Open and solved problems in the stability of mechanical, electromechanical and electronic systems (plenary lecture)

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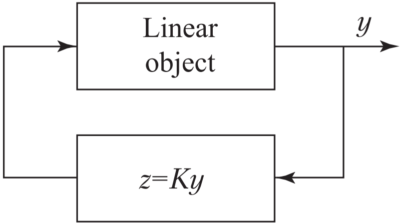
Content

- Linear stationary stabilization: Nyquist criterion (1932)
- Linear nonstationary stabilization:
  - pendulum upper equilibrium - A. Stephenson (1907), P.Kapitsa (1951), V.Chelomey (1956) S.Meerkov (1973), R.Bellman, J.Bentsman, S.Meerkov (1986)
  - rope stabilization - I.Blekhman (2006)
  - Brockett's stabilization problem (2000)
  - low frequency stabilization of crane-container and helicopter-container
- Nonlinear stabilization
  - self-excited and hidden oscillations (2010)
  - counterexamples to Aizerman's (1949) and Kalman's (1957) conjectures on absolute stability
  - hidden oscillations in drilling systems (2009)
  - hidden oscillations in aircraft control system (1992)
  - hidden attractors in Chua's circuits (2010)

I. Linear stabilization

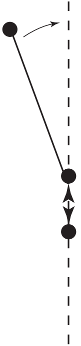


Nyquist criterion and various generalizations

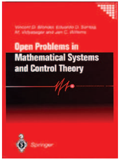
Vibration stabilization and control $K = K(t)$

Stabilization of pendulum upper equilibrium
vertically vibrating point of suspension

- S.Meerkov 1973
- R.Bellman, J.Bentsman, S.Meerkov 1986
- A.Stephenson 1907, P. Kapitsa 1951, V.Chelomey 1956
- I.Blekhman 2006



Brockett stabilization problem

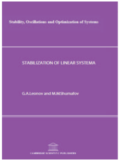


Problem: R.Brockett (MIT; IEEE and AMS Fellow) *Open problems in mathematical systems and control theory*, Springer, **2000**

The problem is to define $K(t)$ for given matrixes A, B, C such that

$$\frac{dx}{dt} = (A + BK(t)C)x$$

is asymptotically stable?



Solution: G.A. Leonov, M.M. Shumafov *Stabilization of Linear Systems*, Cambridge Sci. Press, **2011**

The problem is to define $K(t)$ for given matrixes A, B, C such that

$$\frac{dx}{dt} = (A + BK(t)C)x$$

is asymptotically stable?

In other words: *how the nonstationary stabilization extends the possibilities of stabilization?*

Two approaches for stabilization:

- high frequencies : $K(t) = a \sin \omega t, \;\; \omega \gg 1$
  L.Moreau, D.Aeyels 2004
- low frequencies: $K(t) = a\text{sign} \sin \omega t, \;\; \omega \ll 1$
  G.A.Leonov 2001–2004.

For two dimensional system – necessary and sufficient conditions of stabilization:

Transfer function

$$W(p) = \frac{p+c}{p^2 + b_2 p + b_1} \quad (B \text{ и } C - \text{vectors})$$

Non singularity: $c^2 - b_2 c + b_1 \neq 0$

Stationary stabilization $K(t) \equiv K_0$: $c > 0$, or $c \leq 0,\ a_2 c < a_1$

Let $c \leq 0,\ a_2 c \geq a_1$ (stationary stabilization is impossible)

For nonstationary stabilization **necessary** and **sufficient**

$$c^2 - a_2 c + a_1 > 0.$$

Discrete analog

$$x(k+1) = (A + BK(k)C)x(k)$$

Two-dimensional system can be stabilize by low frequency for almost all parameters ($CA^{-1}B \neq 0,\ W(p)$ – nonsingular).

Low frequency stabilization of crane.

Control of a Parametrically Excited Crane,
J.Collado, C.Vazquez, Mexico 2010

Low frequency stabilization of helicopter-container

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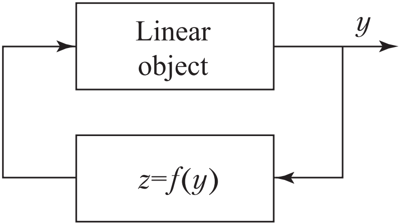
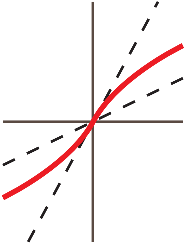
  - hidden attractors in Chua circuits (2010)
  - hidden oscillations in phase locked loops (1961)

II. Nonlinear theory



Hidden oscillations (3d): Aizerman and Kalman conjectures

if $\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{b}k\mathbf{c}^*\mathbf{z}$, is asympt. stable $\forall\, k \in (k_1, k_2)$: $\forall \mathbf{z}(t, \mathbf{z}_0) \to 0$, then
is $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}\varphi(\sigma)$, $\sigma = \mathbf{c}^*\mathbf{x}, \varphi(0) = 0$, $k_1 < \varphi(\sigma)/\sigma < k_2$: $\forall \mathbf{x}(t, \mathbf{x}_0) \to 0$?



$$1949: \quad k_1 < \varphi(\sigma)/\sigma < k_2 \qquad\qquad 1957: \quad k_1 < \varphi'(\sigma) < k_2$$

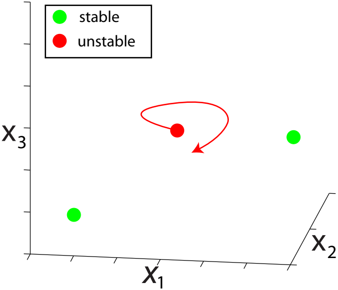
In general, conjectures are not true(Aizerman's :$n \geq 2$, Kalman's :$n \geq 4$)
Periodic solution can exist for nonlinearity from linear stability sector

Aizerman's conj.: N.Krasovsky 1952:$n = 2$, $\mathbf{x}(t) \to \infty$; V.Pliss 1958: $n = 3$, periodic $\mathbf{x}(t)$

Survey: V.O. Bragin, V.I. Vagaitsev, N.V. Kuznetsov, G.A. Leonov (2011) Algorithms for finding hidden oscillations in nonlinear systems. The Aizerman and Kalman conjectures and Chua's circuits, *J. of Computer and Systems Sciences Int., V.50, N4, 511-544 (doi:10.1134/S106423071104006X)*

Engineering analytical-numerical analysis of dynamical system

$$\dot{\mathbf{x}} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dots \\ \dot{x}_n \end{pmatrix} = \begin{pmatrix} f_1(x_1, x_2, .., x_n) \\ f_2(x_1, x_2, .., x_n) \\ \dots \\ f_n(x_1, x_2, .., x_n) \end{pmatrix} = \mathbf{F}(\mathbf{x})$$

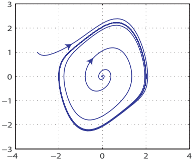


- 1. Find equilibria
  $\mathbf{x}^k \ : \ \mathbf{F}(\mathbf{x}^k) = 0$
  local stability analysis
  $$\dot{\mathbf{y}} = \frac{\mathrm{d}\mathbf{F}(\mathbf{x})}{\mathrm{d}\mathbf{x}}\Big|_{\mathbf{x}=\mathbf{x}^k} \mathbf{y}$$
- 2. Numerical simulation of trajectories with initial data from neighborhoods of unstable equilibria.

self-excited attractor localization: *standard computational procedure* is 1) to find equilibria; 2) after transient process trajectory, starting from a point of unstable manifold in a neighborhood of unstable equilibrium, reaches an self-excited oscillation and localizes it.

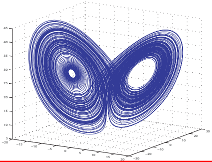
Van der Pol

$$\dot{x} = y$$
$$\dot{y} = -x + \varepsilon(1-x^2)y$$



Lorenz

$$x = -\sigma(x-y)$$
$$y = rx - y - xz$$
$$z = -bz + xy$$

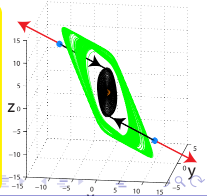


hidden attractor: basin of attraction does not intersect with a small neighborhood of equilibria [Leonov,Kuznetsov,Vagaitsev,Phys.Lett.A,375,2011,2230-2233]

✓ *standard computational procedure* does not work: all equilibria are stable or not in the basin of attraction

✓integration with random initial data does not work: basin of attraction is small, system's dimension is large

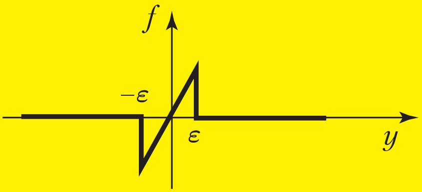
How to choose initial data in the attraction domain?



Nonexistence of oscillations (sufficient conditions):
Popov criteria (1958) $o < \frac{f(y)}{y} < k$

$$\frac{1}{k} + \mathrm{Re}\,(1 + qi\omega)W(i\omega) > 0 \quad W(p) = C(A - pI)^{-1}B, \forall\, \omega,$$

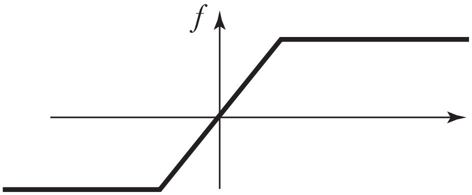
Hidden oscillations:
Pliss counterexample to Aizerman's conjecture(1956)



Counterexample to Aizerman's conjecture with typical engineering nonlinearity

$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}\varphi(\sigma),\ \sigma = \mathbf{c}^*\mathbf{x},\ \varphi(0) = 0,\ \ 0 < \varphi(\sigma) < k: \quad \forall \mathbf{x}(t, \mathbf{x}_0) \to 0?$

Typical nonlinearity $f(y) = sat(y)$



- Bernat J. & Llibre J. 1996
- Leonov G.A., Kuznetsov N.V., Bragin V.O. 2010

Kalman conjecture (Kalman problem) 1957

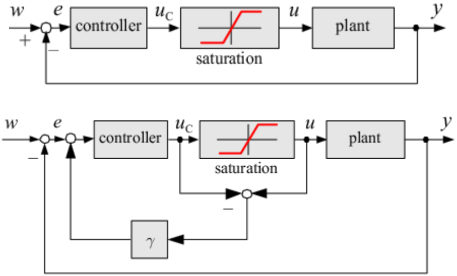
$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}\varphi(\sigma),\ \sigma = \mathbf{c}^*\mathbf{x},\ \varphi(0) = 0,\ \ 0 < \varphi'(\sigma) < k:\ \ \forall \mathbf{x}(t, \mathbf{x}_0) \to 0?$

- **Fitts R. 1966**: series of counterexamples in $\mathbb{R}^4$, nonlinearity $\varphi(\sigma) = \sigma^3$
- **Barabanov N. 1979-1988**: some of Fitts counterexamples are false; analytical 'counterex.' construction in $\mathbb{R}^4$, $\varphi(\sigma)$ 'close' to $\mathrm{sign}(\sigma)$ $(0 \leq \varphi'(\sigma))$
  later 'gaps' were reported by Glutsyuk, Meisters, Bernat & Llibre
- **Leonov G. et al. 1996**: Kalman conj. is true in $\mathbb{R}^3$ (by YKP freq. methods)
- **Bernat J. & Llibre J. 1996**: analytical-numerical 'counterexamples' construction in $\mathbb{R}^4$, $\varphi(\sigma)$ 'close' to $\mathrm{sat}(\sigma)$ $(0 \leq \varphi'(\sigma))$
- **Leonov G., Kuznetsov N., Bragin V. 2010**:
  some of Fitts counterexamples are true;
  smooth counterexample in $\mathbb{R}^4$ with $\varphi(\sigma) = \tanh(\sigma)$: $0 < \tanh'(\sigma) \leqslant 1$;
  analytical-numerical counterexamples construction for any type of $\varphi(\sigma)$;

Survey: V.O. Bragin, V.I. Vagaitsev, N.V. Kuznetsov, G.A. Leonov (2011) Algorithms for finding hidden oscillations in nonlinear systems. The Aizerman and Kalman conjectures and Chua's circuits, *J. of Computer and Systems Sciences Int., V.50, N4, 511-544 (doi:10.1134/S106423071104006X)*

Hidden oscillation in aircraft control systems

Windup – oscillations with increasing amplitude
- Crash - YF-22 Raptor (Boeing) 1992
- Crash - JAS-39 Gripen (SAAB) 1993



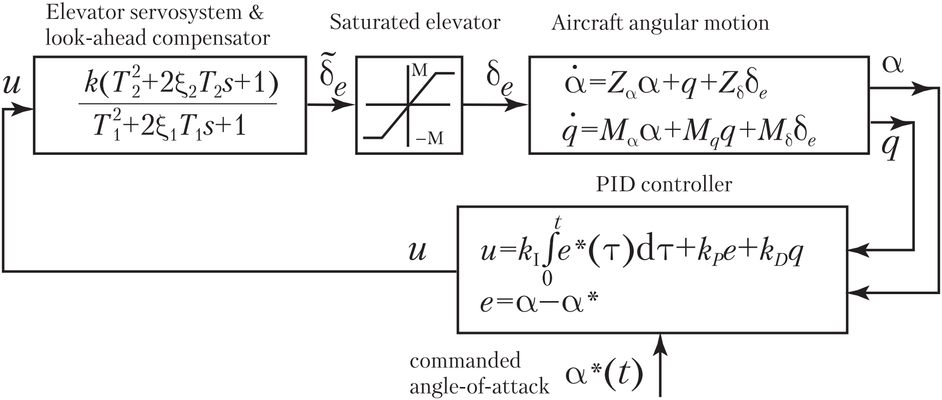
Antiwindup – an additional scheme to avoid windup effect in system with saturation

Lauvdal, Murray, Fossen, Stabilization of integrator chains in the presence of magnitude and rate saturations; a gain scheduling approach, *Proc. of CDC, 1997*: *"Since stability in simulations does not imply stability of the physical control system (an example is the crash of the YF22) stronger theoretical understanding is required"*

B.R. Andrievsky, N.V. Kuznetsov, G.A. Leonov, A.Yu. Pogromsky, Hidden Oscillations in Aircraft Flight Control System with Input Saturation, IFAC Proceedings Volumes (IFAC-PapersOnline), Volume 5(1), 2013, pp. 75-79 (doi: 10.3182/20130703-3-FR-4039.00026)

Hidden oscillation in aircraft control systems

The angle-of-attack control system of the unstable aircraft model in the presence of saturation in magnitude of the control input: stable zero equilibrium coexist with undesired stable oscillation — hidden oscillations



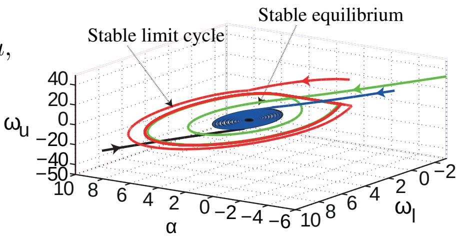
B.R. Andrievskii, N.V. Kuznetsov, G.A. Leonov et. al., Hidden oscillations in aircraft flight control system with input saturation, 5th IFAC International Workshop on Periodic Control Systems, 2013, Volume 5(1), pp. 75-79 (doi:10.3182/20130703-3-FR-4039.00026)

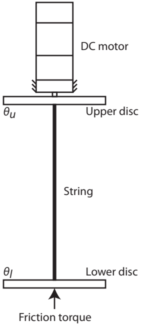
Hidden oscillation: Drilling system

Drill string failure ≈ 1 out of 7 drilling rigs, costs $100 000 (www.oilgasprod.com)

De Bruin, J.C.A. et al. (2009), *Automatica*, **45**(2), 405–415

$$\dot\omega_u = -k_\theta\alpha - T_{fu}(\omega_u) + k_m u,$$
$$\dot\omega_l = k_\theta\alpha - T_{fl}(\omega_l),$$
$$\dot\alpha = \omega_u - \omega_l,$$
$$\omega_{u,l} = \dot\theta_{u,l},\ \ \alpha = \theta_u - \theta_l,$$
T_{fu}, T_{fl} — friction torque





Operating mode: angular displacement between upper and lower discs $\alpha = \theta_u - \theta_l \to$ const.

Hidden oscillation: stable limit cycle coexists with stable equilibrium

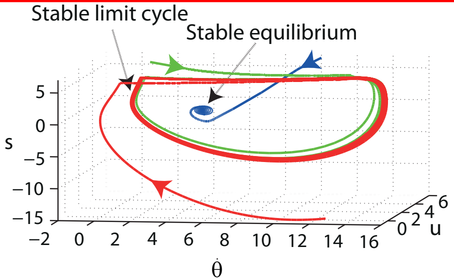
Hidden oscillation of $(\theta_u - \theta_l)$:
- is difficult to find by standard simulation
- may lead to breakdown

Hidden oscillation: Drilling system with induction motor

Hidden oscillation: stable limit cycle coexists with stable equilibrium

Hidden oscillation of $(\theta_u - \theta_l)$:
— is difficult to find by standard simulation;
— may lead to breakdown

$$\dot{y} = -cy - s - xs, \dot{x} = -cx + ys,$$
$$\dot{\theta} = u - s, \; s = -\dot{\theta}_u, \; u = -\dot{\theta}_l$$
$$\dot{s} = \frac{k_\theta}{J_u}\theta + \frac{b}{J_u}(u - s) + \frac{a}{J_u}y,$$
$$\dot{u} = -\frac{k_\theta}{J_l} - \frac{b}{J_l}(u - s) + \frac{1}{J_l}T_{fl}(\omega - u),$$

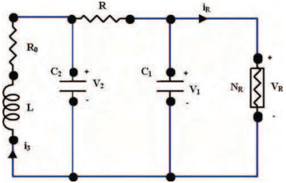


Leonov G.A., Kuznetsov N.V., Kiseleva M.A., Solovyeva E.P., Zaretskiy A.M., Hidden oscillations in mathematical model of drilling system actuated by induction motor with a wound rotor, Nonlinear dynamics, 2014 (doi:10.1007/s11071-014-1292-6)
M.A. Kiseleva, N.V. Kuznetsov, G.A. Leonov, P. Neittaanmaki, Drilling Systems Failures and Hidden Oscillations, NSC 2012 - 4th IEEE International Conference on Nonlinear Science and Complexity, 2012, 109-112 (doi:10.1109/NSC.2012.6304736)

Attractors in Chua's circuits

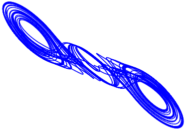
L.Chua (1983)

$$\dot{x} = \alpha(y - x - f(x)),$$
$$\dot{y} = x - y + z,$$
$$\dot{z} = -(\beta y + \gamma z),$$



$$f(x) = m_1 x + \mathrm{sat}(x) = m_1 x + \frac{1}{2}(m_0 - m_1)(|x+1| + |x-1|)$$

Chua circuit is used in chaotic communications



1983–now: computations of Chua self-excited attractors by standard procedure: trajectory from neighborhood of unstable zero equilibrium reaches & identifies attractor. [*Bilotta&Pantano, A gallery of Chua attractors, WorldSci. 2008*]

Could an attractor exists and how lo localize it if equilibrium is stable?

L.Chua, 1990: If zero equilibrium is stable ⇒ no chaotic attractor

Hidden attractor in classical Chua's system

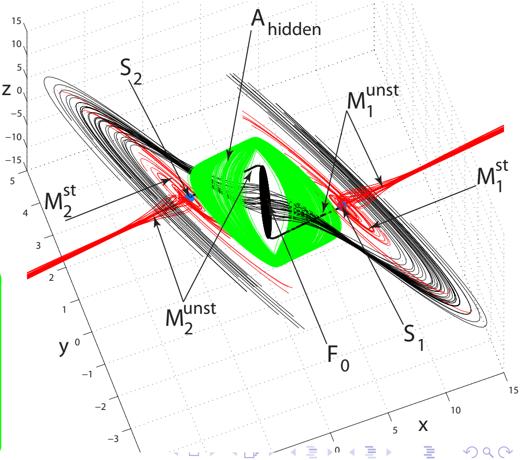
In 2010 the notion of *hidden attractor* was introduced
and hidden chaotic attractor was found for the first time:

[Leonov G.A., Kuznetsov N.V., Vagaitsev V.I, Localization of hidden Chua's attractors, Physics Letters A, 375(23), 2011, 2230-2233]

$$\dot{x} = \alpha(y - x - m_1 x - \psi(x))$$
$$\dot{y} = x - y + z, \dot{z} = -(\beta y + \gamma z)$$
$$\psi(x) = (m_0 - m_1)\mathrm{sat}(x)$$

$\alpha = 8.4562, \beta = 12.0732$
$\gamma = 0.0052$
$m_0 = -0.1768, m_1 = -1.1468$

equilibria: stable zero F_0 & 2 saddles $S_{1,2}$
trajectories: 'from' $S_{1,2}$ tend (black) to zero F_0 or tend (red) to infinity;
Hidden chaotic attractor (in green) with positive Lyapunov exponent



$$\begin{cases} \dot{x} = F(x), \; x \in \mathbb{R}^n, & F(x_0) = 0 \\ x(t) \equiv x_0, A = \frac{dF(x)}{dx}\Big|_{x=x_0} \end{cases} \quad \begin{cases} \dot{y} = Ay + o(y) \\ y(t) \equiv 0, (y = x - x_0) \end{cases} \quad \begin{cases} \dot{z} = Az \\ z(t) \equiv 0 \end{cases}$$

✓stationary: $z(t) = 0$ is exp. stable $\Rightarrow$ $y(t) = 0$ is asympt. stable

$$\begin{cases} \dot{x} = F(x), & \dot{x}(t) = F(x(t)) \not\equiv 0 \\ x(t) \not\equiv x_0, A(t) = \frac{dF(x)}{dx}\Big|_{x=x(t)} \end{cases} \quad \begin{cases} \dot{y} = A(t)y + o(y) \\ y(t) \equiv 0, (y = x - x(t)) \end{cases} \begin{cases} \dot{z} = A(t)z \\ z(t) \equiv 0 \end{cases}$$

? nonstationary: $z(t) = 0$ is exp. stable $\Rightarrow ?$ $y(t) = 0$ is asympt. stable

! Perron effect: $z(t) = 0$ is exp. stable(unst), $y(t) = 0$ is exp. unstable(st)

Positive largest Lyapunov exponent doesn't, in general, indicate chaos

Survey: G.A. Leonov, N.V. Kuznetsov, Time-Varying Linearization and the Perron effects, International Journal of Bifurcation and Chaos, Vol. 17, No. 4, 2007, pp. 1079-1107 (doi:10.1142/S0218127407017732)

Further examples of hidden attractors

J.C. Sprott, A conservative dynamical system with a strange attractor, Physics Letters A, **2014**
Wei Z., A new finding of the existence of hyperchaotic attractors with no equilibria, Mathematics and Computers in Simulation, 100, **2014**
C. Li, J.C. Sprott, Coexisting hidden attractors in a 4-d simplified Lorenz system, IJBC, **2014**
C. Li, J.C. Sprott, Chaotic flows with a single nonquadratic term, Physics Letters A, 378, **2014**
Z. Wei, Hidden hyperchaotic attractors in a modified Lorenz-Stenflo system with only one stable equilibrium, Int. J. of Bif.&Chaos, **2014**
Q. Li, H. Zeng, X.-S. Yang, On hidden twin attractors and bifurcation in the Chua's circuit, Nonlinear Dynamics, **2014**
H. Zhao, Y. Lin, Y. Dai, Hidden attractors and dynamics of a general autonomous van der Pol-Duffing oscillator, Int. J. of Bif.&Chaos, **2014**
S.-K. Lao, Y. Shekofteh, S. Jafari, J.C. Sprott, Cost function based on Gaussian mixture model for parameter estimation of a chaotic circuit with a hidden attractor, Int. J. of Bif.&Chaos, 24(1), **2014**
U. Chaudhuri, A. Prasad , Complicated basins and the phenomenon of amplitude death in coupled hidden attractors, Physics Letters A, 378, **2014**
M. Molaie, S. Jafari, J.C. Sprott, S.M.R.H. Golpayegani, Simple chaotic flows with one stable equilibrium, Int. J. of Bif.&Chaos, 23(11), **2013**
Jafari S., Sprott J., Simple chaotic flows with a line equilibrium, Chaos, Sol & Fra, 57, **2013**
C. Li, J.C. Sprott, Multistability in a butterfly flow, Int. J. of Bif.&Chaos, 23(12), **2013**

Survey: Leonov G.A.,Kuznetsov N.V., Hidden attractors in dynamical systems. From hidden oscillations in Hilbert-Kolmogorov, Aizerman, and Kalman problems to hidden chaotic attractor in Chua circuits, Int. J. of Bifurcation and Chaos, 23(1), **2013**, 1-69 (doi: 10.1142/S0218127413300024)

Other our papers on hidden attractors

✓ Kuznetsov N., Kuznetsova O., Leonov G., Vagaitsev V., Analytical-numerical localization of hidden attractor in electrical Chua's circuit, **Lecture Notes in Electrical Engineering**, Volume 174 LNEE, **2013**, Springer, 149-158 (doi:10.1007/978-3-642-31353-0_11)
✓ Leonov G.A., Kuznetsov N.V., Vagaitsev V.I., Hidden attractors in smooth Chua's systems, **Physica D**, 241(18) **2012**, 1482-1486 (doi:10.1016/j.physd.2012.05.016)
✓ V.O. Bragin, V.I. Vagaitsev, N.V. Kuznetsov, G.A. Leonov Algorithms for finding hidden oscillations in nonlinear systems. The Aizerman and Kalman Conjectures and Chua's Circuits, **J. of Computer and Systems Sciences Int.**,50(4),**2011**, 511-544 (doi:10.1134/S106423071104006X)(survey)
✓ Leonov G.A., Kuznetsov N.V., Vagaitsev V.I, Localization of hidden Chua's attractors , **Physics Letters A**, 375(23), **2011**, 2230-2233 (doi:10.1016/j.physleta.2011.04.037)
✓ G.A. Leonov, N.V. Kuznetsov, O.A. Kuznetsova, S.M. Seledzhi, V.I.Vagaitsev, Hidden oscillations in dynamical systems, **Transaction on Systems and Control**, 6(2), **2011**, 54–67 (survey)
✓ G.A. Leonov, N.V. Kuznetsov, Algorithms for searching for hidden oscillations in the Aizerman and Kalman problems, **Doklady Mathematics**, 84(1), **2011**, 475-481 (doi:10.1134/S1064562411040120)
✓ Leonov G.A., Kuznetsov N.V., Analytical-numerical methods for investigation of hidden oscillations in nonlinear control systems, **IFAC Proceedings Volumes (IFAC-PapersOnline)**, 18(1), **2011**, 2494-2505 (doi:10.3182/20110828-6-IT-1002.03315) (survey paper)
✓ Kuznetsov N.V., Leonov G.A., Seledzhi S.M., Hidden oscillations in nonlinear control systems, **IFAC Proceedings Volumes (IFAC-PapersOnline)**, 18(1), **2011**, 2506-2510
✓ G.A. Leonov, V.O. Bragin, N.V. Kuznetsov, Algorithm for constructing counterexamples to the Kalman problem. **Doklady Mathematics**, 82(1), **2010**, 540-542 (doi:10.1134/S1064562410040101)
✓ G.A. Leonov, V.I. Vagaitsev, N.V. Kuznetsov, Algorithm for localizing Chua attractors based on the harmonic linearization method, **Doklady Mathematics**, 82(1), **2010**, 663-666
✓ N.V. Kuznetsov, G.A. Leonov and V.I. Vagaitsev, Analytical-numerical method for attractor localization of generalized Chua's system, **IFAC Proc. Vol. (IFAC-PapersOnline)**, 4(1), **2010**, 29-33

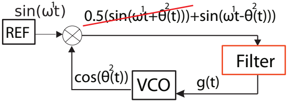
Phase-locked loops (PLL): history

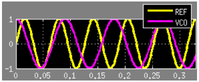
- Radio and TV
  - de Bellescize H., La réception synchrone, L'Onde Électrique, 11, 1932
  - Wendt, K. & Fredentall, G. Automatic frequency and phase control of synchronization in TV receivers, *Proceedings IRE*, 31(1), 1943
- Computer architectures (frequency multiplication)
  Ian Young, PLL in a microprocessor i486DX2-50 (1992)
  (in Turbo regime stable operation was not guaranteed)

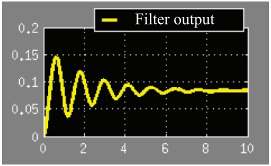
- Theory and Technology
  ✓ F.M.Gardner, *Phase-Lock Techniques*, 1966
  ✓ A.J. Viterbi, *Principles of Coherent Comm.*, 1966
  ✓ W.C.Lindsey, *Synch. Syst. in Comm. and C.*, 1972
  ✓ W.F.Egan, *Freq. Synthesis by Phase Lock*, 2000
  ✓ B. Razavi, *Phase-Locking in High-Perf. Syst.*, 2003
  ✓ R.Best, *PLL: Design, Simulation and Appl.*, 2003
  ✓ V.Kroupa, *Phase Lock Loops and Freq. Synthesis*, 2003

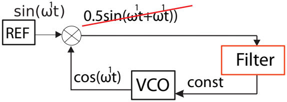
PLL analysis & design

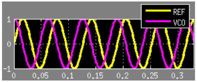
PLL operation: generation of electrical signal (voltage), phase of which is automatically tuned to phase of input reference signal (after transient process the signal, controlling the frequency of tunable osc., is constant)











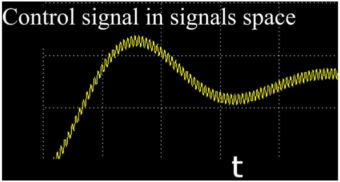
Design: Signals class (sinusoidal,impulse...), PLL type (PLL,ADPLL,DPLL...)

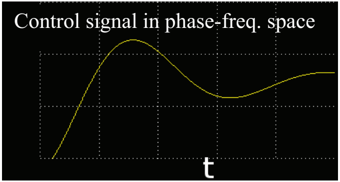
Analysis: Choose PLL parameters (VCO, PD, Filter etc.) to achieve stable operation for the desired range of frequencies and transient time

Analysis methods: simulation, linear analysis, nonlinear analysis of mathematical models in *signal space* and *phase-frequency space*

PLL analysis & design: simulation

D.Abramovitch, ACC-2008 plenary lecture:
Full simulation of PLL in signal/time space is very difficult since one has to observe simultaneously very fast time scale of input signals and slow time scale of signal's phase.
How to construct model of signal's phases?

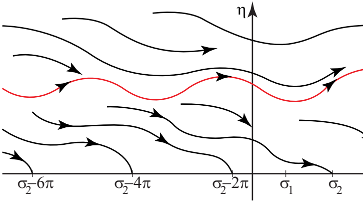




Simulation in *phase space*: Could stable operation be guaranteed for all possible inputs, internal blocks states by simulation?
N.Gubar' (1961), hidden oscillation in PLL

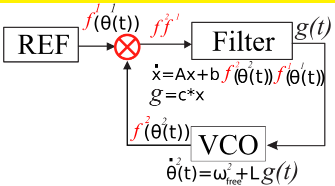
$$\dot{\eta} = \alpha\eta - (1 - a\alpha)(\sin(\sigma) - \gamma),$$
$$\dot{\sigma} = \eta - a(\sin(\sigma) - \gamma)$$

global stability in simulation, but only bounded region of stability in reality



Classical PLL: models in time & phase-freq. domains

?) Control signals in signals and phase-frequency spaces are equal.
?) Qualitative behaviors of signals and phase-freq. models are the same.



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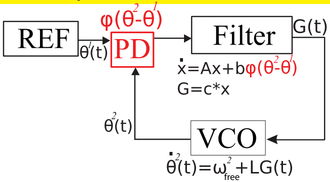


$$\dot{\theta}^j \geq \omega_{min},\ |\dot{\theta}^1-\dot{\theta}^2| \leq \Delta\omega,\ |\dot{\theta}^j(t)-\dot{\theta}^j(\tau)| \leq \Delta\Omega,\ \ |\tau-t| \leq \delta,\ \forall \tau, t \in [0,T]\ \ (*)$$

waveforms: $f^p(\theta) = \sum_{i=1}^{\infty}\left(a_i^p \sin(i\theta) + b_i^p \cos(i\theta)\right), \quad p = 1,2$

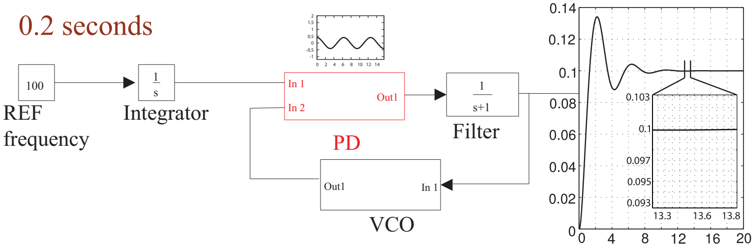
$b_i^p = \frac{1}{\pi}\int_{-\pi}^{\pi} f^p(x)\sin(ix)dx, \quad a_i^p = \frac{1}{\pi}\int_{-\pi}^{\pi} f^p(x)\cos(ix)dx,$

Thm. If (*), $\varphi(\theta)=\frac{1}{2}\sum_{l=1}^{\infty}\left((a_l^1 a_l^2 + b_l^1 b_l^2)\cos(l\theta) + (a_l^1 b_l^2 - b_l^1 a_l^2)\sin(l\theta)\right)$
then $|G(t)-g(t)| = O(\delta),\ \forall\, t \in [0,T]$

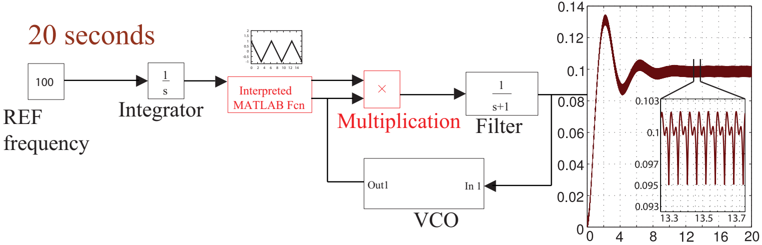
Leonov G.A., Kuznetsov N.V., Yuldahsev M.V., Yuldashev R.V., Analytical method for computation of phase-detector characteristic, IEEE Transactions on Circuits and Systems Part II, vol. 59, num. 10, 2012 (doi:10.1109/TCSII.2012.2213362)

PLL simulation: Matlab Simulink

0.2 seconds



20 seconds



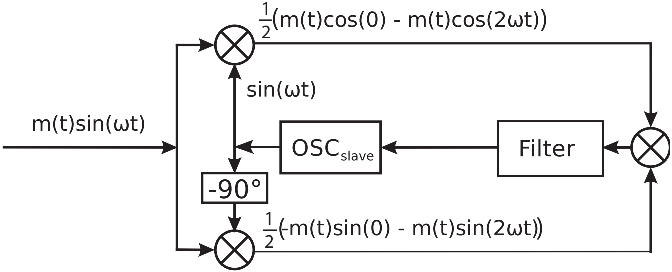
Kuznetsov,Leonov et al., Patent 2449463, 2011

Kuznetsov, Leonov, Neittaanmaki et al., Patent appl. FI20130124, 2013

Costas loop: digital signal demodulation

John P. Costas. General Electric, 1950s.

- signal demodulation in digital communication



$m(t) = \pm 1$ — data,
$m(t)\sin(2\omega t)$ and $m(t)\cos(2\omega t)$
— can be filtered out,
$m(t)\cos(0) = m(t)$,
$m(t)\sin(0) = 0$

- wireless receivers
- Global Positioning System (GPS)

Questions