Efficient a posteriori error estimates for nonconforming approximation of elliptic problems based on Helmholtz type decomposition of the error

S.K. Tomar
Johann Radon Institute for Computational and Applied Mathematics (RICAM),
Austrian Academy of Sciences, Altenbergerstr. 69, A-4040 Linz, Austria
satyendra.tomar@ricam.oeaw.ac.at

(Joint work with S. Repin, St. Petersburg, Russia)

Abstract

In this talk a new type of a posteriori error estimates for nonconforming approximations of elliptic problems will be presented. The method is based on Helmholtz type decomposition of the error expressed in terms of fluxes. Such a decomposition of the nonconforming error has been suggested and studied in [1, 2] where it was shown that the error can be presented as the sum of two functions (a gradient term and a divergence-free term), which are the exact solutions of two auxiliary problems. A similar decomposition of the error is followed in this work. However, the procedure for obtaining computable two-sided bounds of the norms of these solutions is different, for which the method suggested in [4] for conforming approximations is used. A posteriori estimates obtained in this way differ from those which are based on a different type of decomposition of the error and the projection of the nonconforming approximation to the conforming space (see [3, 5]). Numerical experiments confirm that these new estimates provide very accurate error bounds and can be efficiently exploited in practical computations.

Key words: Discontinuous Galerkin FEM; a posteriori error estimates; Helmholtz decomposition.

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