Treatment of Inverse Problems with applications

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In the case of indirect measurements where the quantity of interest of are not direct measurable we deal with an inverse problem. In the past two decades the solution of inverse problems establishes an important field of applied mathematics. In praxis different objectives are pursued: parameter identification, control problems and optimal design. In addition to the numerical difficulties of direct problems the challenge of inverse problem consists in the ill-posedness, this means the violation of at least one of the three properties: existence, uniqueness and stability. Techniques known as regularization methods have been developed to transfer an ill-posed problems into a well-posed one. However, the problem usually remains ill-conditioned, and therefore, a reliable modelling of the direct problem is of particular importance.

After an introduction to the background and difficulties encountered with ill-posed problems the talk will go into regularization strategies differing for linear and nonlinear problems. Generally speaking, a regularization employs objective and subjective a priori information about the solution and the measurement uncertainty of the data. In this connection the choose of the regularization parameter plays a decisive role, the magnitude and the question ‘a priori or a posteriori’. Whereas for linear problems various effective tools exists including error estimation, the situation in nonlinear case remains an open field.

The strategies are demonstrated by three examples of indirect measurements: inverse scatterometry, determination of thermal transport properties, and optical tomography. All underlying direct models are based on partial differential equations and solved by the finite element method. However, differences are in the type of measurements and quantity of interest requiring a different treatment.

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