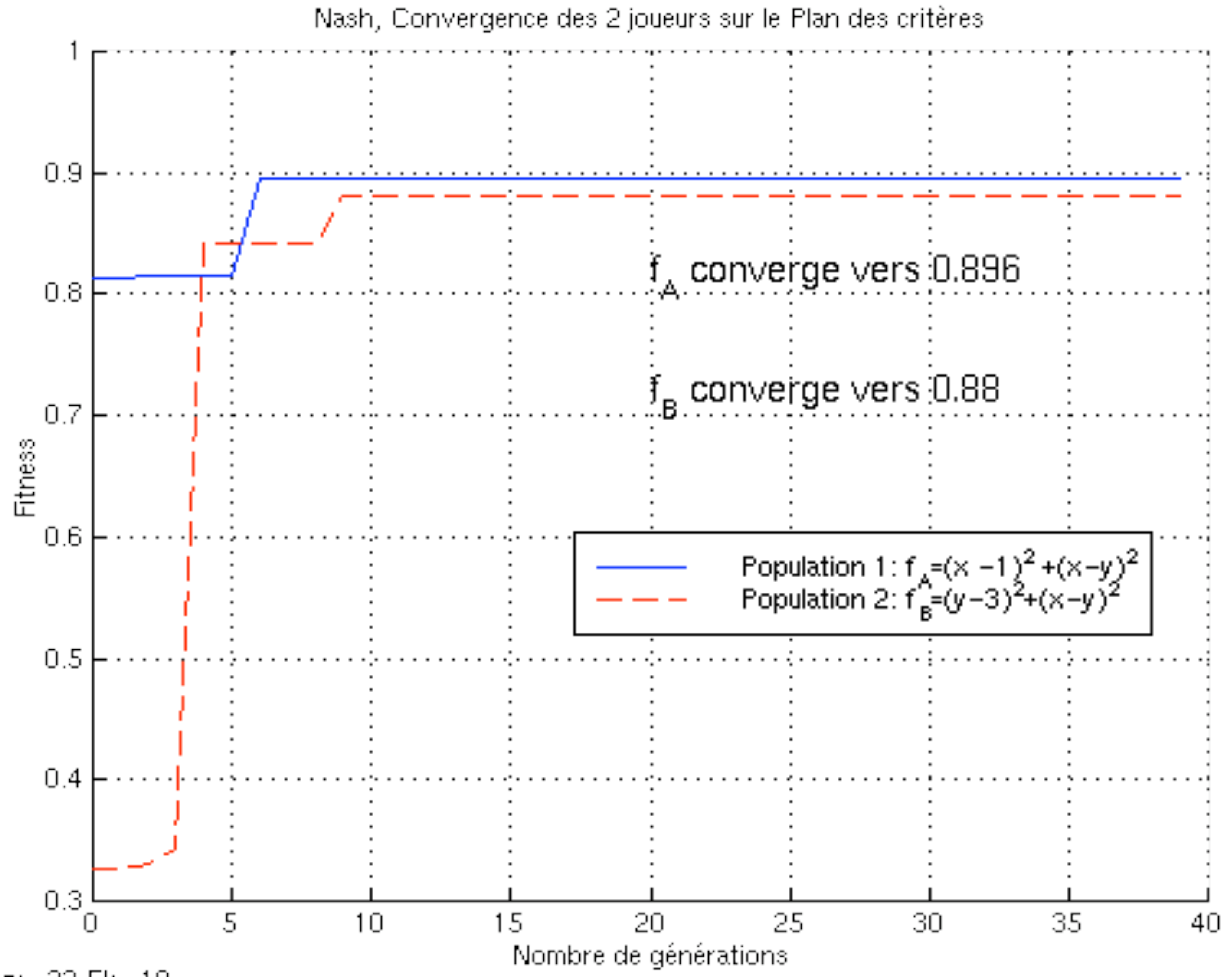
$$\begin{pmatrix} \dfrac{5}{3} \\ \dfrac{7}{3} \end{pmatrix} = \begin{pmatrix} 1.66 \\ 2.33 \end{pmatrix}$$


Equilibres Théoriques
D_A
D_B
Equilibre de Pareto Théorique
Equilibre de Nash Théorique
Stackelberg A leader
Stackelberg B leader
D_B : y=(x+3)/2
D_A : y=2x-1
Pareto Equilibrium

Optimization results with GAs

- Try to optimize the function f_A and f_B with the optimization tools presented earlier
  - With a Pareto/ GA game
  - With a Nash/ GA game
  - With a Stackelberg/ GA game

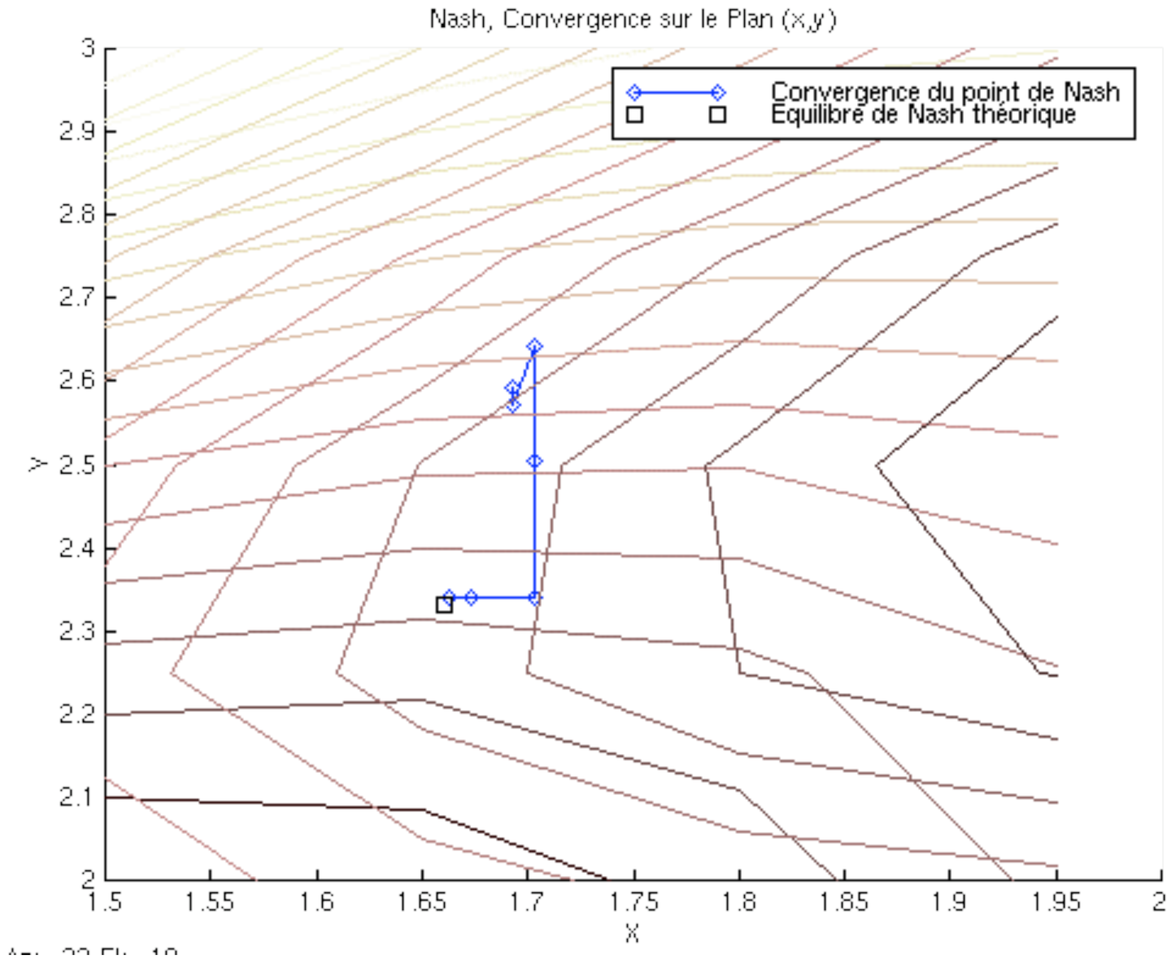
Nash GA : convergence



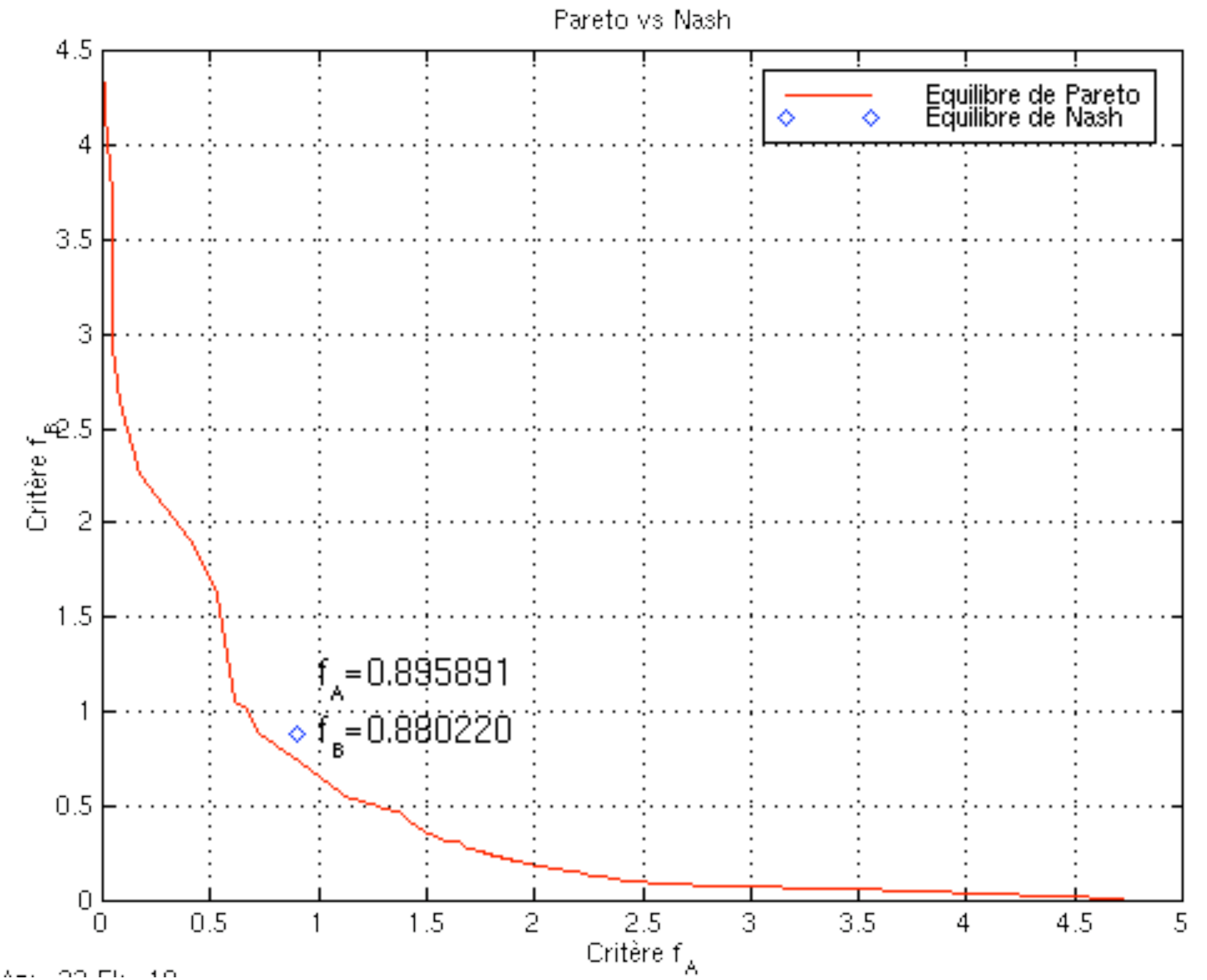
- f_A converges towards 0.896 and f_B towards 0.88
- Both those are the values on the objective plane!
- And we can check that

$$f_A(\frac{5}{3},\frac{7}{3}) = 0.896 \ \text{ and } \ f_B(\frac{5}{3},\frac{7}{3}) = 0.88$$

- So the Nash GA finds the theoretic Nash Equilibrium
- Specifics
  - 2 populations, each of size 30
  - P_c=0.95 P_m=0.01
  - Exchange frequency : every generation
  - (x,y) in [-5,5]x[-5,5]

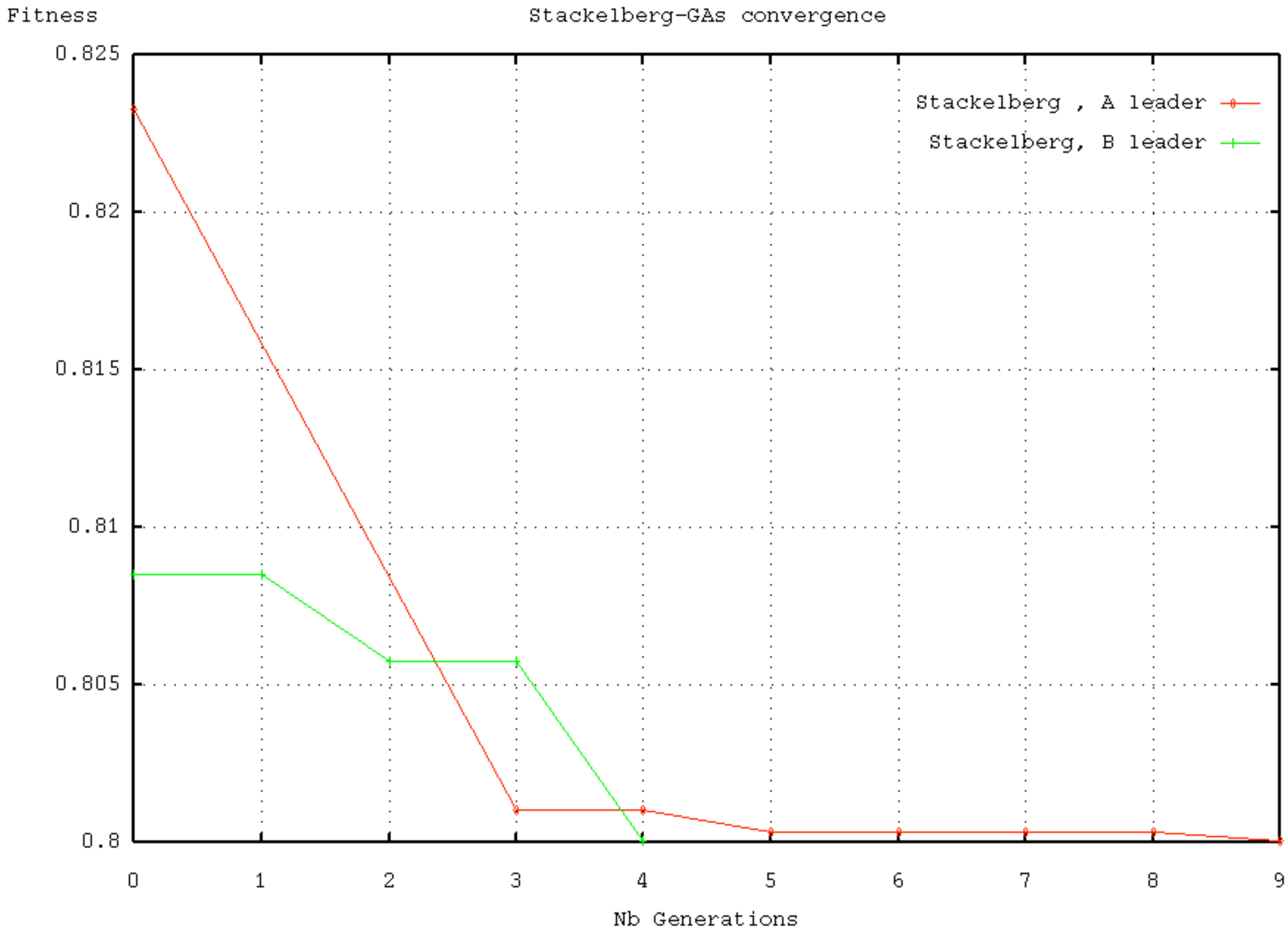

Nash, Convergence sur le Plan (x,y)
Convergence du point de Nash
Equilibre de Nash théorique
y
X
3
2.9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2
1.5
1.55
1.6
1.65
1.7
1.75
1.8
1.85
1.9
1.95
2

Pareto GA



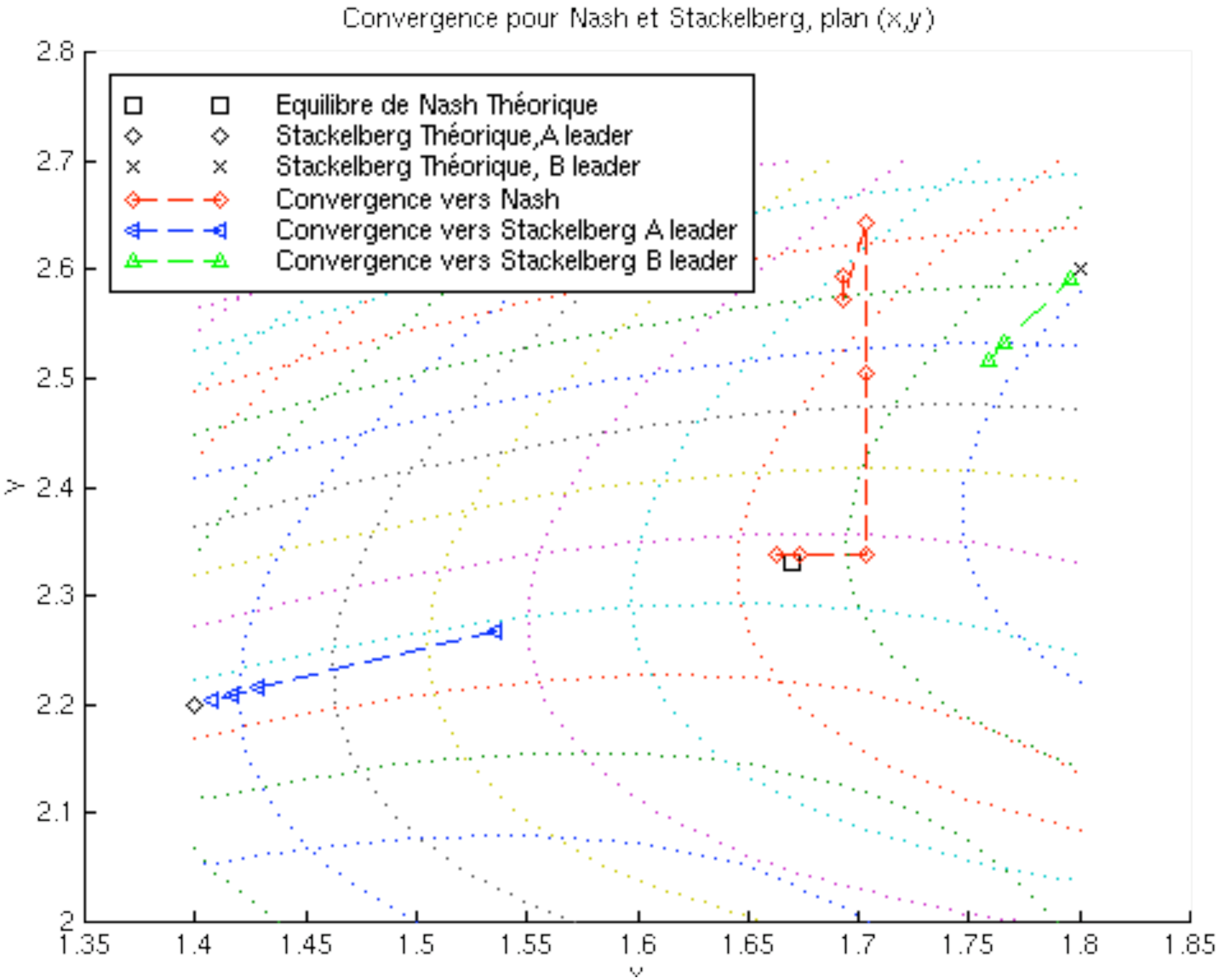
Pareto vs Nash
Equilibre de Pareto
Equilibre de Nash
f_A=0.895891
f_B=0.880220
Critère f_B
Critère f_A
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

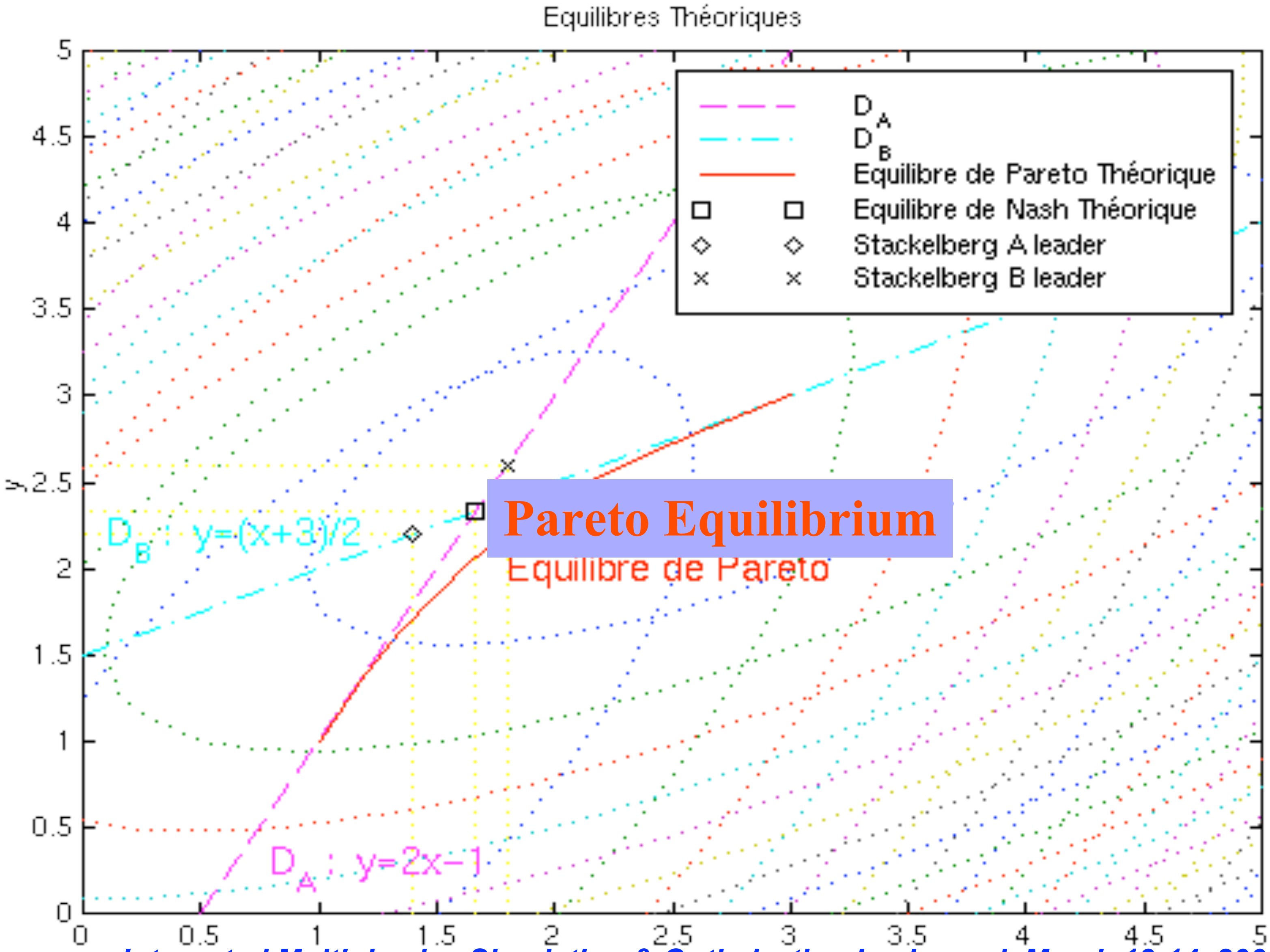
Stackelberg GA : convergence



- In both cases (with either A or B leaders), the algorithms converges towards 0.8. But in the objective plane.
- In the plane (x,y), we can see that the first game converges towards (1.4,2.2) and that the second game converges towards (1.8,2.6)

Converged game solutions for GAs vs analytical approaches



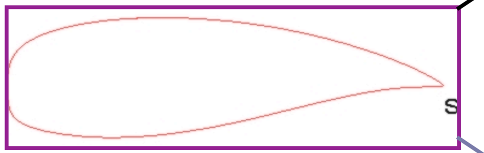

Equilibres Théoriques
Pareto Equilibrium
Equilibre de Pareto
D_B : y=(x+3)/2
D_A : y=2x-1
D_A
D_B
Equilibre de Pareto Théorique
Equilibre de Nash Théorique
Stackelberg A leader
Stackelberg B leader
y
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

MULTI-OBJECTIVE DESIGN with games
Tang Zhili et al, 2004)

Two-objective Inverse Design in Aerodynamics

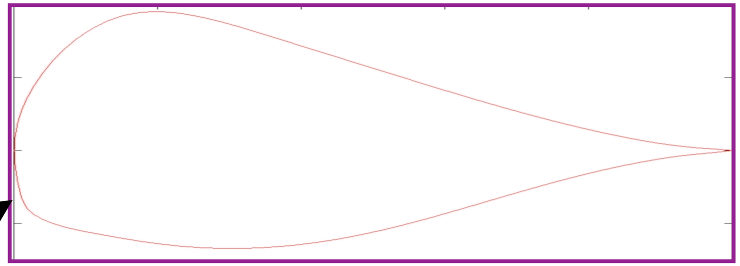
The optimization problem:

Optimization problem :
reconstructing two different
pressure distributions



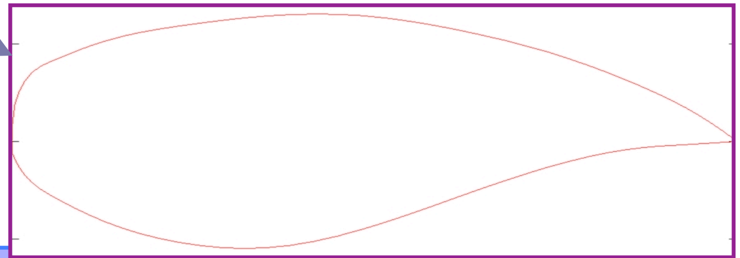
$M_a = 0.2 \qquad \alpha = 10.8^0$

$$\min f_1 = \int_{\Gamma_c} \left(p(w) - p_{sub}\right)^2 ds$$



$M_a = 0.77 \qquad \alpha = 1^0$

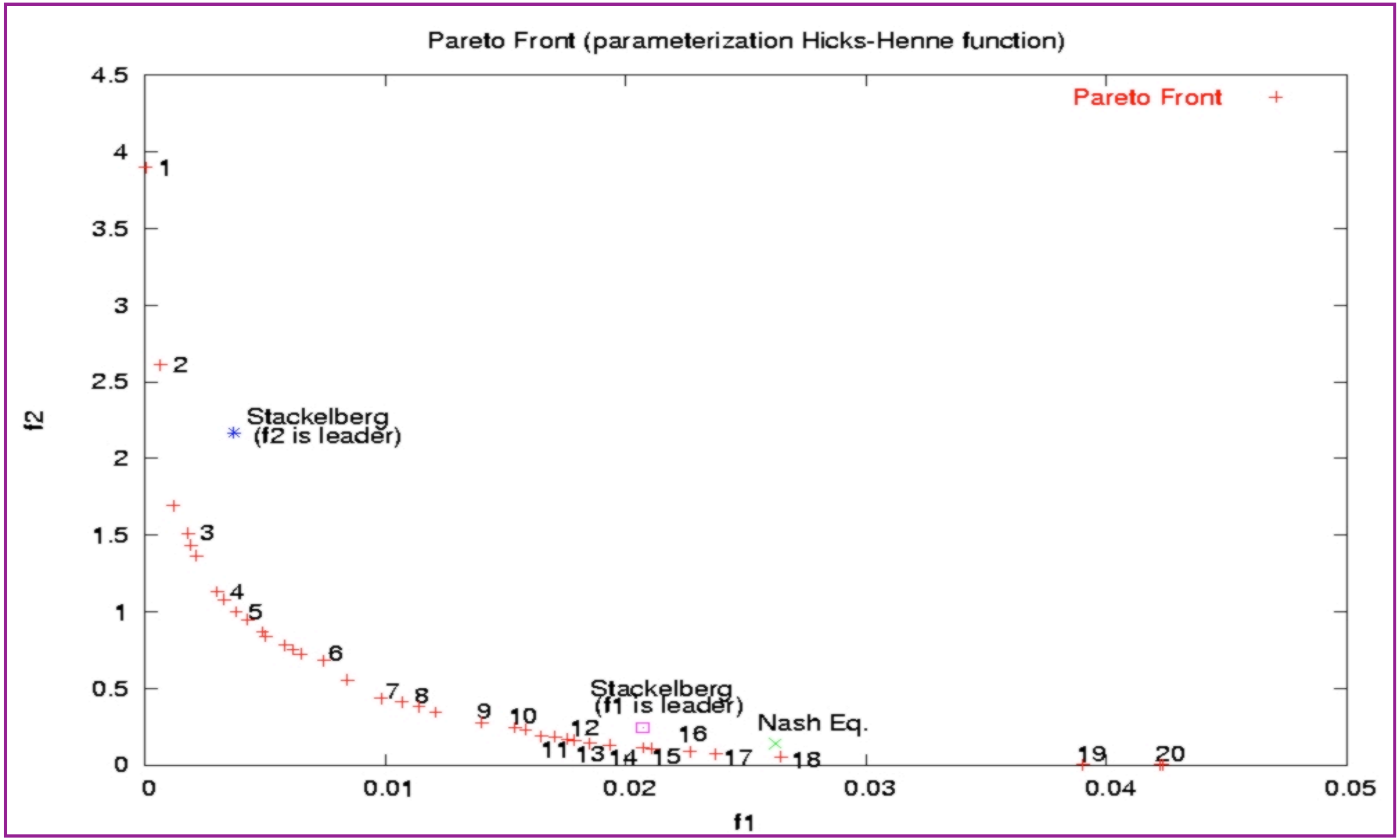
$$\min f_2 = \int_{\Gamma_c} \left(p(w) - p_{tran}\right)^2 ds$$



Problem: find all the profiles existing between the low-drag profile and the high lift profile

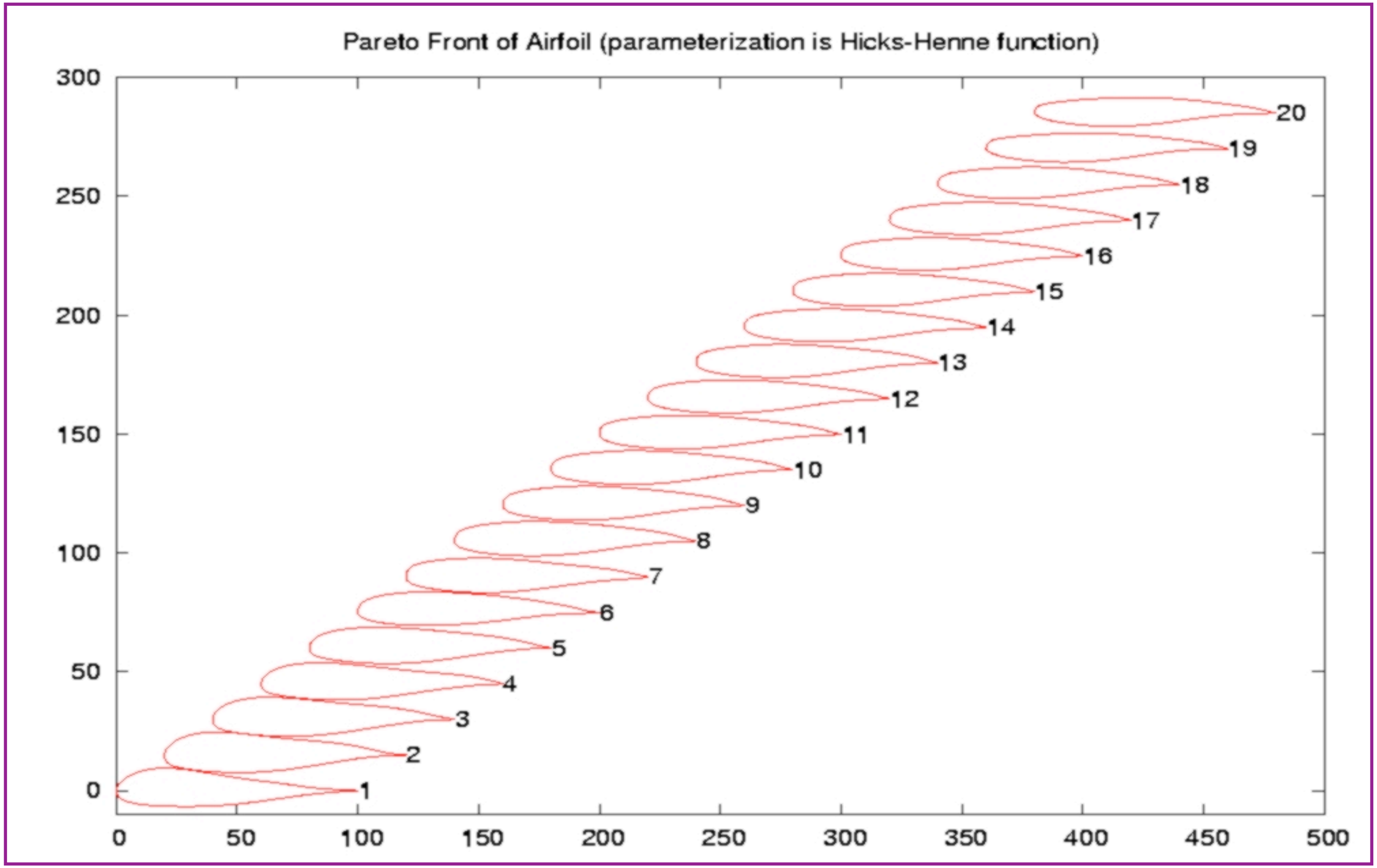
MULTI-OBJECTIVE DESIGN with gradient method

Pareto-front, Stackelberg points,Nash equilibrium(Parameterization with Hicks-Henne functions)



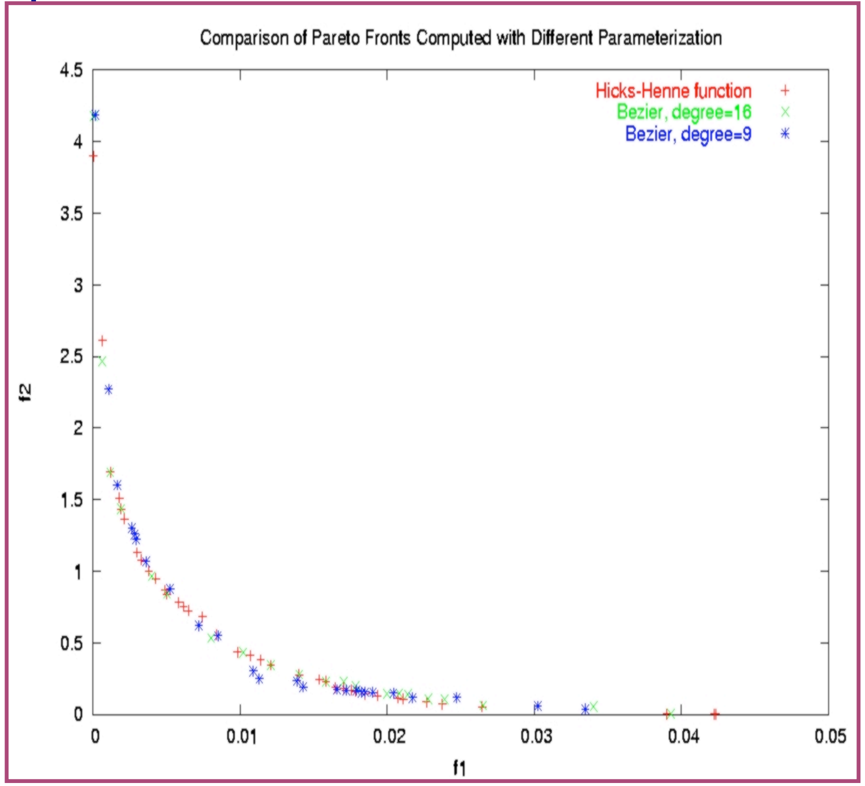
MULTI-OBJECTIVE DESIGN: Pareto solution set

Parameterization with Hicks-Henne functions)

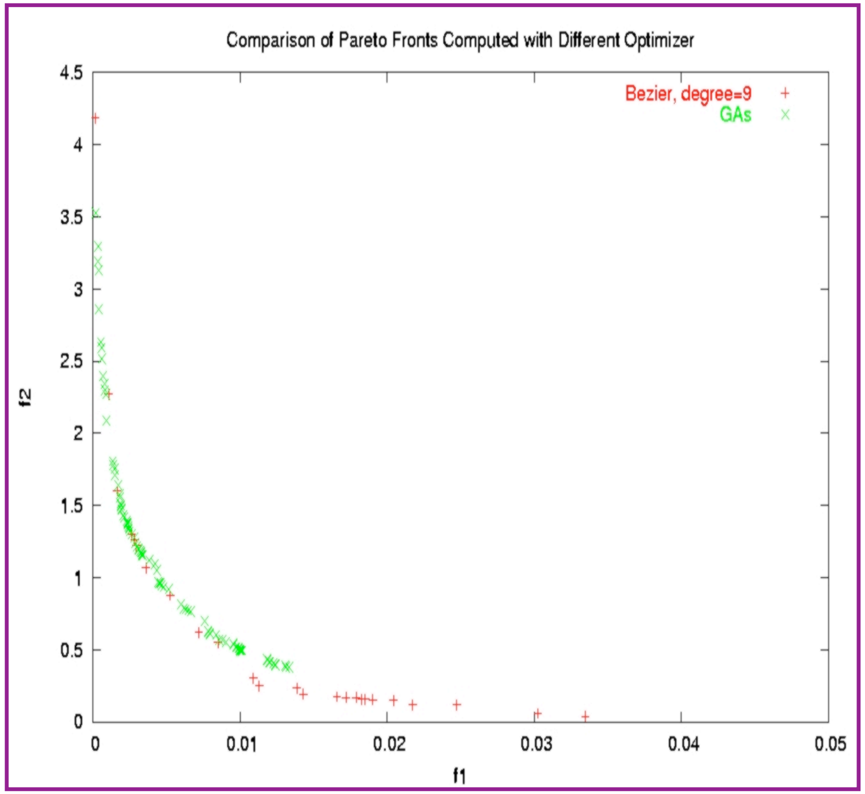


MULTI-OBJECTIVE DESIGN: Comparisons

Comparison of pareto-fronts computed by different parameterization

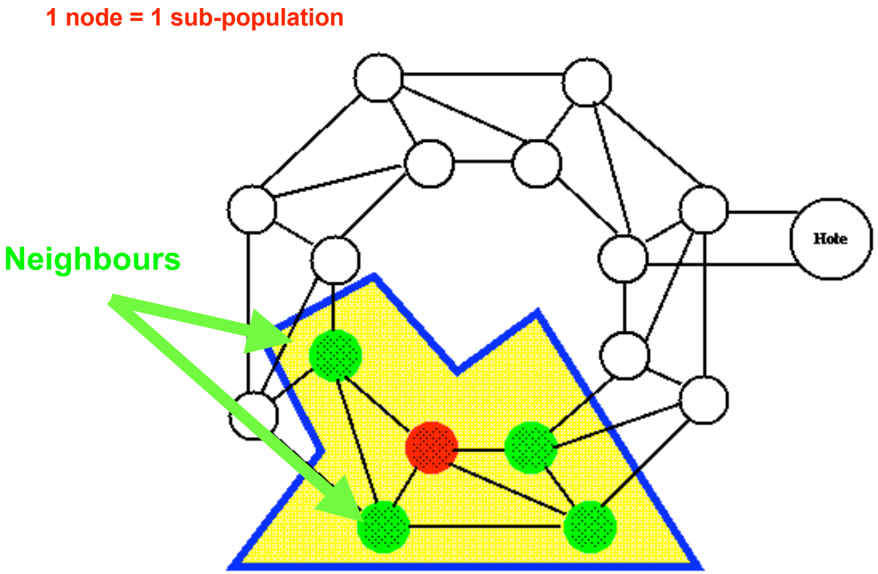


Comparison of pareto-fronts computed by GAs and Deterministic method



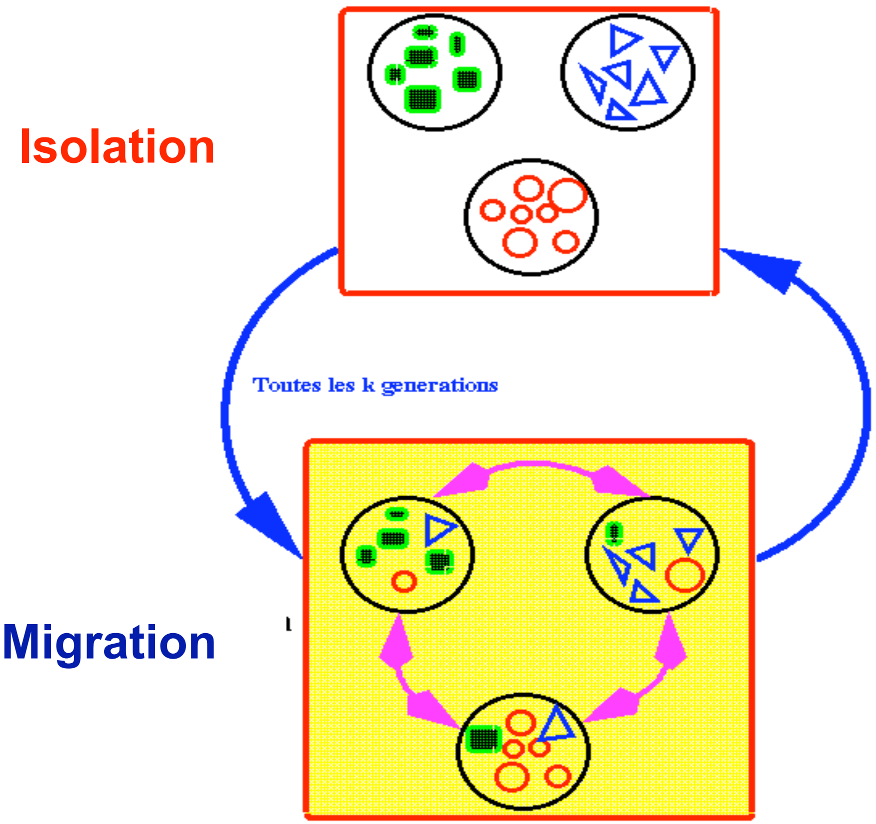
6. Parallel GAs mechanisms

- PGAs : a particular instance of GAs
  - sub-population (H.Muhlenbein, 1989)
  - network of interconnected sub-populations (*Island Model*)
  - smaller sub-populations versus a single large one



Parallel GAs mechanisms : a road map to robustness ! (M. Sefrioui & JP, 1996)

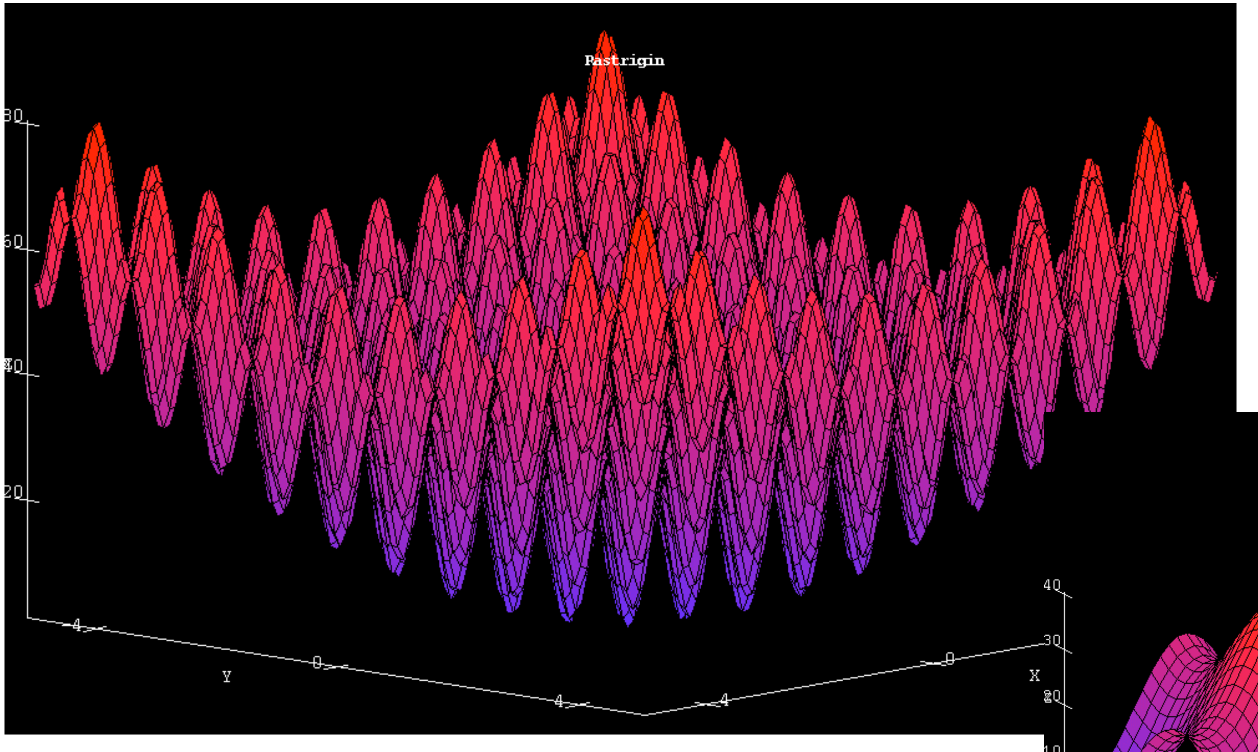
- Isolation and Migration
  - sub-populations evolve independently for a given period of time (epoch)
  - after each epoch, migration between sub-populations before isolation resumes
  - promising solutions shared by sub-populations via their neighbours

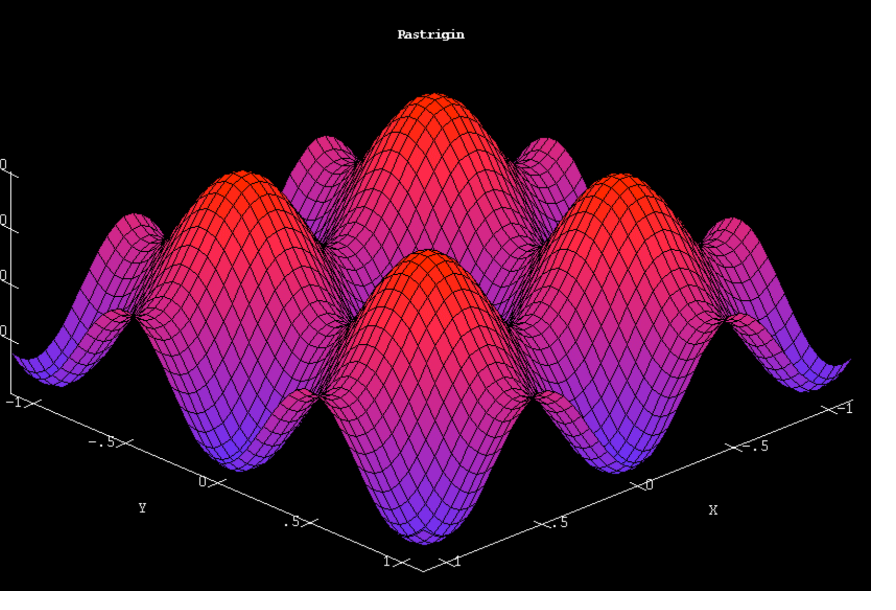


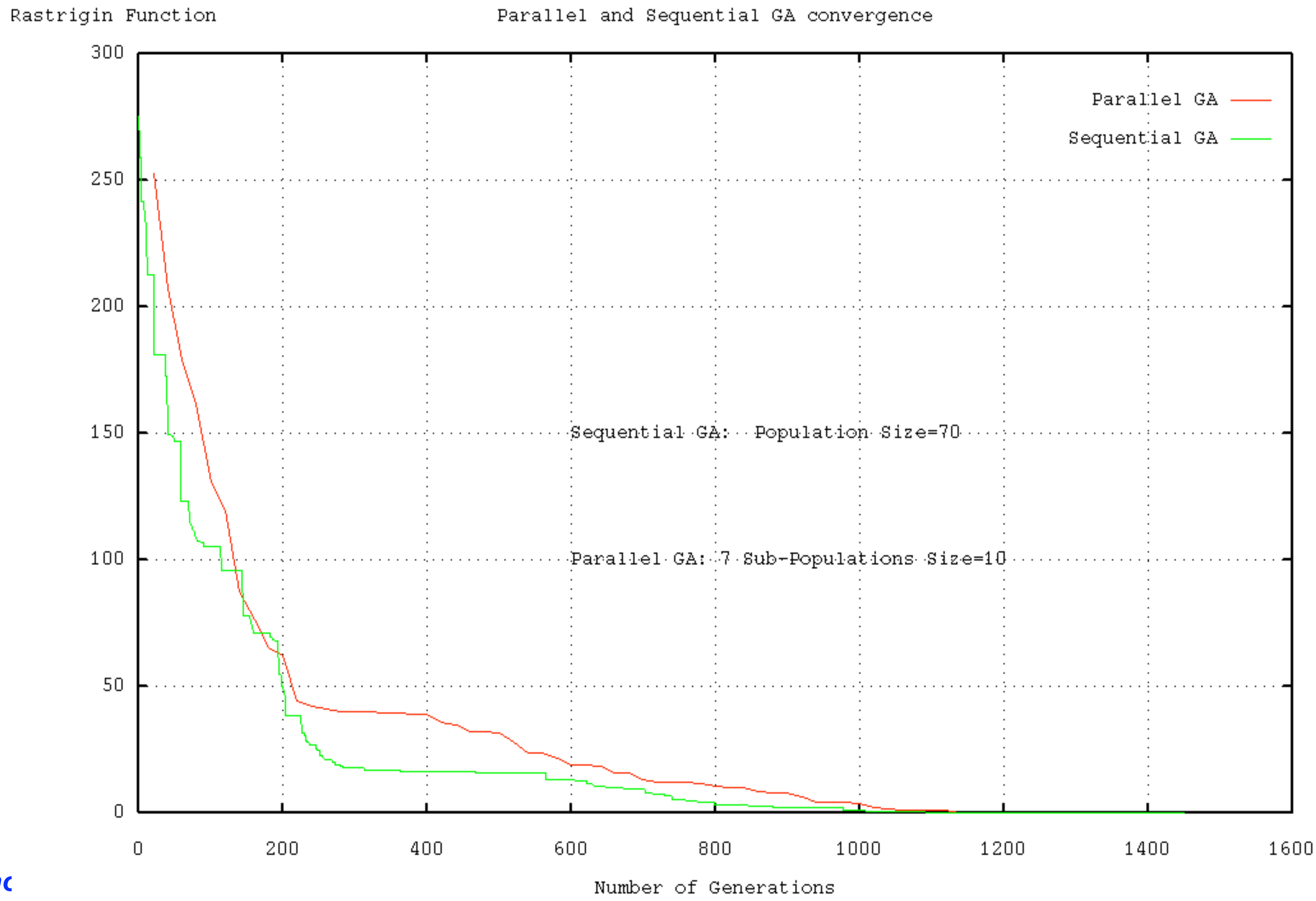
Test-case : Rastrigin Function

$$f = 10 \cdot 20 + \sum_{i=1}^{20} \left(x_i^2 - 10 \cdot \cos(2\pi \cdot x_i) \right)$$

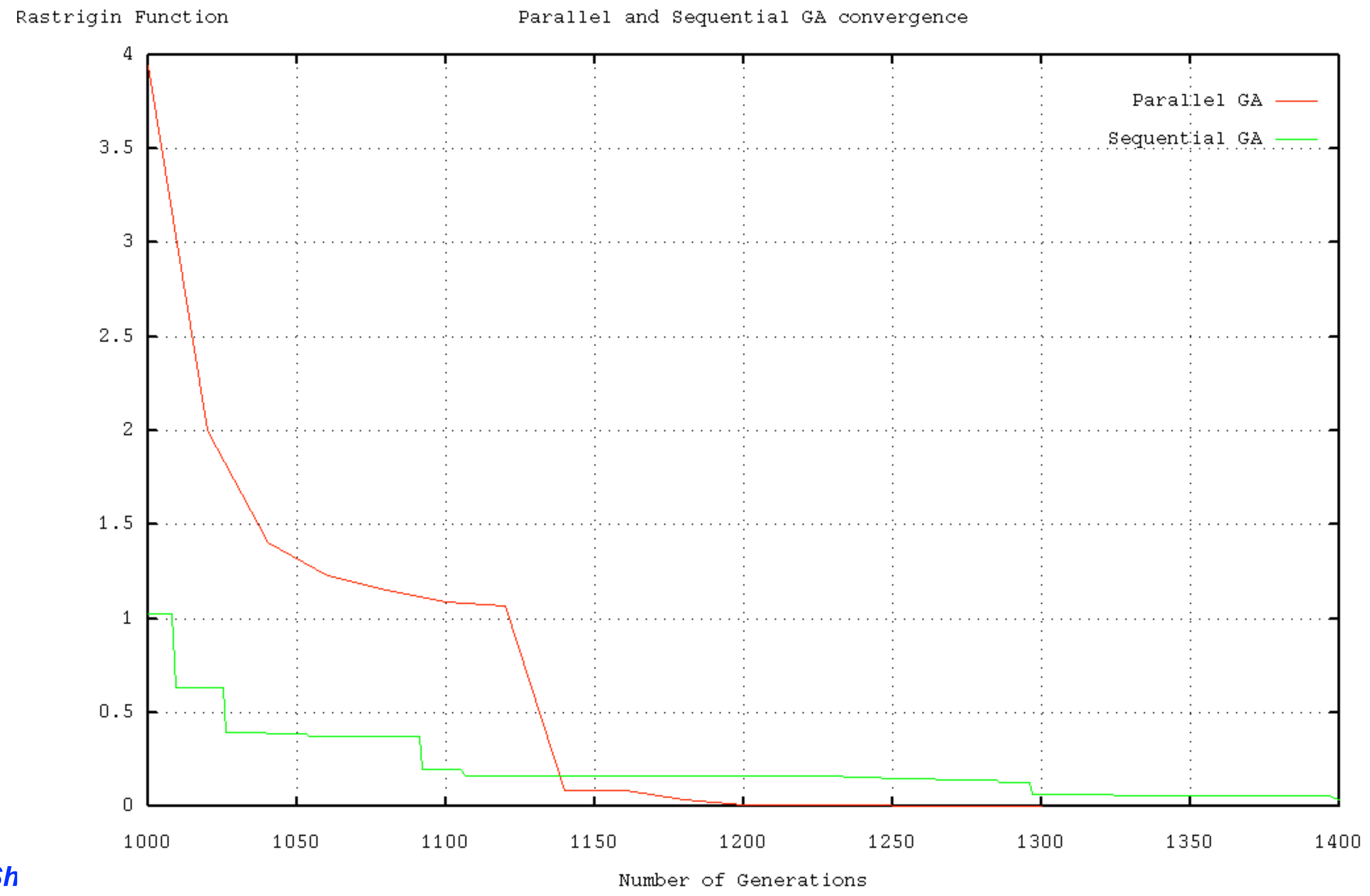
- In dimension 2, the surface is:






Rastrigin Function
Parallel and Sequential GA convergence
Parallel GA
Sequential GA
Sequential GA: Population Size=70
Parallel GA: 7 Sub-Populations Size=10
300
250
200
150
100
50
0
0
200
400
600
800
1000
1200
1400
1600
Number of Generations

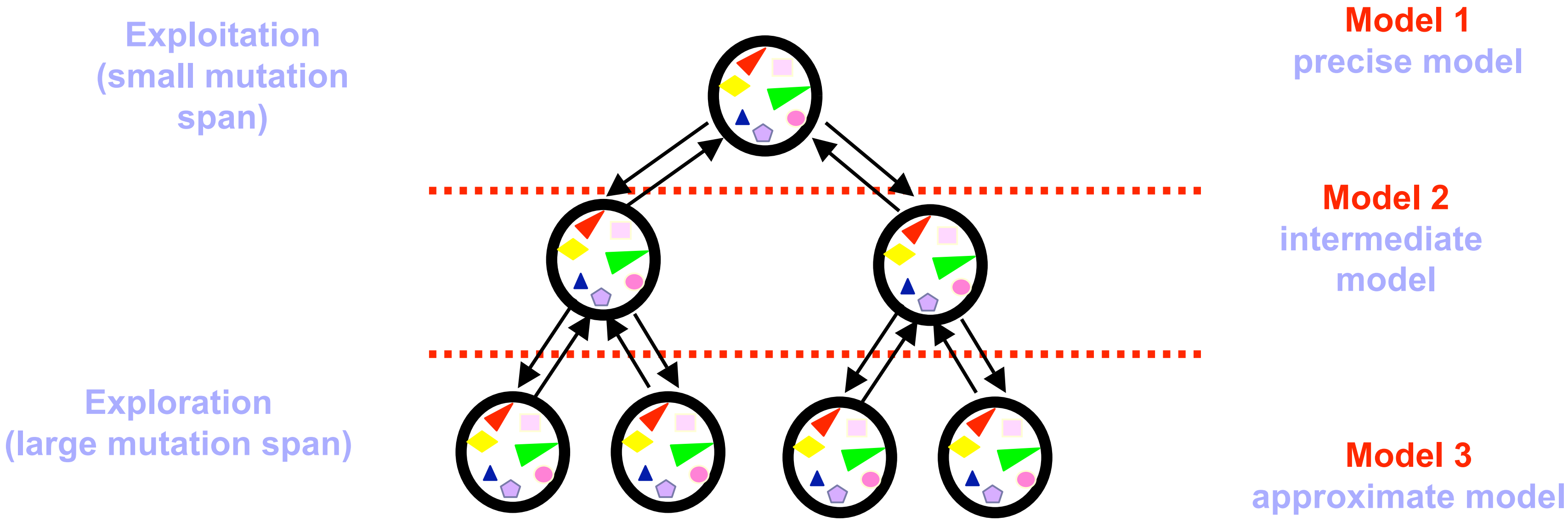
Sho



Rastrigin Function
Parallel and Sequential GA convergence
Parallel GA
Sequential GA
4
3.5
3
2.5
2
1.5
1
0.5
0
1000
1050
1100
1150
1200
1250
1300
1350
1400
Number of Generations

Sh

7. Hierarchical Topology-Multiple Models, Sefrioui et al, 1998



- Interactions of the 3 layers : solutions go up and down the layers.
- The best ones keep going up until they are completely refined.
- No need for great precision during exploration.
- Time-consuming solvers used only for the most promising solutions.
- Think of it as a kind of optimisation and population based *multi grid*.

HIERARCHICAL TOPOLOGY- MULTIPLE MODELS

- Start migration:

  - **Layer 1:** Receive (1/3 population) best solutions from layer 2 reevaluate using type 1 integrated analysis

  - **Layer 2:** Receive (1/3 population) random solutions from layer 1 and best from layer 3 reevaluate them using type 2 integrated analysis

  - **Layer 3:** Receive (1/3 population) random solutions from layer 2 reevaluates them using type 3 integrated analysis.

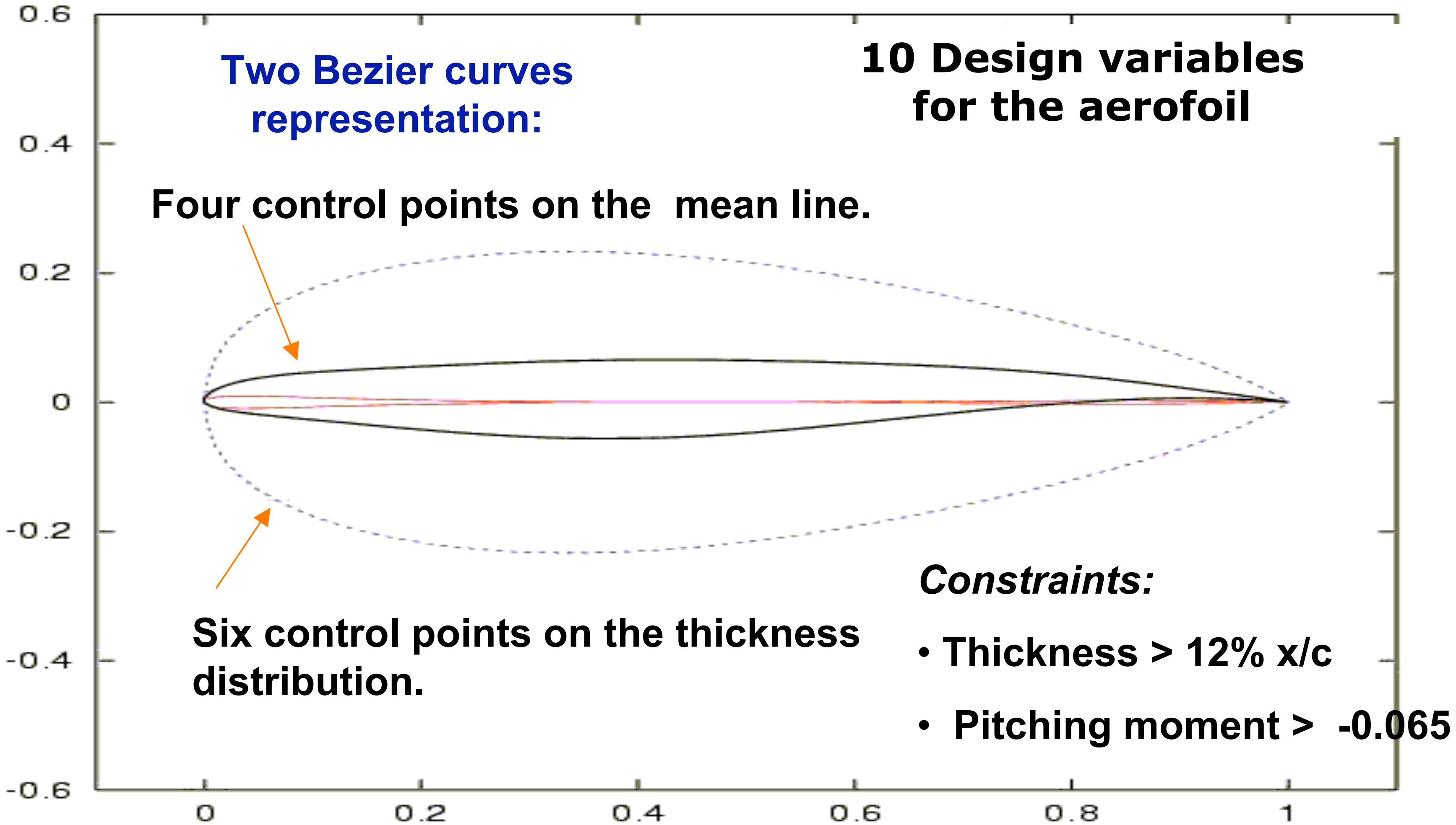
10. Discontinuous Pareto Front industrial test case : Two Objective UAV Airfoil Section Design (Eurogen 2003

- Design of a single element aerofoil for a low-cost UAV application.

- Two subsonic design points considered for optimisation

  - Loitering flight
  - Rapid-transit flight.

Design Variables: Bounding Envelope of the Aerofoil Search Space



Fitness Functions and Design Constraints

Specifications: Z. Johan, Dassault Aviation

Min f1 (Cd transit) Mach=0 .60 and Re= 14.0x10**6,Cm>-.065

Min f2 (Cd loiter) Mach=0 .15 and Re= 3.5x10**6

- Constraints are applied by equally penalizing both fitness values via a penalty method.

- Aerofoil generated outside the thickness bounds of 10% to 15% are rejected immediately, before analysis.

Solver

- XFOIL software written by Drela.
- It comprises a higher order panel method with coupled integral boundary layer.
- We have allowed free transition points for the boundary layer.
- Locally sonic flow will be prevented by checking :

The value of Cp: Cpi< Cp then the candidate is rejected immediately

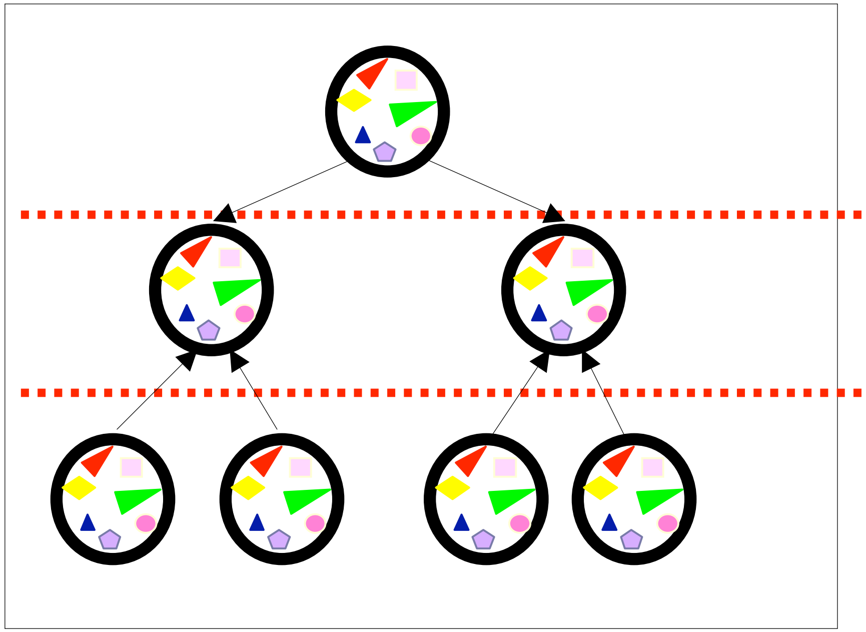
Implementation

Hierarchical Asynchronous Parallel EA (HAPEA)

Exploitation
Population size = 20

Intermediate
Population size = 20

Exploration
Population size = 10



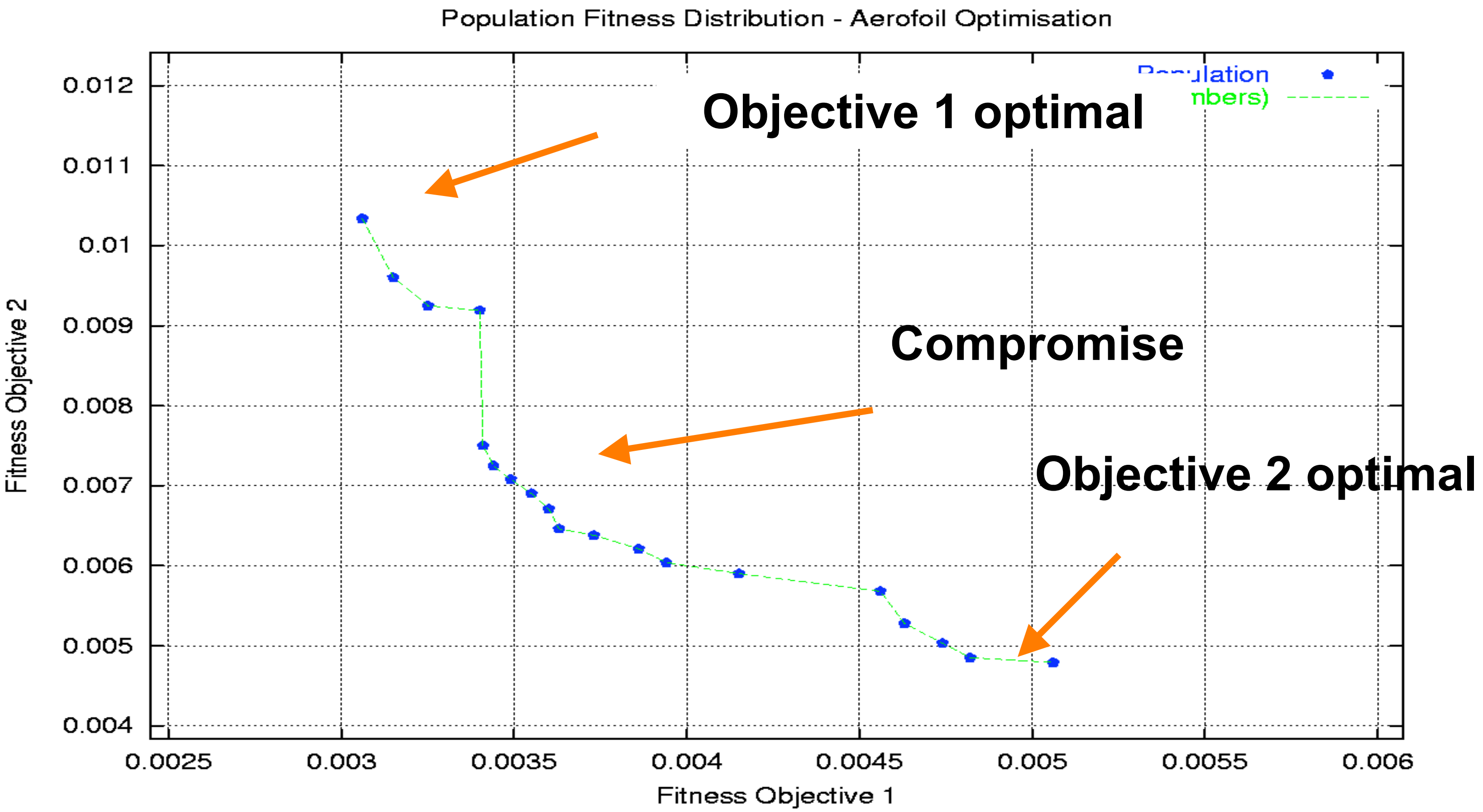
Model 1
Grid= 119 panels

Model 2
Grid=99 panels

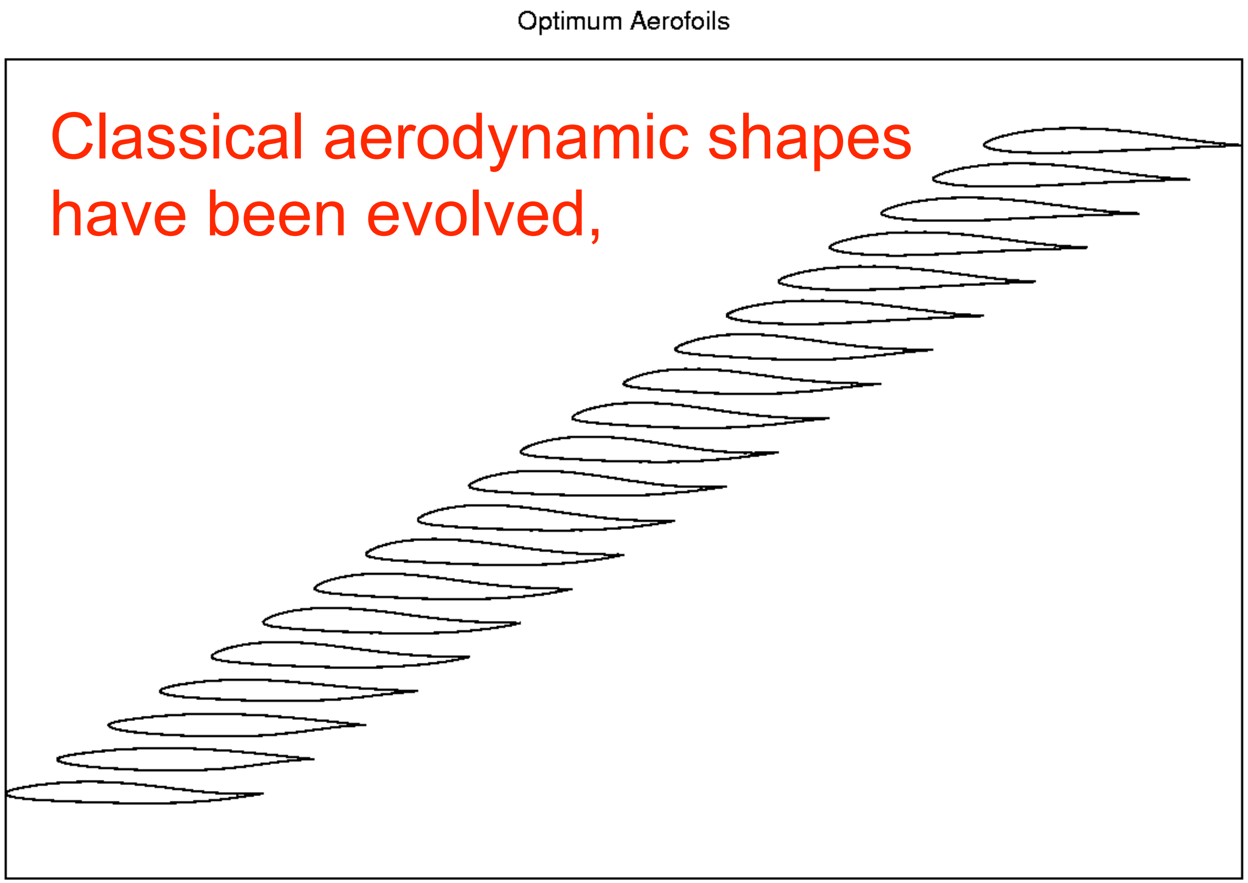
Model 3
Grid= 79 panels

Discontinuous Pareto Front for Aerofoil

This case was run for 5300 function evaluations of the head node, and took approximately four hours on a single 1.0 GHz processor.



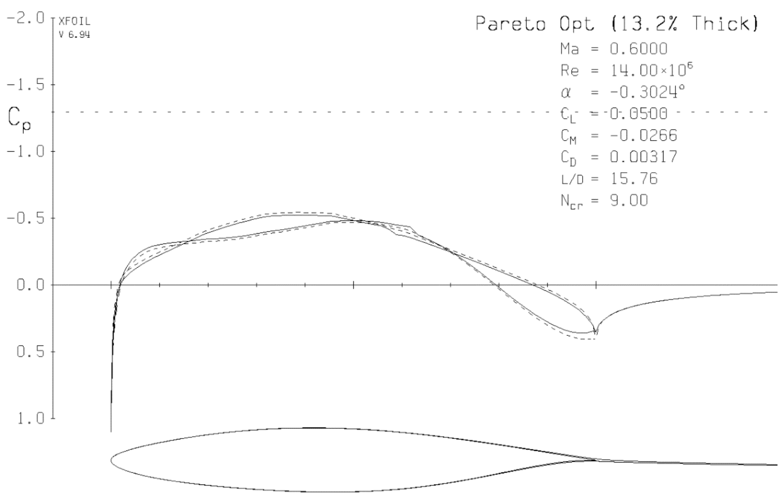
The Ensemble of Pareto



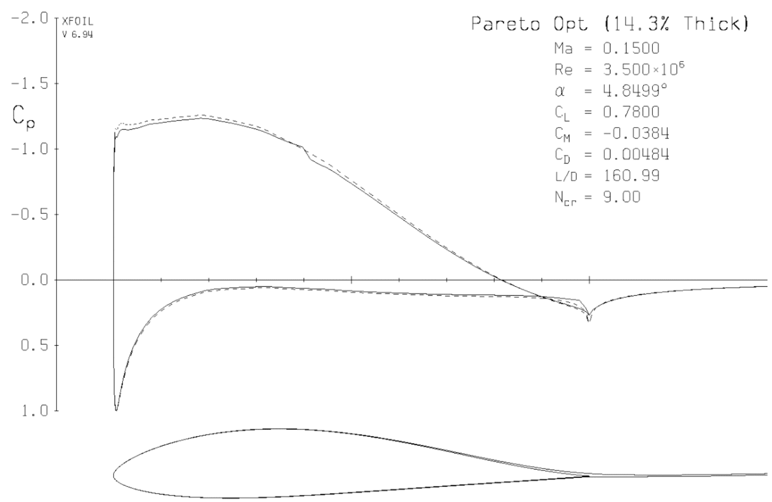
Optimum Airfoils for Cruise and Loiter

Evolved a conventional low-drag pressure distribution and overall form

Classical 'rooftop' type pressure distribution upper surface Almost constant favorable pressure gradient lower surface



Objective 1: Optimal Aerofoil – Cruise *CP* Distribution.

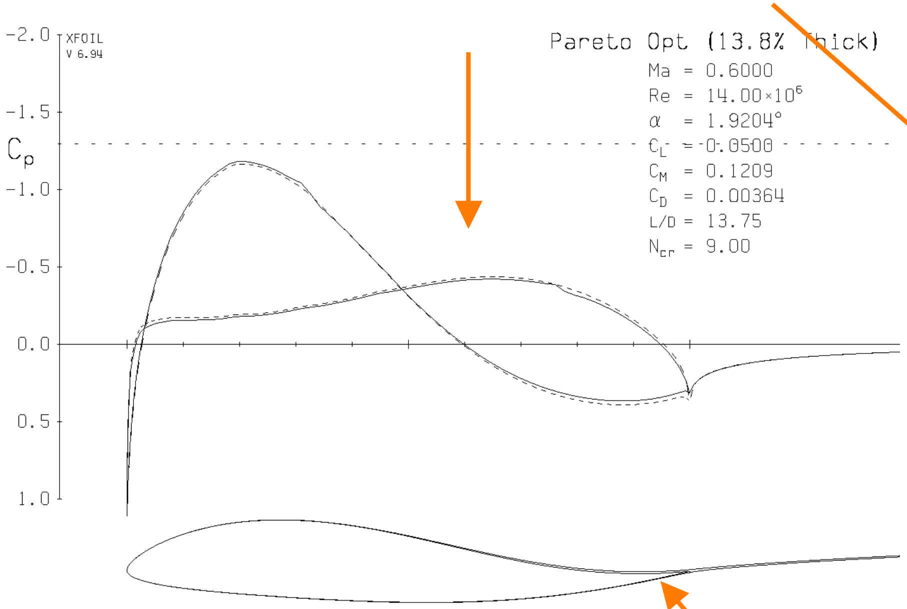


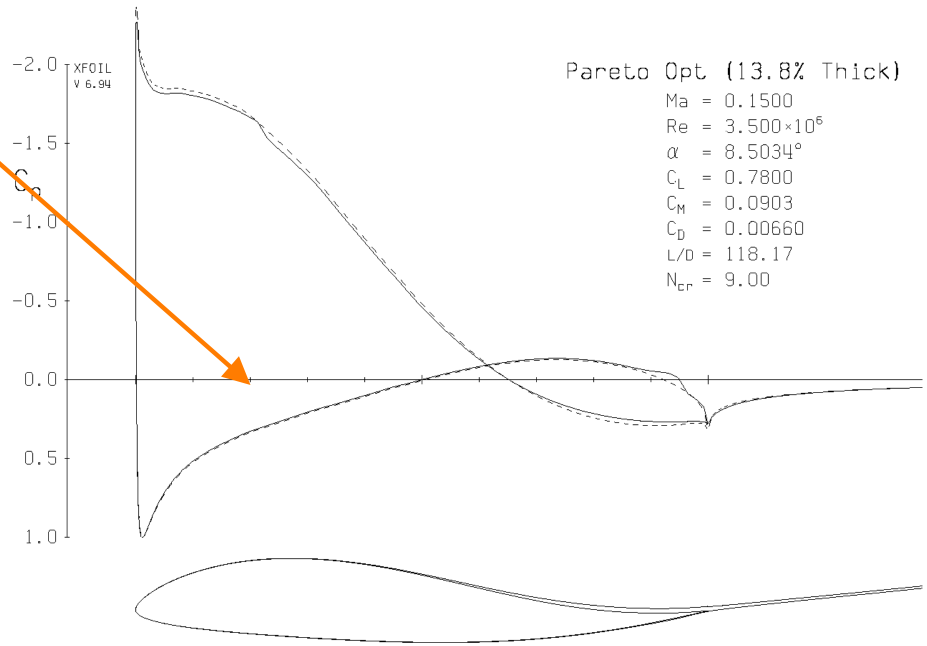
Objective 2: Optimal Aerofoil – Loiter *CP* Distribution.

Compromise Individual

Marked favorable gradient on the lower surface in both flow regimes

Conventional Pressure distribution,





Pronounced S-shaped camber distribution.

Cruise *CP* Distribution

Loiter *CP* Distribution.

8. Asynchronous Evaluation (E. Whitney , 2002)

Why asynchronous ?

Converged PDE solutions to MO and MDO -> variable time to complete

Time to solve non-linear PDE - > depends upon geometry

Ignore any concept of a generation

Solution can be generated in and out of order

Processors – Can be of different speeds
- Added at random

Parallelization Strategy

Classification of our model (S. Armfield, USYD) :

✓ The algorithm : classified as a hierarchical Hybrid pMOEA model [Cantu Paz], uses a Master slave PMOEA but incorporates the concept of isolation and migration through hierarchical topology binary tree structure where each level executes different MOEAs/parameters (heterogeneous)

✓The distribution of objective function evaluations over the slave processors is where each slave performs k objective function evaluations.

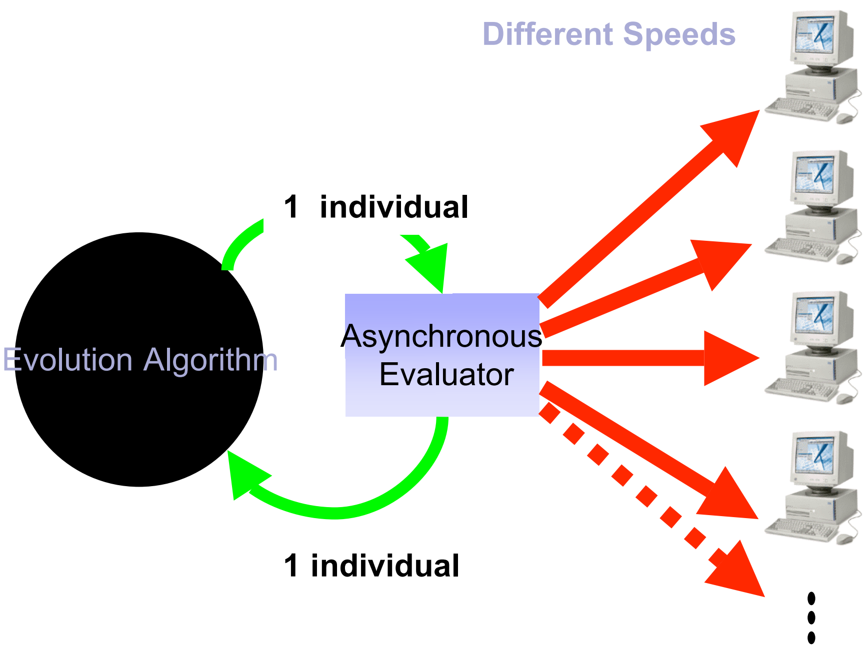
Parallel Processing system characteristics:

✓ Cluster of maximum 18 PCs with Heterogeneous CPUs, RAMs , caches, memory access times , storage capabilities and communication attributes.

Inter-processor communication:

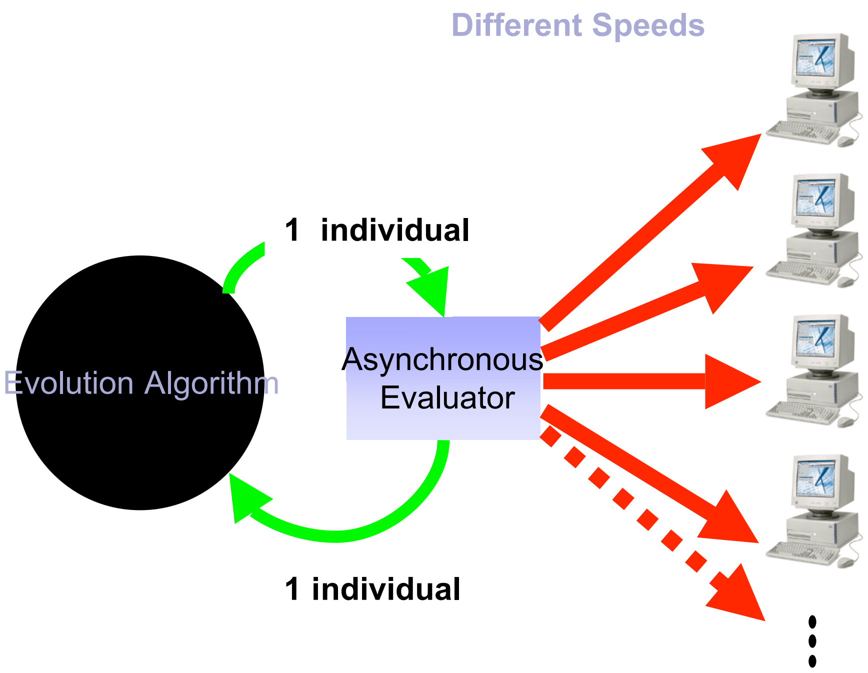
✓Using the Parallel Virtual Machine (PVM)

Asynchronous Evaluation (1)



- Ignores the concept of generation-based solution.
- Fitness functions are computed asynchronously.
- Only one candidate solution is generated at a time, and only one individual is incorporated at a time rather than an entire population at every generation as is traditional EAs.
- Solutions can be generated and returned out of order.

Asynchronous Evaluation (2)



- No need for synchronicity → no possible wait-time bottleneck.
- No need for the different processors to be of similar speed.
- Processors can be added or deleted dynamically during the execution.
- There is no practical upper limit on the number of processors we can use.
- All desktop computers in an organization are fair game.

Results So Far…

- HAPEA technique is approximately three times faster than other similar EA methods.

	Evaluations	*CPU Time*
Traditional EA	2311 ±224	152m±20m
New Technique	504 ±490 (-78%)	48m ±24m

- A test bench for single and multi objective problems has been developed and tested successfully

- We have successfully coupled the optimisation code to different compressible CFD codes and also to some aircraft design codes

CFD

HDASS MSES XFOIL

FLO22 Nsc2ke

Aircraft Design

Flight Optimisation Software (FLOPS)

ADS (In house)

9. Robust design : TAGUCHI METHOD (Uncertainty)

Robust Design method, also called the Taguchi Method (uncertainty), pioneered by Genichi Taguchi in 1978, improves a quality of engineering productivity. An optimisation problem could be defined as:

$$Max \text{ or } Min\, f = f\left(x_1,\dots,x_n,x_{n+1},\dots,x_m\right)$$

Where $x_1,\dots,x_n$ represent design parameters and $x_{n+1},\dots,x_m$ represent uncertainty parameters that are in fine step size.

Taguchi optimization method minimizes the variability of the performance under uncertain operating conditions. Define two different objectives associated to the function to optimise: mean value and variance.

MEAN

$$\overline{f} = \frac{1}{K}\sum_{j=1}^{K} f_j$$

VARIANCE

$$\delta f = \frac{1}{K-1}\left(\sum_{j=1}^{K}\left|f_j - \overline{f}\right|\right)$$

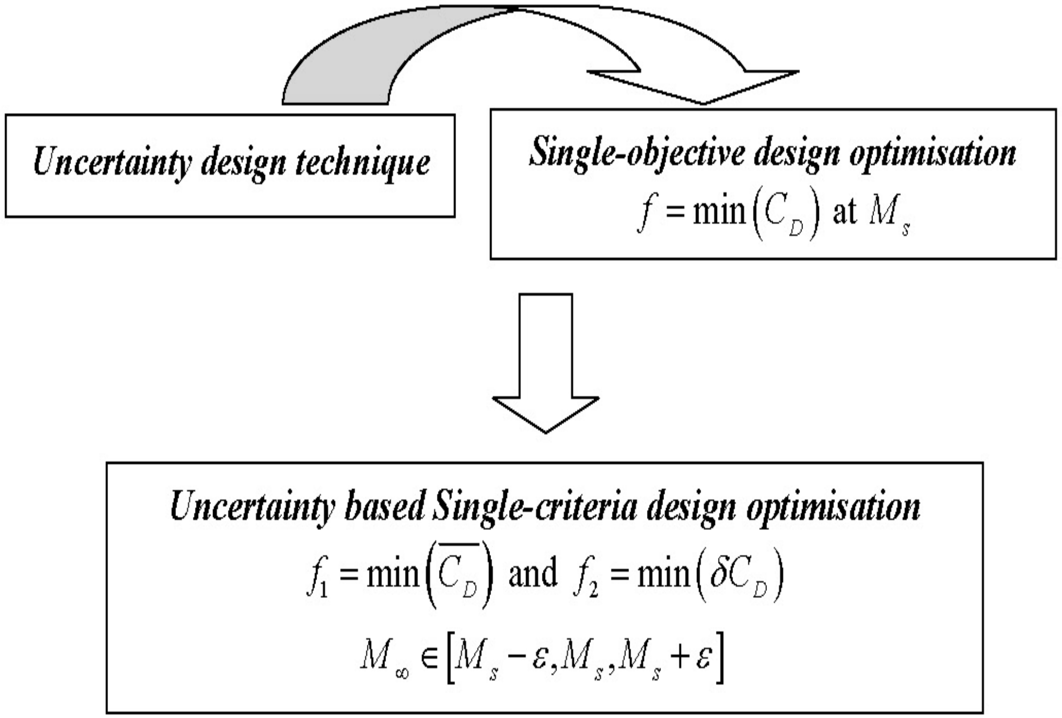
UNCERTAINTY

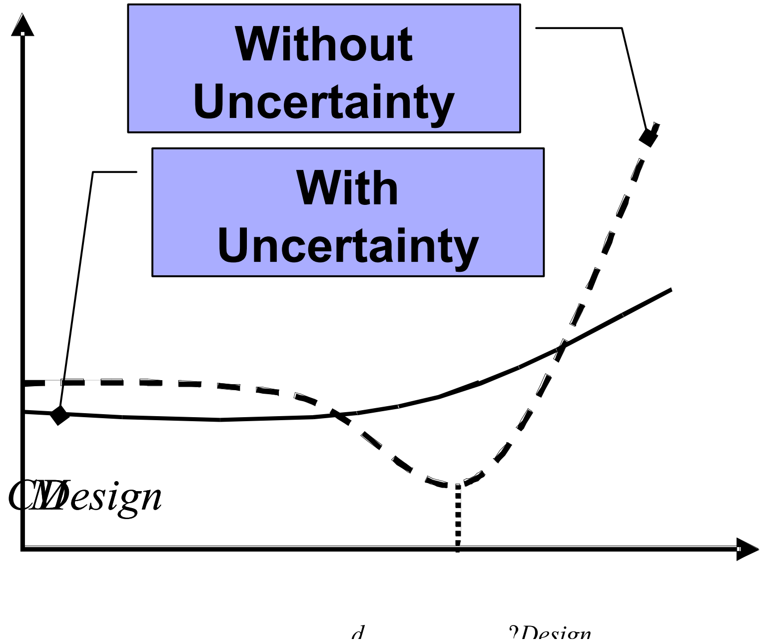
MEAN

$$\bar{f} = \frac{1}{K}\sum_{j=1}^{K} f_j$$

VARIANCE

$$\delta f = \frac{1}{K-1}\left(\sum_{j=1}^{K}\left|f_j - \bar{f}\right|\right)$$





UNCERTAINTY BASED MULTI DISCIPLINARY DESIGN OPTIMISATION OF J-UCAV

- Fitness functions are

$$fitness(f_1) = \min\left(\frac{1}{\overline{\overline{L/D}}}\right)$$

$$fitness(f_2) = \min\left(\delta\frac{L}{D}\right)$$

where $\overline{\frac{L}{D}} = {}^1\!/_K \sum_{i=1}^{K} (L/D_i)\frac{M_{\infty i}^2}{M_S^2}$ and $\delta\frac{L}{D} = \frac{1}{(K-1)}\sum_{i=1}^{K}\left(L/D_i\frac{M_{\infty i}^2}{M_S^2} - \overline{L/D}\right)^2$

$$f_3 = \min\left(RCS_{Quality}\right) = \frac{1}{2}\left[\left(\overline{RCS}_{mono} + \delta RCS_{mono}\right) + \left(\overline{RCS}_{bi} + \delta RCS_{bi}\right)\right]$$

where $\theta = [0^\circ : 3^\circ : 360^\circ]$ and $\phi = [0^\circ : 0^\circ : 0^\circ]$ $(Monostatic)$

where incident angles $\theta = 135^\circ$, $\phi = 90^\circ$ at $\theta = [0^\circ : 3^\circ : 360^\circ]$, $\phi = [0^\circ : 0^\circ : 0^\circ]$ $(Bistatic)$

- Variability of flight conditions and radar frequencies

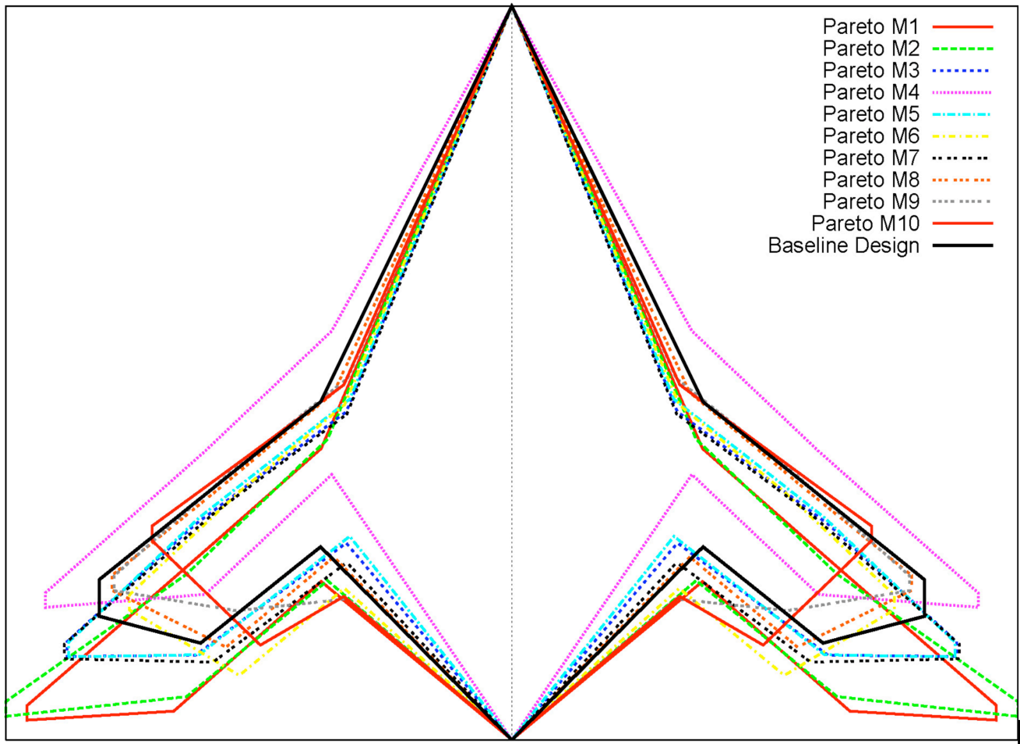
$$M_{\infty} \in \left[0.75, 0.775, M_S = 0.80, 0.825, 0.85\right]$$

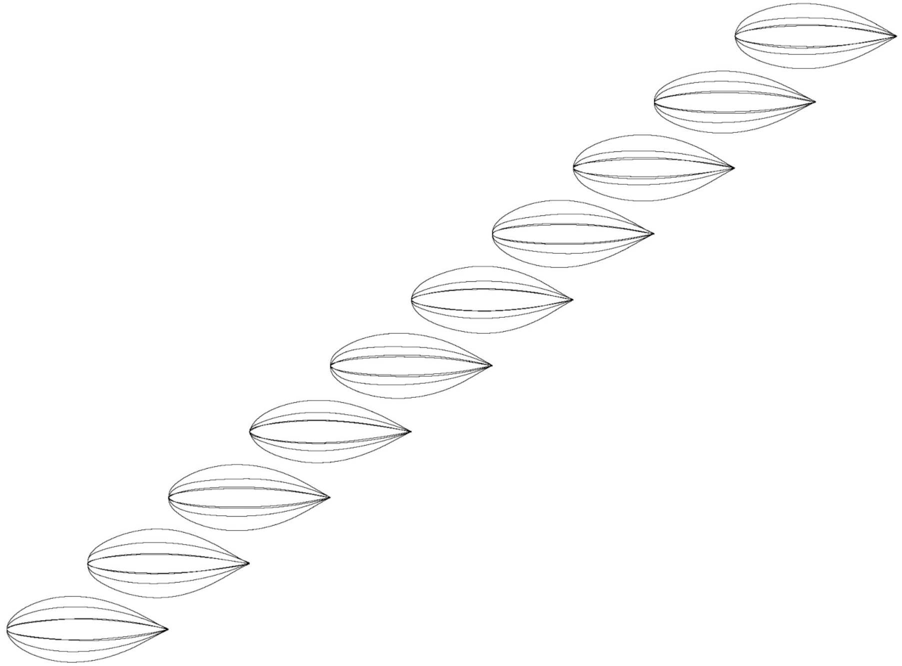
$$\alpha_{\infty} \in \left[4.662, 3.968, \alpha_S = 3.275^\circ, 2.581, 1.887\right]$$

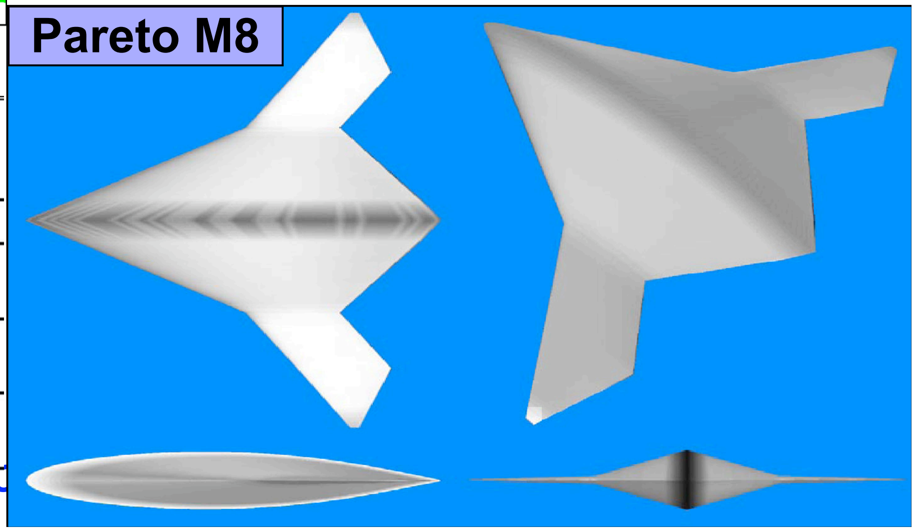
$$ATI_{\infty} \in \left[30062, 25093, ATI_S = 20125 ft, 15156, 10187\right]$$

$$F_{\infty} \in \left[1.0, 1.25, F_S = 1.5 GHz, 1.75, 2.0\right]$$

RESULT: *PARETOSET PLANFORMS and AEROFOIL SECTIONS*







Models	AR	b	λ_{C1} ($\%c_{root}$)	λ_{C2} ($\%c_{root}$)	Λ_{R-C1}	Λ_{C1-C2}	Λ_{C2-T}
Baseline	4.45	18.9	19.7	19.7	55°	29°	29°
ParetoM1 (BO1)	6.02 (+35%)	22.20 (+17%)	18.1 (-8%)	18.1 (-8%)	58.0°	31.2°	30.9°
ParetoM8	4.30 (-3%)	18.32 (-3%)	22.4 (-14%)	22.4 (-14%)	56.3°	30.0°	29.2°
ParetoM10 (BO2-BO3)	3.46 (-22%)	16.47 (-13%)	29.0 (+47%)	27.0 (+37%)	57.26°	27.2°	26.7°

13. CONCLUSION (1)

- This lecture has described the basic concepts of EAs, and a short review of different approaches and industrial needs for MDO presented.
- Details of Evolutionary Algorithms and their specific applications to aeronautical design problems discussed.
- The lecture provided specific details on a particular EA used in this research named HAPEA.
- It is noticed that there are different methods, architectures and applications of optimisation and multidisciplinary design optimisation methods for aeronautical problems.
- However, still further research for alternative methods are still required to address the industrial and academic challenges and needs of aeronautic industry.
- EAs is an alternative option to satisfy some of these needs, as they can be easily coupled, particularly adaptable, easily parallelised, require no gradient of the objective function(s), have been used for multi-objective optimisation and successfully applied to different aeronautical design problems.
- Nonetheless, EAs have seen little application at an industrial level due to the computational expense involved in this process and the fact that they require a larger number of function evaluations, compared to traditional deterministic techniques.
- The continuing research has focused on development and applications of canonical evolution algorithms for their application to aeronautical design problems. It is worth to have a single framework that allows:
  - Solving single and multi-objective problems that can be deceptive, discontinuous, multi-modal.
  - Incorporation of different game strategies-Pareto, Nash, Stackelberg
  - Implementation of multi-fidelity approaches
  - Taking into account uncertainties
  - Parallel Computations
  - Asynchronous evaluations

Conclusion (2): KEY CONCEPTS

- systemic technology like the one required by UAVs will increase in the future (see Part 3)
- In order to obtain true optimised-global solution we need to think multidisciplinary.
- Evolutionary Algorithms are techniques to consider as it provides fruitful and optimal results.
- Simple EAs are not sufficient : the complex task of MO and MDO in aeronautics required advanced EAs

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