

Book of Abstracts  
Computational Methods in  
Applied Mathematics (CMAM-7)  
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University of Jyväskylä, Finland

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# Foreword

The Conference is organized under the aegis of the journal Computational Methods in Applied Mathematics (CMAM) and will be focused on various aspects of mathematical modeling and numerical methods for problems arising in natural sciences and engineering. Biennial CMAM-7 meetings are aimed at fostering cooperation between researchers working in the area of theoretical numerical analysis and applications to modeling, simulation, and scientific computing. Another goal of the CMAM-7 meetings is to improve existing contacts and to establish new ones between scientists from the West and the East. This is the 7th conference following earlier ones in Minsk in 2003, Trakai in 2005, Minsk in 2007, Bedlewo in 2010, Berlin in 2012, and Strobl 2014.

The CMAM-7 conference is organized by the joint forces of Department of Mathematical Information Technology (MIT), University of Jyväskylä, Finland, and St. Petersburg Department of V.A. Steklov Institute of Mathematics of the Russian Academy of Sciences, St. Petersburg, Russia.

The CMAM-7 meeting is dedicated to a remarkable jubilee in the history of computational mathematics: 100 years of Galerkin method. This method is one of the keystones of modern numerical analysis. Also, the ideas encompassed in the Galerkin method have seriously influenced development of the theory of partial differential equations in 20th century. It is planned that plenary and contributed talks of CMAM-7 will expose recent advances and trends in mathematical modeling, many of which bring their origins in the Galerkin method.

# History of CMAM-7

In the millennium year 2000, the Institute of Mathematics of the National Academy of Belarus took the decision to start a new international scientific journal. Their main motivation was to strengthen international cooperation between East and West in the field of numerical analysis. A prominent figure in computational mathematics and mathematical physics, the Russian academician Alexander Samarskii, agreed to head the new project; and a little later, the renowned Swedish numerical analyst Vidar Thomée also lent his support. Belarussian businessman Alex Yakoubenia was instrumental in funding the new project.

At that time it was very challenging to establish a new scientific journal; nevertheless the first issue of the journal, called "Computational Methods in Applied Mathematics" (CMAM), appeared in 2001. The journal's title was suggested by Pieter Hemker, who together with Raytcho Lazarov invested much effort into the establishment and development of the journal. Professors Samarskii and Thomée helped to assemble a prestigious Editorial Board consisting of well-known numerical analysts from all over the world, and CMAM established a high scientific standard under its Managing Editor, Piotr Matus. CMAM's subject areas include initial value and boundary value problems for differential equations and integral equations that are linked to applied mathematics and mathematical physics. Theoretical contributions, numerical algorithms and computer simulations all lie within the scope of the journal.

In 2011 Carsten Carstensen became Editor-in-Chief of CMAM. His energetic leadership and high scientific reputation have established CMAM as a high-quality journal with a short reviewing time and almost no space restrictions. It has been accepted by Thomson Reuters for inclusion in their Web of Science database, starting with volume 15, i.e., from the beginning of 2015; its citations there will be evaluated and will soon lead to an impact factor for the journal. In the MathSciNet database, CMAM has an MCQ (Mathematical Citation Quotient) of 0.56, while the average MCQ over all journals is 0.40.

In 2003 the CMAM Editorial Board decided to hold biennial international conferences entitled "Computational Methods in Applied Mathematics" under the aegis of the journal. The scope of these conferences coincides with the scope of the journal described above. A goal of these CMAM forums is to establish new contacts and to improve existing links between numerical analysts from West and East. The Editors also regard CMAM conferences as a venue for its editorial board members to meet and to discuss further development of the journal.

The previous conferences in this series took place in Minsk, Belarus (2003), Trakai, Lithuania (2005), Minsk, Belarus (2007), Bedlewo, Poland (2010), Berlin, Germany (2012) and Strobl, Austria (2014). At the CMAM-3 conference in Minsk, Vidar Thomée was named Honorary Editor of CMAM in recognition of his strong influence and committed involvement and his pioneering efforts in starting the journal.

The forthcoming CMAM-7 conference will be organized jointly by the Department of Mathematical Information Technology (MIT) at the University of Jyväskylä, Finland, and the St. Petersburg Department of the V.A. Steklov Institute of Mathematics of the Russian Academy of Sciences, St. Petersburg, Russia. This cooperation between West and East is evidence of the increasing popularity of the CMAM conferences, which are becoming leading international forums for numerical analysis.

The current structure of the CMAM Editorial Board is as follows: one Editor-in-Chief (Carsten Carstensen), four Senior Editors (Ivan Gavriluk, Ulrich Langer, Piotr Matus, and Petr Vabishchevich), with Piotr Matus as Managing Editor. In addition the editorial board contains several Associate Editors.

CMAM is a De Gruyter journal. Older papers can be downloaded online and new issues of the journal are distributed by De Gruyter; see <http://www.degruyter.com/view/j/cmam>.

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# General Information

## Registration and Help Desk

The conference registration and help desk are located in the Agora lobby on the first floor. Staff will be available to register participants and answer any queries you may have. The desk will be open at the following times:

- Sunday 31.7. July 17:00-19:00 (registration in Hotel Alba, next to Agora building)
- Monday 1.8. August 9:00-17:30
- Tuesday 2.8. August 8:30-16:30
- Wednesday 3.8. August 8:30-18:30
- Thursday 4.8. August 8:30-16:00
- Friday 5.8. August 8:30-15:00

The local organizing committee and conference assistants will be happy to provide you with information on the conference, identifiable by their blue name badge.

## Getting to Campus

The conference venue is located in Mattilanniemi Campus, Mattilanniemi 2. The city bus station can be found between the Forum shopping centre and the station square on Vapaudenkatu 40. Buses number 5K, 5 and 20 stop at Agora building. A single ticket costs 3 euros. You can get to the campus by foot in 25 minutes from the centrum.

## Conference Venue

The conference will take place in Agora building at the University of Jyväskylä, Mattilanniemi 2, Jyväskylä, Finland.

## Meals

### Lunch

If lunches are included in your registration, you will get lunch tickets with your badge. Lunch will be served in the University canteen Piato, located in Agora lobby. You can choose a dish from four different choices; meat, fish, vegetarian and soup, for vegan dish please ask the canteen staff. The price of lunch is 8,00 EUR / normal, 5,00 EUR / foreign student cards and 2,60 EUR / for ISIC and Finnish student card holders.

### Coffee breaks

Coffee breaks are located in Agora lobby. Coffee, tea, water and small snacks are provided.

### Conference Dinner

If conference dinner is included in your registration, you will get a dinner voucher with your badge. Conference dinner is arranged as a dinner cruise on m/s Rhea. The cruise departs from Jyväskylä Harbor, 15 minute walk from the conference venue. One of our organizers will walk with you down the picturesque promenade beside the lake Jyväsjärvi to the harbor. You can enjoy Rhea's drinks and delicacies as it sails around the beautiful archipelago of Nothern Päijänne.

### Local places to eat

**Pöllöwaari** has been titled as the best restaurant in Jyväskylä. Most dishes are made of ingredients produced in the region. Nordic-style fine dining in a romantic and relaxing atmosphere. Ylipistonkatu 23.

**Viking restaurant Harald** takes you on a voyage back to the age of the Vikings. You can experience Viking-style milieu while enjoying medieval food and beverages. Kauppakatu 33.

**Base Camp** offers you delicious Nepalese cuisine and warm hospitality in the heart of the city. The

restaurant has been praised for its vegetarian food. Yliopistonkatu 38.

*Katriina* is the coziest vegan restaurant in town, favored by numerous students due to its fair student prices and fresh ingredients. Kauppakatu 11.

*Teeleidi* offers a heaven for tea lovers, with diverse collection of teas from around the world. Over a century old wooden house brings you the right atmosphere to relax and try out their freshly made cakes and cookies. Schaumanin puistotie 2 A 2.

In addition to these examples, more restaurants can be found from Tripadvisor Jyväskylä.

## Important Numbers

TAXI: +358 100 6900

Emergency: 112

Help Desk: +358400247458

## Internet Access

A personal guest username and password for jyu-guest network will be given to all conference participants with their badge. The University of Jyväskylä is an eduroam Wi-Fi location. Visitors whose institution uses eduroam Wi-Fi can use this service at the University of Jyväskylä. However, before arrival you are strongly advised to set up your laptop or smartphone for eduroam at your home institution. You should be able to log in using your home institution username and password. In some areas of the venue there is free access to agora-open network.

## Cash Machines

You can exchange currency effortlessly at Forex Bank in the city centre, Kauppakatu 7. Automated teller machines are located in the centrum, identifiable by orange "Otto" sign.

## Post Offices

A post office can be found in the centrum of Jyväskylä in Vapaudenkatu 48-50. The post office is open Mon-Fri 8.00-20.00 and Saturdays 10.00-15.00. Please note that the opening hours might change during the summer season, so remember to check the opening hours.

## Certificate of Attendance

If you require a certificate of attendance please ask at the help desk. A receipt for payment have been emailed to you at the time of booking.

## Optional Social Program

If the weather permits, a guided walk around the campus of the university will be arranged on Saturday 6th August. You can ask more information from the registration desk.

## Attractions in Jyväskylä

Jyväskylä's compact city centre makes your congress experience as our guest even more rewarding. Campuses, vibrant city centre, hotels and event venues are all located within walking distance of one another, surrounded by beautiful lake scenery. Below you will find just a few of those many interesting points in Jyväskylä, and all except for the museums are free to enjoy:

- Harju ridge and Vesilinna observation tower. Harju is a pine forest haven in the middle of the city with many beautiful paths and trails for pedestrians. The main route from the city centre to Harju runs through the majestic Nero stairs at the top end of Gummeruksenkatu. On top of Harju, Vesilinna tower serves as an observation tower with magnificent views in all directions, as well as the location of Vesilinna Restaurant and the Natural History Museum of Central Finland. Opening hours for the museum Tue-Fri from 11am to 6pm, Sat-Sun from 12 noon to 5pm. Opening hours for the restaurant Mon-Fri from 11am to 23pm, Sat from 12 noon to 23pm. Free entrance.
- Jyväskylä Harbour, located in Satamakatu. From the conference venue, walk down the pictur-

esque promenade beside the lake Jyväsjärvi to the harbour, just a short hop from the centre of Jyväskylä. Besides boats and sauna rafts, the harbour offers cafés, bike rental and boat cruises around the stunning lakes of Jyväskylä area. Most cruises leave in the afternoon and evening. The harbour café is open Mon-Sun 8am-11pm.

- Jyväskylä marketplace offers a wide range of fresh vegetables and berries, fish, pastries and local handicrafts. A perfect place to try seasonal produce and local delicacies. Located in Yliopistonkatu 15. Open Mon-Fri from 6.30am to 6pm, Sat from 6.30am to 3pm
- Toivola Old Courtyard is one of the most unique attractions in Jyväskylä, taking you back in time to the late 19th century. Toivola Old Courtyard consists of seven beautiful wooden buildings, serving as artisans' workshops, boutiques and a museum. In addition they offer workshops, guided tours for groups, exhibitions and bike rental. Café Muisto serves light lunch and an impressive selection of fresh pastries and cakes. Located in Cygnaeuksenkatu 2, open Mon-Fri from 10am to 5pm, Sat from 10am to 3 pm. Free entrance.
- Tourujoki Nature Trail lets you experience the serenity of wild Finnish nature just a hop away from the centrum. Though being only 700m long, the trail and nature reserve give home to diverse variety of plants and small animals. The trail has many stairs, as well as duckboards. Located in Tourukatu 25, 30 minute walk from conference venue, 10 minutes from the centrum.

## Museums

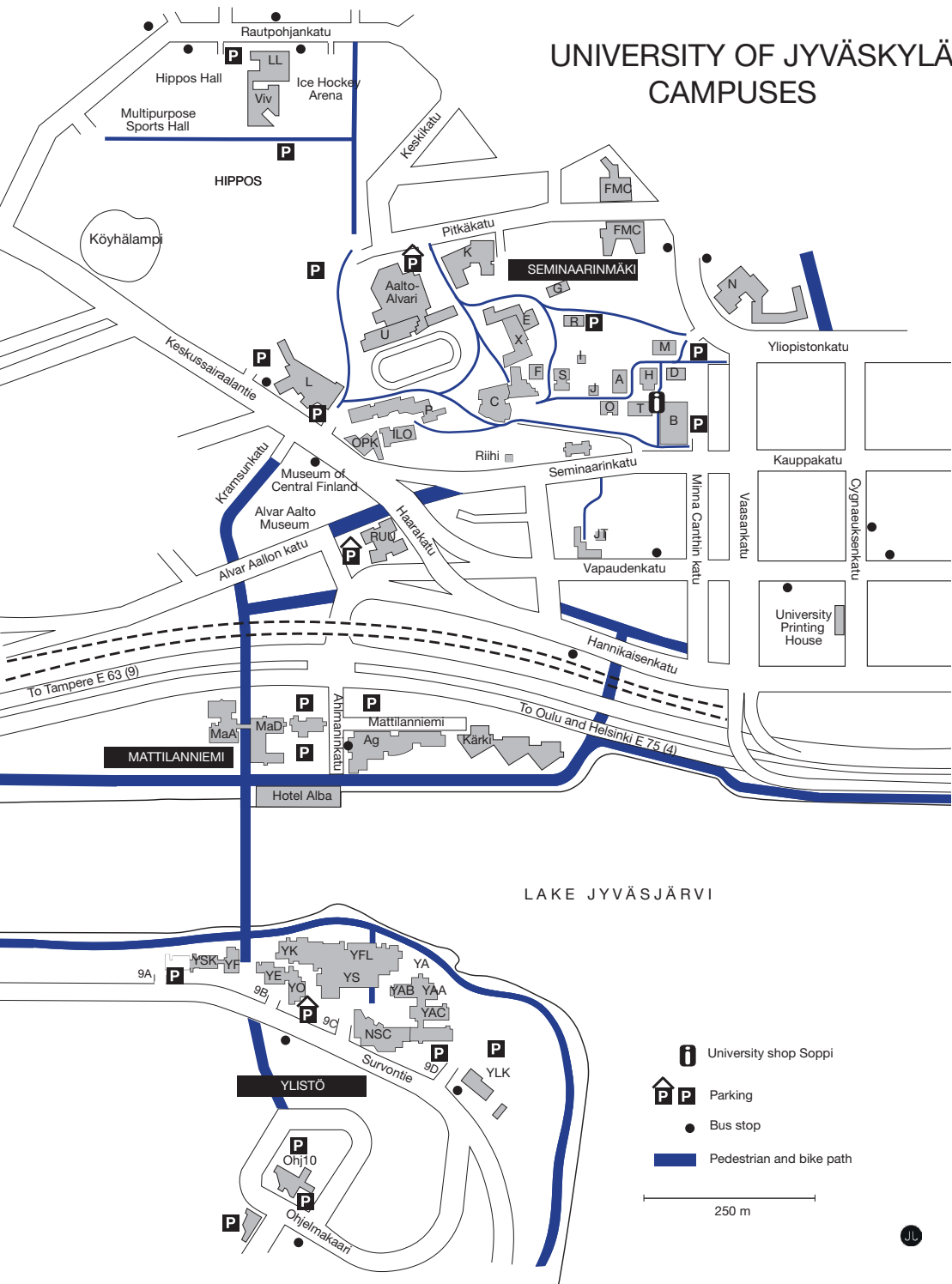
- Alvar Aalto Museum focuses on fostering the heritage of Alvar Aalto, the famous Finnish architect and designer. All furniture in the museum is designed by Aalto. Located in Alvar Aallon katu 7, opening hours Tue-Fri from 10am to 6pm, Sat-Sun 11am to 6pm. Admission 6 EUR (regular) / 4 EUR (students, unemployed, senior citizens)
- Craft Museum of Finland is a partly interactive museum specialized in different techniques of handicraft, from house building to lace making. Some parts of the exhibition let you use all your senses - you can touch, smell and taste - and some give you a chance to do things for yourself. The museum is also the centre for national costumes and textiles. From the museum shop you can find quality Finnish craft products and traditional souvenirs. Open Tue-Fri, Sun from 11am to 6pm, Sat 11am - 4pm, admission 6 EUR. Free admission for under 18's and on Fridays.
- Museum of Central Finland serves as the town museum of Jyväskylä and the provincial museum of Central Finland. Specializing in cultural history, the museum exhibits in a most illustrative way the history of Central Finland from prehistory to our time. Open from Tuesday to Sunday 11am-6pm. Admission 6 EUR/adult, 3 EUR/student. Free admission for under 18's.
- Jyväskylä University Museums, founded over a century ago, collects and exhibits material related to the past and the present of the University of Jyväskylä as well as the diversity of nature in Central Finland. The cultural history section is located in Seminaarinkatu 15 Building G, 10min walk from conference venue. Open Wed-Fri 12am-5pm, Sat 12am-4pm. The natural history section is located in Vesilinna observation tower, Ihantolantie 5. Open Tue-Fri 11am-6pm, Sat-Sun 12am-5pm. Free admission.

## Saunas

- Rantasipi is a relaxing Spa Hotel in Laajavuori, Laajavuorentie 30, 5 kilometers from the conference venue. You can enjoy the pool area and different saunas for 3 hours, Mon-Fri 7.00-14.00 13,5eur/adult and Sat-Sun or daily after 14.00 18eur/adult. Discount with official student card. Swimsuit rental 5eur, towel rental 3,5eur.
- SS Kippari and M/S Löyly are sauna rafts that operate on lake Jyväsjärvi from the harbor. You can book the rafts for personal two hour cruise with a sauna. Maximum capacity 12 persons. Prices range from 290 to 420 euros.
- Several hotels in Jyväskylä offer sauna facilities for their guests. Ask from the reception!

Jyväskylä Tourist Information - Asemakatu 7, opening hours Mon-Fri 9am - 5pm.

# University campus and Jyväskylä map



# Plenary Sessions

# PETROV-GALERKIN FINITE ELEMENT METHOD FOR FRACTIONAL CONVECTION-DIFFUSION EQUATIONS

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## ABSTRACT

Boundary value problems for fractional order convection-diffusion equations arise in mathematical modeling of asymmetric subdiffusion and transport processes in highly heterogeneous media. In this work we continue the line of study in [1, 2, 3] by developing variational formulations of Petrov-Galerkin type for the one-dimensional case involving either Riemann-Liouville or Caputo fractional derivatives of order  $\alpha \in (3/2, 2)$  in the leading term.

First, the well-posedness of the formulations and sharp regularity pickup of the weak solutions are established. Based on [2] we present a novel finite element method, which employs continuous piecewise linear finite elements and “shifted” fractional powers for the trial and test space, respectively. The new approach has a number of distinct features: (1) it allows deriving optimal error estimates in both  $L^2$ - and  $H^1$ -norms and (2) it produces well conditioned linear systems, since for uniform meshes the leading term of the stiffness matrix is diagonal. Further, in the case of Riemann-Liouville derivative, an enriched FEM is proposed to improve the convergence. Finally, extensive numerical results are presented to verify the theoretical analysis and robustness of the numerical scheme.

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**Raytcho Lazarov** Professor, Department of Mathematics, Texas A&M, College Station, TX, USA.



# GEOMETRIC DECOMPOSITION OF FINITE ELEMENT SPACES

Ragnar Winther and Richard S. Falk

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## ABSTRACT

The purpose of this talk is to discuss properties of geometric decompositions of finite element spaces and some consequences. In addition to most of the familiar finite element spaces of scalar and vector valued functions, the discussion will also cover the standard spaces of differential forms appearing in finite element exterior calculus. Geometric decompositions of finite element spaces are closely tied to the degrees of freedom, which appear to be fundamentally linked to the degree of the piecewise polynomial spaces. However, a surprising fact is that the underlying geometric structure is independent of both polynomial degree and the index of the differential forms. This fact motivates the construction of the bubble transform, a new tool for the analysis of finite element spaces, with potential applications both to  $h$  and  $p$ -methods. Important applications are the constructions of local projection operators, and the construction of local bases and frames.



Professor **Ragnar Winther** (Department of Mathematics, University of Oslo) received his Ph.D. from Cornell University, USA, with Professor James H. Bramble as adviser. Most of his research is related to numerical methods for partial differential equations. In particular, he is known for his work on preconditioning, and for his collaboration with D. N. Arnold and R.S. Falk on the development of finite element exterior calculus.

# ON THE MOTION OF RIGID SOLID PARTICLES IN INCOMPRESSIBLE VISCOELASTIC LIQUIDS: A NUMERICAL APPROACH

Roland Glowinski and T.W. Pan

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## ABSTRACT

Our main goal in this lecture is to discuss the numerical solution of the system of ordinary and partial differential equations modelling the motion of rigid solid particles in a region filled with an incompressible viscoelastic liquid. The above system is a coupling between the unsteady Navier-Stokes equations modelling the flow of the liquid, the Newton-Euler equations modelling the motion of the particles, and the differential equation modelling the evolution of the elastic component of the stress tensor. The viscoelastic models to be considered are of the Oldroyd-B and FENE-CR (for Chilcott & Rallison) types. The key ingredients of the methodology we employ to achieve the numerical simulation of the above mixtures are:

- (i) An operator-splitting scheme a la S. Lie for the time discretization.
- (ii) A wave-like reformulation of the advective parts of the model.
- (iii) The Lozinski-Owens approach for the treatment of the elastic component of the stress tensor (it relies on the Cholesky factorization of the elasticity tensor).
- (iv) A distributed Lagrange multiplier based fictitious domain method to take into consideration the deformation of the flow region as  $t$  varies.
- (v) Lagrangian finite element approximations of the velocity, pressure and elasticity tensor fields.

The results of numerical experiments will be presented: they concern the settling of rigid solid particles subjected to gravity in an infinitely long vertical cylinder filled with an incompressible visco-elastic liquid. These experiments show, as expected, particle chaining phenomena taking place in the cylinder, with Oldroyd-B and FENE-CR fluids having significantly different chaining behaviors.

The variational approach taken in several of the fractional steps makes this presentation appropriate for a conference dedicated to the *100 Years of the Galerkin Method*.



**Roland Glowinski** is a Cullen Professor of Mathematics at University of Houston, a Distinguished Emeritus Professor at University P. & M. Curie in Paris, a Docent Professor at University of Jyväskylä and a Distinguished Visiting Professor at the Hong Kong Baptist University. He is also a Member of the French National Academy of Sciences, of the French National Academy of Technology, and of the Academia Europaea. He is a Honorary Doctor of the University of Jyväskylä, and a Honorary Professor of the Fudan University, Shanghai, China. Dr Glowinski is a SIAM Fellow, a Fellow of the AMS. He is a laureate of the SIAM Von Kármán Prize and has authored or co-authored 10 books and more than 450 articles, his most recent book being: *Variational Methods for the Numerical Solution of Nonlinear Elliptic Problems*, SIAM, Philadelphia, PA, 2015 a publication in the spirit of this Galerkin dedicated conference.

# DIRECTIONAL $\mathcal{H}^2$ -MATRICES FOR HELMHOLTZ INTEGRAL OPERATORS

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## ABSTRACT

Boundary Element Methods (BEM) are an important tool for the numerical solution of acoustic and electromagnetic scattering problems. These BEM matrices are fully populated so that data-sparse approximations are required to reduce the complexity from quadratic to log-linear. For the high-frequency case of large wavenumber, standard blockwise low-rank approaches are insufficient. One possible data-sparse matrix format for this problem class that can lead to log-linear complexity are *directional  $\mathcal{H}^2$ -matrices*. We present a full analysis of a specific incarnation of this approach.

Directional  $\mathcal{H}^2$ -matrices are blockwise low rank matrices, where the block structure is determined by the so-called parabolic admissibility condition. In order to achieve log-linear complexity with this admissibility condition, a nested multilevel structure is essential that provides a data-sparse connection between clusters of source and target points on different levels. We present a particular variant of directional  $\mathcal{H}^2$  matrices in which all pertinent objects are obtained by polynomial interpolation. This allows us to rigorously establish exponential convergence in the block rank in conjunction with log-linear complexity.

The work presented here is joint with S. Börm (Kiel).

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**Markus Melenk** studied mathematics at the University of Stuttgart. After the Vordiplom, he moved to the University of Maryland, where he obtained his Master of Arts and his PhD (1995) under the supervision of Ivo Babuska. After his PhD, he moved to ETH Zurich, where he obtained his habilitation (2000). He then joined the research group of Prof. Hackbusch at the Max-Planck-Institute for Mathematics in the Sciences in Leipzig. In 2004, he became a faculty member at the University of Reading. In 2005, joined Technische Universität Wien as the professor of computational mathematics.

# EFFICIENT PRECONDITIONERS FOR EDGE ELEMENT SYSTEMS FOR VARIOUS MAXWELL EQUATIONS

Jun Zou

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## ABSTRACT

In this talk we shall present several efficient preconditioners for solving the edge element systems arising from the discretization of various Maxwell systems, including the  $H(\text{curl})$  system, the time-harmonic Maxwell equations and the PML systems for Maxwell scattering problems. The motivations and derivations of the preconditioners shall be discussed, and the spectral estimates of the preconditioned systems will be presented.

The main results of the talk are based on several joint works with Qiya Hu (Chinese Academy of Sciences), Shi Shu (Xiangtan University), Na Huang (Fujian Normal University) and Hua Xiang (Wuhan University), and the research projects have been substantially supported by Hong Kong RGC grants (projects 14306814 and 405513).

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**Jun Zou** is a professor at Department of Mathematics, the Chinese University of Hong Kong. Before taking up his current position in Hong Kong, he had worked for two years (93-95) in University of California at Los Angeles as a post-doctoral fellow and a CAM Assistant Professor, worked for two and a half years (91-93) in Technical University of Munich as a Visiting Research Scholar and an Alexander von Humboldt Research Fellow, and worked for two years (89-91) in Chinese Academy of Sciences (Beijing) as an Assistant Professor. Jun Zou's major research interests include numerical solutions of Maxwell equations, interface problems, ill-posed and inverse problems, as well as domain decomposition methods and preconditioned iterative methods.

# AXIOMS OF ADAPTIVITY: RATE OPTIMALITY OF ADAPTIVE ALGORITHMS WITH SEPARATE MARKING

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Mixed finite element methods with flux errors in  $H(\text{div})$ -norms and div-least-squares finite element methods require the separate marking strategy in obligatory adaptive mesh-refining. The refinement indicator  $\sigma_\ell^2(K) = \eta_\ell^2(K) + \mu^2(K)$  of a finite element domain  $K$  in a triangulation  $\mathcal{T}_\ell$  on the level  $\ell$  consists of some residual-based error estimator  $\eta_\ell$  with some reduction property under local mesh-refining and some data approximation error  $\mu_\ell$ . Separate marking (SAFEM) means either Dörfler marking if  $\mu_\ell^2 \leq \kappa \eta_\ell^2$  or otherwise an optimal data approximation algorithm run with controlled accuracy as established in [CR11, Rab15] and reads as follows

```

for  $\ell = 0, 1, \dots$  do
  COMPUTE  $\eta_\ell(K), \mu(K)$  for all  $K \in \mathcal{T}_\ell$ 
  if  $\mu_\ell^2 := \mu^2(\mathcal{T}_\ell) \leq \kappa \eta_\ell^2 \equiv \kappa \eta_\ell^2(\mathcal{T}_\ell)$  then
    |  $\mathcal{T}_{\ell+1} := \text{Dörfler\_marking}(\theta_A, \mathcal{T}_\ell, \eta_\ell^2)$ 
  else
    |  $\mathcal{T}_{\ell+1} := \mathcal{T}_\ell \oplus \text{approx}(\rho_B \mu_\ell^2, \mathcal{T}_0, \mu_\ell^2)$ 

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The enfolded set of axioms (A1)–(A4) and (B1)–(B2) plus (QM) simplifies and generalizes [CFPP14] for collective marking, treats separate marking in an axiomatic framework for the first time, generalizes [CP15] for least-squares schemes, and extends [CR11] to the mixed FEM with flux error control in  $H(\text{div})$ .

The presented set of axioms guarantees rate optimality for AFEMs based on collective and separate marking and covers existing literature of rate optimality of adaptive FEM. Separate marking is necessary for least-squares FEM and mixed FEM with convergence rates in  $H(\text{div}, \Omega) \times L^2(\Omega)$ .

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**Carsten Carstensen** studied civil engineering and mathematics in Hanover, received a PhD in numerical linear algebra, and a habilitation on the coupling of finite and boundary element methods awarded with the von-Mises prize in 1995. He has been full professor for over 20 years with positions in Kiel and Vienna before he stabilized in Berlin and published over 200 papers in applied mathematics and computational sciences. He has served at the editorial boards of Math Comp and SINUM and is a correspondent member of the Akademie der Wissenschaften und der Literatur Mainz. His visiting professorships included positions in Budapest and Mumbai as well as a part time affiliation with the Yonsei University in Seoul. His broad area of activities range from the design and the mathematical foundation of adaptive algorithms to novel discretization schemes such as discontinuous Petrov Galerkin schemes in computational partial differential equations and the applied analysis of nonlinear phenomena in mechanics.

# NONCONFORMING MIXED FE METHODS ON POLYHEDRAL MESHES

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## ABSTRACT

The talk is based on the results of my 20 years collaboration with scientists and engineers at Exxon-Mobil Upstream Research Company and Los Alamos National Laboratory. The target was to develop and investigate mixed finite element discretizations for diffusion and advection-diffusion equations in strongly heterogeneous media on general polyhedral meshes. In practice, the meshes as well as the discretization methods are always nonconforming. The talk includes the most recent theoretical and experimental results.



### Kuznetsov Yuri

M.S.-Novosibirsk State University  
Ph.D.-USSR Academy of Sciences  
USSR/Russian Academy of Sciences: 1965-1997  
University of Houston, Texas: since 1997

#### *Honors*

Invited speaker on numerical analysis at the International Congress of Mathematicians, Warsaw, 1983;  
Distinguished Chair of Mathematics at the University of Houston, 2001



# WAVE-NUMBER EXPLICIT CONVERGENCE ANALYSIS FOR GALERKIN-TYPE DISCRETIZATIONS OF THE HELMHOLTZ EQUATION

Stefan Sauter

joint work with M. Melenk and I. Graham.

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## ABSTRACT

The efficient numerical discretization of highly indefinite Helmholtz problems is a challenging task in the simulation of scattering problems. In our talk we will present an overview over recent results on finite element approximations to these types of problems. We focus on convergence results for hp-discretizations of conventional variational formulations of the Helmholtz problem, least squares methods, as well as a priori and a posteriori error estimates. We also discuss in this talk very recent stability results for non-constant wave speed.



**Stefan Sauter** (since 1999 full professor for "Angewandte Mathematik", Universität Zürich), received his Ph. D in 1993. Title of thesis: "On the efficient use of the Galerkin method to solve Fredholm integral equations".  
In 1998-1999 full professor for "Numerik und Wissenschaftliches Rechnen", Universität Leipzig;  
in 1994-1999 assistant, Universität Kiel;  
in 1993-1994 Post-doc, Univ. of Maryland at College Park.

# WAVE-NUMBER EXPLICIT CONVERGENCE ANALYSIS FOR GALERKIN-TYPE DISCRETIZATIONS OF THE HELMHOLTZ EQUATION

Rolf Stenberg

joint work with Tom Gustafsson and Juha Videman

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## ABSTRACT

Stabilized mixed finite element methods were introduced around thirty years ago by Brezzi, Pitkäranta, Hughes, Franca, . . . They are attractive since the pair of finite element spaces does not have to satisfy the Babuska-Brezzi inf-sup condition. The analysis of these methods have suffered from the shortcoming that full regularity of the exact solution has been assumed. In a recent paper [1] we have used an idea of Gudi [2] to improve the analysis of methods for the Stokes problem. The stabilization amounts to introducing properly weighted residuals of differential equations into the weak form. Since residual error estimators yield a lower bound for the error, one obtains a quasi-optimal a priori estimate. The a posteriori estimates are obtained by the same reason. We next consider stabilized methods for enforcing Dirichlet boundary conditions and show that the same analysis applies. Due to the close connection between the stabilized formulation and Nitsche's methods [3], we obtain new results for that as well. The third application is the obstacle problem. We derive a priori and a posteriori estimate for a new stabilized formulation, and we also present extensive numerical results.

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**Rolf Stenberg** During the years 1978-79 Rolf Stenberg worked as a research engineer at the Philips corporation in Eindhoven, the Netherlands. He then returned to Finland and received his Ph.D. from Helsinki University of Technology in 1984. His post-doctoral period he mostly spent at INRIA-Rocquencourt, France. After that he had various research and teaching positions at Helsinki University of Technology and the Academy of Finland. During the years 1996-99 he was professor of Mathematics at the University of Innsbruck, Austria. He is now professor of Mechanics at the Aalto University, Finland. His research field is the mathematical analysis of finite element methods with applications in continuum mechanics.

# RANK-STRUCTURED TENSOR APPROXIMATION OF MULTI-DIMENSIONAL PDES

Boris N. Khoromskij

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## ABSTRACT

The breaking through approach to low-parametric representation of multivariate functions and operators is based on the principle of separation of variables which can be realized by using approximation in rank-structured tensor formats [2]. This allows the linear complexity scaling in dimension  $d$ , hence breaking the "curse of dimensionality". The method of quantized tensor train (QTT) approximation is proven to provide the logarithmic data-compression on a wide class of discretized functions and operators [1,2].

We discuss how the tensor numerical methods based on the low-rank canonical, Tucker, TT and QTT approximation apply to calculation of electrostatic potential of many-particle systems like in the protein modeling, in the (post) Hartree-Fock calculations for large 3D lattice-structured molecules [3,4,5], to elliptic equations with highly-oscillating coefficients (geometric homogenization theory) [6], and to parametric and stochastic elliptic PDEs [7]. The other direction is related to tensor approaches for parabolic equations in space-time  $\mathbf{d} + 1$  formulation.

The efficiency of the tensor approach is demonstrated by numerical examples.

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**Boris N. Khoromskij**, DrSci, PhD

Current position: Senior Scientist at the Max-Planck-Institute for Mathematics in the Sciences (MPI MiS), Leipzig, Germany.

# COUPLED MATHEMATICAL MODELS AND MULTISCALE PHENOMENA AT THE NANOSCALE WITH THEIR APPLICATIONS

Roderick Melnik

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## ABSTRACT

As matter is organized at the nanoscale, we want to understand, control, and take advantage of the phenomena that take place at that level. Not only we aim at utilizing the unique properties of matter that naturally occur at small elementary scales, but also we seek fundamental knowledge of matter and its interactions with other phenomena such as light. Mathematics is a discipline that contributes to this knowledge, and the development of novel mathematical models plays a key role in this process. To model and control the physical, biological and engineering systems assembled from nanomaterials with sufficient accuracy, accounting for critical physical effects, we inevitably arrive at the development of multiscale physics-based mathematical models.

Coupled mathematical models are essential in describing most natural phenomena, processes, and man-made systems. From large scale mathematical models of climate to modelling of quantum mechanical effects, coupling and nonlinearity go often hand and hand. Coupled dynamic systems of partial differential equations (PDEs) often provide a foundation for the description of many such systems, processes, and phenomena. In majority of cases, however, their solutions are not amenable to analytical treatments and the development, analysis, and applications of effective numerical approximations for such models become a core element in their studies. The undoubted attractiveness of ab initio atomistic approaches and first principles calculations in such cases is necessarily accompanied by severe computational limitations. As a result, multiscale methodologies based on quantum-continuum coupling become one of the most promising tools in this area. Coupling in this case can be done in several different ways, e.g., the information from the atomistic scale can be passed and built in into continuum models; it can be achieved via entropy maximization or in an iterative manner aiming at the complete quantum-continuum coupling. Size effects, electron-phonon scattering, and coupling of the fields of different physical nature (such as electrical, mechanical, thermal, magnetic) lead to additional non-trivial difficulties. We provide a brief overview of such multiscale coupling schemes and associated challenges.

In this talk we will focus on three examples of coupled mathematical models. Starting from nanoscale low dimensional systems such as quantum dots (QDs), we'll move to mesoscopic models for phase transformations (PTs), and we'll conclude with coupled multiscale problems in studying biological structures constructed from ribonucleic acid (RNA), focusing on the fundamental scale at which much of biology occurs. As we go, we will also provide further insight into their application areas and the development of computationally efficient procedures.



**Roderick Melnik, PhD**, is Professor in the Department of Mathematics at Wilfrid Laurier University, Canada, where he is also Tier I Canada Research Chair in Mathematical Modeling. His affiliations include also the University of Waterloo. He is internationally known for his research in computational and applied mathematics, numerical analysis, and mathematical modeling for scientific and engineering applications. Dr. Melnik is the recipient of many awards, including a number of prestigious fellowships outside of Canada, in Italy, Denmark, England and Spain. He has published over 300 refereed research papers and has served on editorial boards of many international journals and book series. Currently, Dr. Melnik is the Director of the MS2Discovery Interdisciplinary Research Institute in Waterloo, Canada.

# MS 1: Numerical Methods for Fractional Order PDEs

# NUMERICAL SOLUTION OF BOUNDARY VALUE PROBLEMS WITH FRACTIONAL BOUNDARY CONDITIONS

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## ABSTRACT

In the Hilbert space  $H(\Omega) = L_2(\Omega)$ , we define the scalar product and norm in the standard way:

$$\langle u, v \rangle = \int_{\Omega} u(\mathbf{x})v(\mathbf{x})d\mathbf{x}, \quad \|u\| = \langle u, u \rangle^{1/2}.$$

Similarly, in  $H(\Gamma) = L_2(\Gamma)$ ,  $\Gamma \equiv \partial\Omega$ :

$$\langle u, v \rangle_{\Gamma} = \int_{\Gamma} u(\mathbf{x})v(\mathbf{x})d\mathbf{x}, \quad \|u\|_{\Gamma} = \langle u, u \rangle_{\Gamma}^{1/2}.$$

We define the bilinear form

$$a(u, v) = \int_{\Omega} (k \operatorname{grad} u \operatorname{grad} v + c uv) d\mathbf{x}.$$

Then, for the solution  $u \in H^1(\Omega)$  of a Neumann problem, we have

$$a(u, v) = \langle g, v \rangle_{\Gamma}, \quad v \in H^1(\Omega).$$

Let us define the operator  $S$  as

$$\langle Su, v \rangle_{\Gamma} = a(u, v).$$

In the present work, we consider the equation

$$S^{\alpha}u = g, \quad 0 < \alpha < 1,$$

supplemented with fractional boundary conditions. Similarly to the work [1], we construct a numerical algorithm using an auxiliary evolutionary problem.

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# A DIFFERENCE SCHEME FOR THE TEMPERED TIME FRACTIONAL DIFFUSION EQUATION

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## ABSTRACT

We construct a difference analog of the Caputo tempered fractional derivative. The basic properties of this difference operator are investigated and on its basis some difference schemes generating approximations of the second and fourth order in space and the  $2 - \alpha$  - th order in time for the tempered time-fractional diffusion equation with variable coefficients are considered. Stability of the suggested schemes and also their convergence in the grid  $L_2$  - norm with the rate equal to the order of the approximation error are proved. The obtained results are supported by the numerical calculations carried out for some test problems.

## 1 THE DIRICHLET BOUNDARY VALUE PROBLEM FOR THE TEMPERED TIME-FRACTIONAL DIFFUSION EQUATION

In a rectangle  $\bar{Q}_T = \{0 \leq x \leq 1, 0 \leq t \leq T\}$  consider the Dirichlet boundary value problem for the tempered time-fractional diffusion equation with variable coefficients

$$\partial_{0t}^{\alpha, \lambda} u = \mathcal{L}u + f(x, t), \quad 0 < x < 1, \quad 0 < t \leq T, \quad (1)$$

$$u(0, t) = 0, \quad u(1, t) = 0, \quad 0 \leq t \leq T, \quad u(x, 0) = u_0(x), \quad 0 \leq x \leq 1, \quad (2)$$

where

$$\mathcal{L}u = \frac{\partial}{\partial x} \left( k(x, t) \frac{\partial u}{\partial x} \right) - q(x, t)u,$$
$$\partial_{0t}^{\alpha, \lambda} u(x, t) = \frac{1}{\Gamma(1 - \alpha)} \int_0^t e^{-\lambda(t-\eta)} (t - \eta)^{-\alpha} \frac{\partial u}{\partial \eta}(x, \eta) d\eta$$

is the Caputo tempered fractional derivative of order  $\alpha$ ,  $0 < \alpha < 1$  and parameter  $\lambda \geq 0$ ;  $0 < c_1 \leq k(x, t) \leq c_2$ ,  $q(x, t) \geq 0$  for all  $(x, t) \in \bar{Q}_T$ .

Suppose that a solution  $u(x, t) \in C^{4,2}(\bar{Q}_T)$  of the problem (1)–(2) exists, and the coefficients of Eq. (1) and the functions  $f(x, t)$  and  $u_0(x)$  satisfy the conditions, required for the construction of difference schemes with the order of approximation  $O(h^2 + \tau^{2-\alpha})$ .

# NUMERICAL SOLUTION OF FRACTIONAL ORDER MODEL OF HIV-1 INFECTION OF $CD4^+$ T-CELLS BY USING LAPLACE ADOMIAN DECOMPOSITION METHOD

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## ABSTRACT

In this paper , we introduce fractional-order into a model of HIV-1 infection of  $CD4^+$  T-cells. We study the effect of the changing the average number of viral particles  $N$  with different sets of initial conditions on the dynamics of the presented model. The Laplace Adomian Decomposition Method is implemented to examine the dynamic behaviors in the fractional-order HIV-1 infection model. Numerical results show that the Laplace ADM approach is simple and accurate for solving fractional-order HIV-1 infection model. We obtain the solutions of the fractional differential equations involved in the model in the form of infinite series. The concerned series rapidly converges to its exact value. Then, we compare our results with the results obtained by Runge-kutta method of order-4 in case of integer order derivative.

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# A PSEUDO SPECTRAL MODIFIED QUASILINEARIZATION FOR FRACTIONAL PERTURBATION-DIFFERENTIAL EQUATIONS

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## ABSTRACT

In this paper, we employ the method of modified monotone quasilinearization to prove the existence and uniqueness of a nonlinear fractional order functional differential equation. A pseudo spectral method has been proposed to solve the problem numerically. The numerical simulation supports the fact that the proposed scheme produces decent accuracy and is easy to implement.

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# NUMERICAL APPROXIMATION OF A VARIATIONAL PROBLEM ON A BOUNDED DOMAIN INVOLVING THE FRACTIONAL LAPLACIAN

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## ABSTRACT

The mathematical theory and numerical analysis of non-local operators has been a topic of intensive research in recent years. One class of applications come from replacing Brownian motion diffusion by diffusion coming from a symmetric  $\alpha$ -stable Levy process, i.e., the Laplace operator is replaced by a fractional Laplacian.

In this talk, we propose a numerical approximation of equations with this type of diffusion terms posed on bounded domains. We focus on the simplest example of an elliptic variational problem coming from the fractional Laplacian on a bounded domain with homogeneous Dirichlet boundary conditions. Although it is conceptually feasible to study the Galerkin approximation based on a standard finite element space, such a direct approach is not viable as the exact computation of the resulting stiffness matrix entries is not possible (at least in two or more spatial dimensions).

Instead, we will develop a non-conforming method by approximating the action of the stiffness matrix on a vector (sometimes referred to as a matrix free approach). The bilinear form is written as an improper integral involving the solution of parameter dependent elliptic problems on  $R^d$ . We compute an approximate action of stiffness matrix by applying a SINC quadrature rule to the improper integral, replacing the problems on  $R^d$  by problems on parameter dependent bounded domains, and the application of the finite element method to the bounded domain problems. The entire procedure can be implemented using standard finite element tools, e.g., the DEAL-II library. The analysis of the resulting algorithm is discussed. In addition, the results of numerical computations on a model problem with known solution are given.

# DISSIPATIVITY AND CONTRACTIVITY OF FRACTIONAL-ORDER SYSTEMS AND THEIR NUMERICAL SIMULATION

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## ABSTRACT

For the dissipativity and contractivity of the Caputo fractional initial value problems, we prove that the systems have an absorbing set under the same assumptions as the classic integer-order systems. This directly extends the dissipativity from integer-order systems to the Caputo fractional-order ones. The fractional dissipativity conditions can be satisfied by many fractional chaotic systems and the systems from the spatial discretization of some time-fractional partial differential equations. The fractional-order systems satisfying the so-called one-sided Lipschitz condition are also considered in a similar way, and the contractivity property of their solutions is proved. Two numerical examples are provided to illustrate the theoretical results.

For the dissipativity and stability of the Caputo nonlinear fractional functional differential equations (F-FDEs) with order  $0 < \alpha < 1$ . The fractional generalization of the Halanay-type inequality is proposed, which plays a central role in studies of stability and dissipativity of F-FDEs. Then the dissipativity and the absorbing set are derived under almost the same assumptions as the classical integer-order functional differential equations (FDEs). The asymptotic stability of F-FDEs are also proved under the one-sided Lipschitz conditions. Those extend the corresponding properties from integer-order FDEs to the Caputo fractional ones. The results can also be directly applied to some special cases of fractional nonlinear equations, such as the fractional delay differential equations (F-DDEs), fractional integro-differential equations (F-IDEs) and fractional delay integro-differential equations (F-DIDEs). The fractional Adams-Bashforth-Moulton algorithm is employed to simulate the F-FDEs, and several numerical examples are given to illustrate the theoretical results.

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# FRACTIONAL EQUATIONS AND DIFFERENCE SCHEMES

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## ABSTRACT

In this talk we continue our investigations [1] on approximation of differential equations of fractional order in time.

Some time ago, in [2] and [3], were considered the relation between well-posed Cauchy problems

$$v'(t) = A^l v(t) + g(t), t > 0, v(0) = u^0,$$

and

$$(\mathbf{D}_t^{1/l} u)(t) = Au(t), t > 0, u(0) = u^0.$$

Moreover, they have shown that for such kind problems with the operator  $A$  which generates  $C_0$ -semigroup one has  $v(t) \equiv u(t)$  for any  $t > 0$  as soon as  $l = 2$  and special choice of  $g(t)$ .

In this talk, we would like to use such kind of relations for approximation of differential equations of fractional order in abstract spaces.

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# APPROXIMATE ANALYTICAL SOLUTION OF SHARMA-TASSO-OLVER AND BURGERS-KDV EQUATIONS WITH HOMOTOPY ANALYSIS METHOD AND CONFORMABLE FRACTIONAL DERIVATIVE

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## ABSTRACT

The main goal of this paper is finding the approximate analytical solution of Sharma-Tasso-Olver and Burgers-KdV equations which have great role in the applied sciences with newly defined conformable fractional derivative by using homotopy analysis method (HAM). To show the efficiency of the proposed method with conformable derivative, obtained approximate analytical solutions are compared with the exact solutions and comparative tables are given.

**Keywords:** Burgers-KdV equation; Sharma-Tasso-Olver equation; Conformable Fractional Derivative; Homotopy Analysis Method.

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## MS 2: Reliable Computational Methods

# IMPLEMENTATION OF A FUNCTIONAL-TYPE A POSTERIORI ERROR ESTIMATE FOR REISSNER-MINDLIN PLATES

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## ABSTRACT

A recent a posteriori error estimate [1] for control of accuracy of solutions for Reissner-Mindlin plates is investigated. This reliable majorant is based on the functional approach [2–4]. It is applicable to any conforming approximate solution of the problem and several types of boundary conditions. In comparison to the majorants obtained earlier in [5–7], the main advantage of the new result consist in less restrictive sets of admissible fields. For justification of the approach, some numerical tests with consequent mesh refinements are performed.

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# RELIABLE COMPUTATIONAL METHODS IN LIMIT ANALYSIS OF ELASTIC-PERFECTLY PLASTIC BODIES

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## ABSTRACT

The contribution is concerned with guaranteed and computable bounds of the limit (or safety) load, which is one of the most important quantitative characteristics of elastic-perfectly plastic models. We introduce two innovative ideas for getting such bounds. The first idea is based on the fact that limit analysis is much simpler for models with bounded yield surfaces. In such a case, a guaranteed and easily computable upper bound of the limit load is derived. Consequently, the truncation method for unbounded yield surfaces is used. This technique leads to a lower bound of the limit load. The second idea is related to the incremental method of limit analysis. Instead of the standard load parameter,  $\lambda$ , we suggest to control the loading process through an auxiliary parameter  $\alpha$  satisfying  $\alpha \rightarrow +\infty$  as  $\lambda \rightarrow \lambda^*$  where  $\lambda^*$  denotes the unknown limit load parameter. The function  $\psi : \alpha \rightarrow \lambda$  is nondecreasing and can be used to find other bounds of  $\lambda^*$ . Both ideas are independent of type of discretization method. For the standard finite element method, we introduce several convergence results. We illustrate the new techniques on numerical examples including the von Mises, the Drucker-Prager, and the Mohr-Coulomb yield criteria.

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# ADAPTED NUMERICAL METHODS FOR THE POISSON EQUATION WITH NON-SMOOTH BOUNDARY DATA AND EMPHASIS ON NON-CONVEX DOMAINS

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## ABSTRACT

This talk is concerned with adapted numerical methods for the Poisson equation with  $L^2$  boundary data and emphasis on non-convex domains. Due to the rough boundary data, the equation needs to be understood in the very weak sense. For a standard finite element discretization with regularized boundary data, a convergence order of  $1/2$  in the  $L^2(\Omega)$ -norm can be proved, provided that the domain is convex. However, in non-convex domains the convergence rate is reduced although the solution remains to be contained in  $H^{1/2}(\Omega)$ . The reason for this is a singularity that appears in the solution of the dual problem. In this talk, as a remedy, both a standard finite element method with mesh grading and a dual variant of the singular complement method are proposed and analyzed in order to retain a convergence rate of  $1/2$  also in non-convex domains. Finally, numerical experiments are presented in order to illustrate the theoretical results.

This is joint work with Thomas Apel (Universität der Bundeswehr München) and Serge Nicaise (Université de Valenciennes et du Hainaut Cambrésis)

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# A POSTERIORI ERROR BOUNDS FOR APPROXIMATIONS OF THE STOKES PROBLEM WITH NONLINEAR BOUNDARY CONDITIONS

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## ABSTRACT

The classical Stokes problem consists of finding a velocity field  $u \in \mathbb{S}_0(\Omega) + u_D$  and a pressure field  $p \in \tilde{L}_2(\Omega)$  which satisfy the relations

$$-\text{Div}(\nu \nabla u) + \nabla p = f \quad \text{in } \Omega, \quad (1)$$

$$\text{div} u = 0 \quad \text{in } \Omega, \quad (2)$$

$$u = u_D \quad \text{on } \Gamma_D, \quad (3)$$

$$\sigma n = F \quad \text{on } \Gamma, \quad (4)$$

where  $f \in L_2(\Omega, \mathbb{R}^d)$ ,  $n$  denotes the unit outward normal vector to the boundary,  $u_D$  is a given divergence free function, and  $\sigma = \nu \nabla u$  is the stress tensor.

In this presentation, we are concerned with friction type boundary conditions, which are suitable, when modeling some fragile state of the surface, that allows the fluid to slip on the surface, but as long as the the pushing force is below a threshold, the fluid does not slip.

We consider the system (1)–(3) with the following nonlinear boundary conditions on  $\Gamma$ :

$$u_n = 0, \quad -\sigma_t \in g \partial |u_t| \text{ on } \Gamma, \quad (5)$$

where  $g \geq 0$  is a constant (in general a scalar valued function),  $u_n := u \cdot n$  and  $u_t := u - nu_n$  (normal and tangential components of the velocity).

We derive computable and fully guaranteed estimates of the difference between exact solutions (velocity) of Stokes problems with nonlinear (friction type) boundary conditions and vector functions from the admissible energy space. The estimates are valid for any function satisfying the main boundary conditions and possessing first generalized derivatives. The estimates can be used for a posteriori error control of numerical solutions obtained by various numerical methods.

# A POSTERIORI ERROR ESTIMATES FOR A POROELASTIC MEDIUM

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## ABSTRACT

The work is dedicated to the topic of a posteriori error estimates for poroelastic models, in particular the so-called Biot system (see, e.g., [1, 6]), which contribute to wide range of application areas including simulation of oil reservoirs, preventing environmental changes, predicting the liquefaction of the soil in the earthquake engineering as well as various studies in biomechanics. In particular, it deals with a construction of the upper bound of the approximation error in iterative algorithms for coupling of the flow and the geomechanics [2]. The resulting bound is based on combination of Ostrowski's estimates [3] (derived for error control in the contractive iteration algorithm) as well as the a posteriori error majorants of functional type (see [4, 5]).

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# **A FEM APPROXIMATION OF A TWO-PHASE OBSTACLE PROBLEM AND ITS A POSTERIORI ERROR ESTIMATE**

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## **ABSTRACT**

This paper is concerned with the two-phase obstacle problem, a type of a variational free boundary problem. We recall the basic estimates of Repin and Valdman (2015) and verify them numerically on two examples in two space dimensions. A solution algorithm is proposed for the construction of the finite element approximation to the two-phase obstacle problem. The algorithm is not based on the primal (convex and nondifferentiable) energy minimization problem but on a dual maximization problem formulated for Lagrange multipliers. The dual problem is equivalent to a quadratic programming problem with box constraints. The quality of approximations is measured by a functional a posteriori error estimate which provides a guaranteed upper bound of the difference of approximated and exact energies of the primal minimization problem. The majorant functional in the upper bound contains auxiliary variables and it is optimized with respect to them to provide a sharp upper bound. A space density of the nonlinear related part of the majorant functional serves as an indicator of the free boundary. This is a join work with Farid Bozorgnia (Lisbon).

# RELIABLE AND EFFICIENT A POSTERIORI ERROR CONTROL FOR BEM-BASED FEM ON POLYGONAL MESHES

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## ABSTRACT

Only a few numerical methods can treat boundary value problems on polygonal and polyhedral meshes. The BEM-based Finite Element Method is one of the new discretization strategies, which make use of and benefits from the flexibility of these general meshes that incorporate hanging nodes naturally.

The presentation addresses the residual based error estimate for high order BEM-based FEM. Its reliability and efficiency is discussed on general meshes involving non-convex elements, where important analytical tools are quasi-interpolation operators and uniform bounds for the Poincaré constant for patches of star-shaped elements.

Such a posteriori error estimates can be used to gauge the approximation quality and to implement adaptive FEM strategies. Numerical experiments show optimal rates of convergence for meshes with non-convex elements on uniformly as well as on adaptively refined meshes.

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# NONSYMMETRIC AND NONSTANDARD GALERKIN METHODS FOR NONLINEAR PROBLEMS

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## ABSTRACT

The work is devoted to some nonsymmetric and nonstandard Galerkin Finite Element Methods (GFEM) for solving the model of heat structures - the nonlinear reaction-diffusion problem [1]:

$$u_t = \operatorname{div} (u^\sigma \operatorname{grad} u) + u^\beta, \quad x \in \mathbb{R}^N, \quad t > 0, \quad \sigma > 0, \quad \beta > 1, \\ u(0, x) = u_0(x) \geq 0, \quad u_0 \not\equiv 0, \quad \sup_{x \in \mathbb{R}^N} u_0 < \infty.$$

For the radially symmetric variant of this problem

$$u_t = \frac{1}{r^{N-1}} (r^{N-1} u^\sigma u_r)_r + u^\beta, \quad 0 < r < \infty, \quad t > 0, \\ u(0, r) = u_0(r) \geq 0, \quad r \geq 0, \quad u_0 \not\equiv 0, \quad \sup_{r \geq 0} u_0 < \infty$$

and for the corresponding non-linear self-similar problem

$$-\frac{1}{\xi^{N-1}} (\xi^{N-1} \theta^\sigma \theta')' + \frac{(\beta - \sigma - 1)}{2(\beta - 1)T_0} \xi \theta' + \frac{1}{(\beta - 1)T_0} \theta - \theta^\beta = 0, \quad 0 < \xi < \infty, \\ \theta'(0) = 0, \quad \theta(\xi) \rightarrow \infty, \quad \xi \rightarrow \infty,$$

$N \geq 3$ , the nonsymmetric GFEM [2] is used to deal successfully with the singularity at the origin.

For the nonlinear reaction-diffusion problems in the radially symmetric and in the essentially 2D case the Kirchhoff transformation and interpolation of the nonlinear coefficients, combined with a special mesh adaptation consistent with the self-similar law, are used.

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# INTERVAL ESTIMATION OF POLYNOMIAL SPLINES OF THE FIFTH ORDER

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## ABSTRACT

In this paper we present an algorithm for approximating the range of the function when the values of the function, the values of its first derivative, and the values of the integral over the net intervals are given. The algorithm that we propose is based on the method of approximation function using the integro-differential splines. We construct the approximation separately on every grid interval and furthermore we exploit techniques from interval analysis. Numerical examples are given.

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## MS 3: Galerkin Methods for Nonlinear Problems

# **FITTED ALE SCHEME FOR TWO-PHASE NAVIER–STOKES FLOW**

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## **ABSTRACT**

We present a novel fitted ALE scheme for two-phase Navier–Stokes flow problems that uses piecewise linear finite elements to approximate the moving interface. The meshes describing the discrete interface in general do not deteriorate in time, which means that in numerical simulations a smoothing or a remeshing of the interface mesh is not necessary. We present several numerical experiments for our numerical method, which demonstrate the accuracy and robustness of the proposed algorithm.

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# NUMERICAL CONVERGENCE OF ITERATIVE COUPLING FOR NON-LINEAR BIOT'S MODEL

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## ABSTRACT

Coupled flow and geomechanics models have several applications including  $CO_2$  sequestration, geothermal energy, and subsidence phenomena. The Biot's model is often used to model this coupled behaviour and robust, efficient and accurate numerical methods to solve this model has recently attracted a lot of attention. In particular, splitting techniques that decouple the flow and mechanics problems have been used and analysed [1, 2]. So far the rigorous analysis of the convergence of the splitting techniques has been obtained only for the linear problems (both flow and mechanics). In this work, we extend this to non-linear problems. More specifically, we consider the case when the Bulk modulus (Lame coefficient  $\lambda$ ) and the fluid compressibility are non-linear satisfying certain structural conditions.

We provide a fully discrete linearised scheme to solve the non-linear couple problem and prove its convergence. We discretize in space by using MFEM for the flow and Galerkin FE for the mechanics. The backward Euler method is used for the temporal discretization. The convergence analysis uses the techniques developed in [3, 4], namely showing that the scheme is a contraction in appropriate norms thereby also providing the convergence rates. The splitting scheme has been tested successfully on Mandel's problem and a non linear extension on Mandel's problem. Computational results are presented.

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# CONVERGENT FINITE DIFFERENCE SCHEME FOR COMPRESSIBLE NAVIER-STOKES IN THREE SPATIAL DIMENSIONS

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## ABSTRACT

Motivated by works of Karper, we propose a finite difference scheme for the system of compressible Navier-Stokes equations. Using the techniques developed by Lions in his analytic proof of existence of a weak solution, we show the convergence of the numerical solutions to a weak solution of the problem. The scheme has been implemented, some results in two spatial dimensions will be presented.

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# CONSERVATION LAWS IN BLOW-UP PROBLEMS FOR NONLINEAR PARABOLIC EQUATIONS

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## ABSTRACT

Conditions for the wide class of nonlinear source function, coefficients and input data under which the Dirichlet problem for quasilinear parabolic equation blows up, were found using integral conservation laws [1]. Our approach allows to find blow-up of the solution even for positive energy [2]. The estimate of the blow-up time was obtained.

Consider initial boundary value problem

$$\frac{\partial u}{\partial t} = \sum_{\alpha=1}^m \frac{\partial}{\partial x_{\alpha}} \left( k(u) \frac{\partial u}{\partial x_{\alpha}} \right) + f(u), \quad (x, t) \in Q_T, \quad (1)$$

$$u(x, t) = \mu, \quad (x, t) \in \partial\Omega \times (0, T], \quad u(x, 0) = u_0(x), \quad x \in \bar{\Omega}, \quad (2)$$

where  $\Omega = \{x = (x_1, x_2, \dots, x_m)\} \subset R^m$  is bounded connected domain with smooth boundary  $\partial\Omega$ ,  $\bar{\Omega} = \Omega \cup \partial\Omega$ ,  $Q_T = \Omega \times (0, T]$ ,  $\mu = \text{const} \geq 0$ ,  $u_0(x) \in C(\bar{\Omega})$ ,  $u_0(x) \geq \mu$   $x \in \Omega$ ,  $f(u)$  is continuous and nonnegative under  $u \geq \mu$ ,  $k(u)$  is continuous, nonnegative under  $u \geq \mu$  and differentiable under  $u > \mu$ ,  $\varphi(u)$  is twice continuously differentiable positive function under  $u > \mu$ . Let  $\Phi(u)$  be an antiderivative of the function  $\varphi(u)$ .

The following conservation law

$$E(t) = E(0), \quad 2 \left( k(u), \left( \frac{\partial u}{\partial t} \right)^2 \right) + \frac{d}{dt} \sum_{\alpha=1}^m \left\| \frac{\partial}{\partial x_{\alpha}} \varphi(u) \right\|^2 = \frac{d}{dt} (F(u), 1) \quad (3)$$

corresponds to the problem (1)-(2), where  $F(u)$  is an antiderivative of the function  $2k(u)f(u)$  and  $(u, v) = \int_{\Omega} uv dx$ ,  $\|u\|^2 = (u, u)$ .

**Theorem.** Suppose that the following conditions are satisfied:

1.  $G(v) = \varphi(\Phi^{-1}(v))f(\Phi^{-1}(v)) - F(\Phi^{-1}(v))$  is convex continuous function under  $v \geq v_0$  and  $T_1 = \int_{v_0}^{\infty} \frac{dw}{G(w)-E(0)} < \infty$ ,  $v_0 = v(0) = \frac{1}{mes\Omega} \int_{\Omega} \Phi(u(x, 0)) dx$ ;
2.  $E(0) < \min_{v \geq v_0} G(v)$ ;

then solution of the problem (1)-(2) blows up, i. e.  $\lim_{t \rightarrow T_b} \sup_{x \in \Omega} u(x, t) = \infty$ , and  $T_b \leq T_1$ .

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# ENTROPY STABLE WENO SPECTRAL COLLOCATION SCHEMES FOR THE NAVIER-STOKES EQUATIONS

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## ABSTRACT

High-order numerical methods that satisfy a discrete analog of the entropy inequality are uncommon. Indeed, no proofs of nonlinear entropy stability currently exist for high-order Weighted Essentially Non-Oscillatory (WENO) finite volume or weak-form finite element methods. In this talk, we present a new family of fourth-order WENO spectral collocation schemes. The new schemes are similar to strong form, nodal discontinuous Galerkin methods, but are provably stable in the entropy sense for the three-dimensional Navier-Stokes equations. Individual spectral elements are coupled using penalty type interface conditions. The resulting entropy-stable WENO spectral collocation schemes achieve design order accuracy, maintain the WENO stencil biasing properties across element interfaces, and satisfy the summation-by-parts (SBP) operator convention, thereby ensuring nonlinear entropy stability in a diagonal norm. Numerical results demonstrating accuracy and non-oscillatory properties of the new schemes are presented for both continuous and discontinuous flows.

# A GALERKIN METHOD FOR THE MONGE-AMPERE PROBLEM WITH TRANSPORT BOUNDARY CONDITIONS

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## ABSTRACT

To motivate the talk, I will briefly outline a "general audience" time on the historical and mathematical background of the Monge-Ampere partial differential equation and its relation to optimal mass transportation and geometric problems (including classical geometric optics as a surprising link between the two disciplines). In the second, more technical part, I will brief on the various numerical methods, with a focus on the Nonvariational Finite Element Method (NVFEM) for linear problems in nondivergence form, and fully nonlinear problems. I will then illustrate the use of this method to solve the Monge-Ampere problem via a Newton-Raphson iterative method including the mass transportation boundary conditions. Based on joint work with Tristan Pryer (Reading GB) and Elly Kawecki (Oxford GB).

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# A PRIORI FEEDBACK ESTIMATES FOR MULTISCALE REACTION-DIFFUSION SYSTEMS

Martin Lind joint work with Adrian Muntean

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## ABSTRACT

We study the multiscale approximation of a reaction-diffusion system posed on both macroscopic and microscopic space scales. The coupling between the scales is done via micro-macro flux conditions. Our target system has a typical structure for reaction-diffusion-flow problems in media with distributed microstructures (also called, double porosity materials). Besides ensuring basic estimates for the convergence of two-scale semi-discrete Galerkin approximations, we provide a set of *a priori* feedback estimates and a local feedback error estimator that help in designing a distributed-high-errors strategy to allow for a computationally efficient zooming in and out from microscopic structures. The error control on the feedback estimates relies on two-scale-energy, regularity, and interpolation estimates as well as on a fine bookkeeping of the sources responsible with the propagation of the (multiscale) approximation errors. The working technique based on *a priori* feedback estimates is in principle applicable to a large class of systems of PDEs with dual structure admitting strong solutions.

# ALE-FEM FOR INCOMPRESSIBLE FLOWS AND TRANSPORT PROBLEMS

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## ABSTRACT

We present a finite element method for the flow of two immiscible, incompressible fluids in two and three dimensions. Thereby the presence of soluble and insoluble species (surfactant) is considered, which can be ad- and desorbed by the interface and alter the surface tension. The finite element method uses the Arbitrary Lagrangian Eulerian (ALE) technique, which tracks the interface by moving grids. We use second order finite elements and a second order interface approximation, which allows precise incorporation of surface tension forces and Marangoni forces. Furthermore, Local Projection Stabilisation (LPS) is introduced for both the species transport in the bulk and on the surface.

We consider a bounded domain  $\Omega \subset \mathbb{R}^d$ ,  $d = 2, 3$ , filled with the two fluids, which occupy at time  $t$  the domains  $\Omega^i(t)$ ,  $i = 1, 2$ . Let  $\partial\Omega^i(t)$  denote the boundary of  $\Omega^i(t)$ ,  $i = 1, 2$ , and  $\Gamma(t) = \partial\Omega^1(t) \cap \partial\Omega^2(t)$  the interface of  $\Omega^1(t)$  and  $\Omega^2(t)$ . Our model consists of the Navier-Stokes equations for the flow fields  $u^i$  and pressure fields  $p^i$  in the phases  $\Omega^i(t)$ ,  $i = 1, 2$ , a convection diffusion equation for the bulk surfactant  $c^i$  in  $\Omega^i(t)$ ,  $i = 1, 2$  and a convection diffusion equation on the moving manifold  $\Gamma(t)$  for the surface surfactant  $c_\Gamma$ , completed with various coupling terms. The full model reads:

$$\varrho_i(\partial_t u^i + (u^i \cdot \nabla) u^i) - \nabla \cdot \mathbb{S}(u^i, p^i) = f, \quad \nabla \cdot u^i = 0 \quad \text{in } \Omega^i(t), \quad (1)$$

$$[-\mathbb{S}]n = \sigma(c_\Gamma)\kappa n + \nabla_\Gamma \sigma(c_\Gamma), \quad [u] = 0, \quad V = u \cdot n \quad \text{on } \Gamma(t), \quad (2)$$

$$\partial_t c^i - D^i \Delta c^i + (u^i \cdot \nabla) c^i = 0 \quad \text{in } \Omega^i(t), \quad (3)$$

$$[D \partial_n c] = -S(c^1, c^2, c_\Gamma) \quad \text{on } \Gamma(t), \quad (4)$$

$$\partial_t c_\Gamma - D_\Gamma \Delta_\Gamma c_\Gamma + \nabla_\Gamma \cdot (c_\Gamma u|_\Gamma) = S(c^1, c^2, c_\Gamma) \quad \text{on } \Gamma(t), \quad (5)$$

for  $i = 1, 2$ .  $\mathbb{S}$  is the usual stress tensor for Newtonian fluids,  $f$  describes gravitational forces,  $[h] := h^1 - h^2$  denotes a jump of quantity  $h$  across the interface,  $\sigma(c_\Gamma)$  is the surface tension coefficient,  $\kappa$  denotes the mean curvature of the interface,  $D^i$  is the diffusion coefficient for the bulk  $\Omega^i(t)$ ,  $D_\Gamma$  is the surface diffusion coefficient,  $\nabla_\Gamma$  and  $\Delta_\Gamma$  are the surface version of the corresponding differential operators,  $S$  describes ad- and absorption of surfactant at the interface and while  $\partial_t$  denotes the time derivative,  $\partial_n$  denotes the spatial derivative in normal direction and  $\varrho_i$  the fluid density in phase  $i$ .

## MS 4: Error analysis and computational aspects of PDE eigenvalue problems

# APPROXIMATION OF POLYHARMONIC EIGENVALUE PROBLEMS

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## ABSTRACT

The approximation of the polyharmonic eigenvalue problem

$$(-1)^m \Delta^m u = \lambda (-1)^n \Delta^n u \quad \text{for integers } 0 \leq n < m \quad (1)$$

with standard finite elements requires  $C^{m-1}$ -conforming finite elements. Mixed methods are used to circumvent this difficulty by splitting the original partial differential equation (PDE) in a system of low-order PDEs. However, the stability of mixed finite element discretizations for high-order PDEs often hinges on certain regularity assumptions. Depending on properties of the domain or the boundary conditions, these assumptions may fail to hold and some traditional mixed schemes may converge towards ghost solutions, i.e., functions that do not solve the original problem. Recently, in [1] a new splitting of  $2m$ -th order elliptic problems equations into  $2(m-1)$  problems of Poisson type and one generalized Stokes problem was established for any space dimension  $d \geq 2$  and any integer  $m \geq 1$ . The splitting allows for a numerical approximation based on standard (possibly low-order) finite elements that are suited for the Poisson equation and the Stokes system. This approach does not require any additional regularity of the solution. This talk discusses the application of this methodology to polyharmonic eigenproblems of the type (1).

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# ARNOLD-WINTHER MIXED FINITE ELEMENTS FOR STOKES EIGEN VALUE PROBLEMS

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## ABSTRACT

In this paper, we are presenting the Arnold-Winther mixed finite element formulation for the two dimensional Stokes eigenvalue problem using the stress-velocity formulation. A priori error estimates for eigenvalue and eigenfunction are shown. To improve the approximation for both eigenvalue and eigenfunction, we use local postprocessing. A posteriori error estimates for the proposed method are also obtained. Several numerical experiments are discussed to validate the theoretical results.

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# INEXACT AND PRECONDITIONED LINEAR SOLVES IN ITERATIVE EIGENVALUE METHODS.

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## ABSTRACT

We are interested in the numerical computation of interior eigenvalues of large and sparse algebraic eigenvalue problems  $Ax = \lambda x$ , where the main emphasis lies on nonsymmetric  $A$ . Prominent state-of-the-art methods are, e.g., Jacobi-Davidson and shift-and-invert Arnoldi methods but we draw our focus to prototypes of thereof: the Rayleigh quotient and inverse iteration. There, in each iteration step a linear system of the form

$$(A - \sigma I)x_{k+1} = x_k$$

has to be solved for  $x_{k+1}$ . Due to the large size of these systems, iterative Krylov subspace methods come into play and we investigate the behavior of the Rayleigh quotient and inverse iteration if the above linear systems are only solved to a low or moderate accuracy [1]. We also investigate preconditioners which are specially tailored to the outer eigenvalue algorithm and can significantly boost the performance of the applied Krylov subspace methods [1]. We further consider the case when preconditioned GMRES is used for dealing with the linear systems. Motivated by [2], for this case some novel convergence bounds for GMRES are presented that nicely underline the influence of the right hand side  $x_k$  and the used preconditioner on the performance of GMRES when used within eigenvalue iterations. If time permits, generalizations for generalized and two-sided eigenvalue problems [3] and the more sophisticated methods like shift-and-invert Arnoldi and Jacobi-Davidson are mentioned.

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# A STABILIZED FINITE ELEMENT METHOD FOR THE TWO-FIELD AND THREE-FIELD STOKES EIGENVALUE PROBLEMS

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## ABSTRACT

In this work, the stabilized finite element approximation of the Stokes eigenvalue problems is considered for both the two-field (displacement-pressure) and the three-field (stress-displacement-pressure) formulations. The method presented is based on a subgrid scale concept, and depends on the approximation of the unresolvable scales of the continuous solution. In general, these techniques consist in the addition of a residual based term to the basic Galerkin formulation. Naturally, the application of a standard residual based stabilization method to a linear eigenvalue problem, leads to a quadratic eigenvalue problem in discrete form which is physically inconvenient. As a distinguished feature of the present study, we take the space of the unresolved subscales orthogonal to the finite element space, which promises a remedy to the above mentioned complication. In essence, we put forward that only if orthogonal projection is used, the residual is simplified and the use of term by term stabilization is allowed. Thus, we do not need to put the whole residual in the formulation, and the linear eigenproblem form is recovered properly. We prove that the method applied is convergent, and present the error estimates for the eigenvalues and the eigenfunctions. We report several numerical tests in order to illustrate that the theoretical results are validated. The work to be presented has been submitted for publication in the paper “A stabilized finite element method for the two-field and three-field Stokes eigenvalue problems”.

# ADAPTIVE FINITE ELEMENT SCHEMES FOR EIGENVALUE PROBLEMS: FROM MIXED LAPLACIAN TO MAXWELL'S EQUATIONS

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## ABSTRACT

In [3] we proved that an adaptive procedure based on standard mixed schemes (such as Raviart–Thomas finite elements) is optimally convergent for the approximation of the eigensolution of the Laplace eigenvalue problem. The result is cluster robust and holds in two and three space dimensions.

It is well-known that the Maxwell eigenvalue problem has an equivalent mixed formulation that has some analogies with the mixed formulation for the laplacian (see [2] and [1]). This equivalence makes it possible the design of a robust error indicator for the Maxwell eigenvalue problem (see [4]).

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# A POSTERIORI ESTIMATES FOR EIGENPROBLEMS USING AUXILIARY SUBSPACE TECHNIQUES

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## ABSTRACT

A posteriori error estimators based on auxiliary subspace techniques for second order elliptic eigenproblems in  $\mathbb{R}^d$  ( $d \geq 2$ ) and thin structure vibrations are considered. Auxiliary subspace techniques are a variant of hierarchic estimators extending the work of Grubišić and Owall[1] but with sufficiently unique features such that a separate identity is warranted[2].

The very nature of the construction makes it especially suitable for  $hp$ -FEM. The auxiliary subspace can be defined using the natural degrees of freedom, namely the edge and interior modes in  $\mathbb{R}^2$  and in addition face modes in  $\mathbb{R}^3$ . Theoretical basis for the construction is briefly discussed. The main focus of the talk is on numerical examples supporting the theoretical results. For detailed references visit `eigenproject.math.aalto.fi`.

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# ON A PRIORI ERROR ESTIMATES FOR EIGENVALUE PROBLEMS

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## ABSTRACT

In this talk, we consider the Rayleigh-Ritz approximation of eigenvalues and eigenvectors of positive symmetric differential operators. We derive a priori convergence estimates for the difference between exact and approximate eigenvalues and the gap between the exact and approximate eigenspaces. The topic is classical and the form of the convergence results is well known, see e.g. [1].

The novelty of our approach is in a new technique used to derive the estimates. We interpret the continuous eigenvalue and the elliptic projection of the related eigenspace as approximations of the corresponding discrete eigenvalue and eigenspace. This allows us to derive convergence estimates using elementary techniques in the discrete setting. The applied technique give sharp estimates both for simple and multiple eigenvalues. However, the technique cannot exclude spurious modes and should be used together, for example, with simple a priori convergence results based on Courant-Fisher min-max theorem. Somewhat similar technique has been used by Wheeler to derive convergence estimates for finite element approximation of parabolic problems, [2].

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# MAXIMUM NORM ESTIMATES WITH APPLICATION TO NEUMANN BOUNDARY CONTROL PROBLEMS

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## ABSTRACT

This talk deals with the computation of the numerical finite element solution of boundary value problems with Neumann boundary data and optimal control problems with Neumann control in polygonal domains. Due to the corners of the domain, the convergence rate of the numerical solutions can be lower than in case of smooth domains. As a remedy one can use local mesh refinement near the corners. In order to prove optimal error estimates regularity results in weighted Sobolev spaces are exploited. In such a case the convergence rate of  $|\ln h|^{3/2}h^2$  using piecewise linear ansatz functions can be shown for the state variable as well as the adjoint variable in the domain and the control variable on the boundary. Similar results for boundary value problems with Dirichlet boundary conditions and optimal control problems with distributed control were obtained by Th. Apel, A. Rösch and D. Sirch (2009).

# MS 5: Tensor Numerical Approximation of Multi-Dimensional Functions, Operators and PDEs

# TENSOR-STRUCTURED METHOD FOR FAST CALCULATION OF THE EXCITATION ENERGIES FOR COMPACT MOLECULES

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## ABSTRACT

The Bethe-Salpeter equation (BSE) is a reliable model for estimating the absorption spectra in molecules and solids on the basis of accurate calculation of the excitation energies from first principles. The necessary prerequisites for generation of matrices in the BSE system are ab-initio ground state energy calculations. The tensor-structured Hartree-Fock solver [2, 3], provides a full set of molecular orbitals and eigenvalues and an efficient low-rank Cholesky factorization for the 4th order tensor of two-electron integrals (TEI) [1, 3]. We present a new approach [4, 5] to computation of the Bethe-Salpeter excitation energies with relaxation of the numerical costs to  $O(N^3)$  in size of atomic orbitals basis set,  $N$ , instead of practically intractable  $O(N^6)$  for conventional diagonalization of the BSE matrix. The diagonal plus low-rank tensor approximations to the fully populated blocks in the large BSE matrix are constructed gaining from the low-rank form of TEI, thus enabling easier partial eigenvalue solver for a large auxiliary system but with a simplified block structure. Then a small subset of eigenfunctions from the auxiliary BSE problem is selected to solve the Galerkin projection of the initial spectral problem onto the reduced basis set. The numerical tests on BSE calculations for a number of molecules confirm  $\varepsilon$ -rank bounds for the blocks of BSE matrix. They show that the above scheme for the numerical solution of the BSE eigenvalue problem enables fast calculation of the low part of the excitation spectra for compact molecules up to small amino acids and finite atomic chains [4, 5].

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# A COMBINATION OF ALTERNATING LEAST SQUARES AND LOW-RANK CROSS APPROXIMATION FOR SOLUTION OF PARAMETRIC PDES

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## ABSTRACT

Partial differential equations with stochastic or parameter-dependent coefficients constitute an important uncertainty quantification task and a challenging high-dimensional problem. It has been approached with many techniques, such as Monte Carlo, Sparse Grids, Reduced Basis and tensor decompositions. The latter two offer potentially a lower complexity by finding a tailored low-rank solution approximation. However, generic low-rank methods, such as Alternating Least Squares (ALS), can disturb the sparsity of the original system. As a result, realistic scenarios with fine unstructured grids and large ranks resisted treatment with these algorithms. We propose to combine the ALS steps for the spatial variables and the cross approximation steps for the other parameters. This scheme respects the sparsity (in fact, block-diagonality) of the system matrix and allows to reuse dedicated solvers for the deterministic problem. We show that the new algorithm can be significantly faster than the Sparse Grids and Quasi Monte Carlo methods for smooth random coefficients.



# USING TENSOR DECOMPOSITION IN ISOGEOMETRIC ANALYSIS

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## ABSTRACT

The use of tensor methods in the field of numerical simulation was explored the last decade, with the aim to overcome the curse of dimensionality, ie. the exponential complexity with respect to the spatial dimension of the computational domain. With the advent of Isogeometric Analysis (IGA) during the same period of time, the very same difficulty of dimensionality has appeared, in particular in the task of matrix assembly. Indeed, this task is more challenging than in the case of traditional finite element methods. This is due to factors such as the increased degree and the larger supports of the ansatz functions (tensor-product B-splines), that burden the sparsity pattern and bandwidth of the system matrix.

We report on recent developments on using tensor methods to accelerate the assembly process. The method relies on tensor decomposition of integral kernels which arise in the isogeometric Galerkin matrices. In particular, we obtained a compact representation of these matrices a small number of Kronecker products of auxiliary matrices, which are defined by univariate integrals. This representation, which is based on a low-rank tensor approximation of certain parts of the integrands, made it possible to achieve a significant speedup of the assembly process without compromising the overall accuracy of the simulation. The talk will describe our recent progress towards the extension of these methods to the multivariate case (i.e., to any dimension). We will also discuss further perspectives for the use of numerical low-rank tensor methods in isogeometric analysis.

This is joint work with Bert Jüttler, Ulrich Langer and Boris Khoromskij.

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# FAST SOLVERS FOR OPTIMAL CONTROL PROBLEMS CONSTRAINED BY PDES WITH UNCERTAIN INPUTS

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## ABSTRACT

Optimization problems constrained by deterministic steady-state partial differential equations (PDEs) are computationally challenging. This is even more so if the constraints are deterministic unsteady PDEs since one would then need to solve a system of PDEs coupled globally in time and space, and time-stepping methods quickly reach their limitations due to the enormous demand for storage [4]. Yet, more challenging than the afore-mentioned are problems constrained by unsteady PDEs involving (countably many) parametric or uncertain inputs. This class of problems often leads to prohibitively high dimensional saddle-point system with tensor product structure, especially when discretized with the stochastic Galerkin finite element method (SGFEM) [3]. Moreover, a typical model for an optimal control problem with stochastic inputs (SOCP) will usually be used for the quantification of the statistics of the system response - a task that could in turn result in additional enormous computational expense.

In this talk, we consider two prototypical model SOCPs and discretize them with SGFEM. We derive and analyze robust Schur complement-based block diagonal preconditioners for solving the resulting stochastic Galerkin systems with all-at-once low-rank solvers. The developed solvers are quite efficient in the reduction of temporal and storage requirements of the high-dimensional linear systems [2, 1]. Finally, we illustrate the effectiveness of our solvers with numerical experiments.

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# ON MISTER R METHOD FOR SOLVING LINEAR EQUATIONS WITH SYMMETRIC MATRICES

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## ABSTRACT

We treat with Krylov subspace method for efficiently solving linear equations with symmetric matrices, and design a minimal residual method using coupled two-term recurrences, by which the residual vector and approximate solution are updated, formulated by Rutishauser [3]. Our proposed method is referred to as Mister R (abbreviated as MrR). Coefficients of the residual polynomial are determined by imposing the  $A$ -orthogonality on the residuals in our proposed MrR as well as in the Conjugate Residual (CR) method [4]. MrR is mathematically equivalent to CR and the Minimized Residual method based on the Three-term Recurrence formula of Conjugate Gradient([2])-Type (abbreviated as MRTR) [1], but the implementations are different. Thus the convergence behavior among MrR, CG and MRTR is supposed to be different. From our experience it appears that the residual norms of MrR converge faster than those of CG and MRTR. Numerical experiments on linear equations with symmetric matrices show that MrR is more effective than CG and MRTR.

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# SUPERCONVERGENCE RESULTS FOR NEUMANN BOUNDARY CONTROL PROBLEMS ON LOCALLY REFINED MESHES

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## ABSTRACT

Considered are finite element error estimates for optimal Neumann boundary control problems. A full discretization using a piecewise constant approximation for the control and a continuous and piecewise linear approximation for the state and co-state variable is presented. Due to the polynomial degree used for the control approximation one would obtain a convergence rate of at most one in the  $L^2(\Gamma)$ -norm, but the rate two can be achieved for another control approximation which can be computed in a postprocessing step by evaluating the optimality condition point-wise [2]. The new results presented in this talk are error estimates for the improved control for the case that the computational domain is polyhedral. Due to the presence of singularities in the vicinity of edges and vertices of the domain we observe a reduced convergence rate on quasi-uniform meshes depending on the structure of the domain. With the aim to restore the optimal convergence rate, locally refined meshes are considered and we show refinement criteria that guarantee the desired convergence rates [1, 3]. In the proof of the desired estimates two technical results have to be derived. Firstly, a new error estimate for the trace of the finite element approximation of the co-state in the  $L^2(\Gamma)$ -norm is proved. Secondly, an estimate for the discrete state variable in the  $L^2(\Omega)$ -norm is derived which follows from superconvergence properties and a supercloseness principle for the midpoint interpolant.

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## MS 6: Recent Developments on Computational Electromagnetism and Related Applications

# STAGGERED DISCONTINUOUS GALERKIN METHODS FOR MAXWELL'S EQUATIONS

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## ABSTRACT

In this talk, a new type of staggered discontinuous Galerkin methods for the three dimensional Maxwell's equations is presented. The spatial discretization is based on staggered Cartesian grids so that many good properties are obtained. First of all, our method has the advantages that the numerical solution preserves the electromagnetic energy and automatically fulfills a discrete version of the Gauss law. Moreover, the mass matrices are diagonal, thus time marching is explicit and is very efficient. Our method is high order accurate and the optimal order of convergence is rigorously proved. It is also very easy to implement due to its Cartesian structure and can be regarded as a generalization of the classical Yee's scheme as well as the quadrilateral edge finite elements. Furthermore, a superconvergence result, that is the convergence rate is one order higher at interpolation nodes, is proved. Numerical results are shown to confirm our theoretical statements, and applications to problems in unbounded domains with the use of PML are presented. A comparison of our staggered method and non-staggered method is carried out and shows that our method has better accuracy and efficiency. This research is partially supported by Hong Kong RGC General Research Fund Project 400813.

# A CONVERGENT ADAPTIVE FINITE ELEMENT METHOD FOR CATHODIC PROTECTION

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## ABSTRACT

In this talk, I introduce an adaptive finite element method for the numerical approximation of a cathodic protection problem, which is governed by a steady-state diffusion equation with a nonlinear boundary condition. Under a general assumption on the marking strategy, it is shown that the algorithm generates a sequence of discrete solutions that converges strongly to the exact solution in  $H^1$ -norm and the sequence of error estimators has a vanishing limit. Numerical results are also presented to illustrate the convergence and efficiency of the adaptive algorithm. This is a joint work with Guanglian Li from University of Bonn, Germany.

# NON-STANDARD PARTIAL INTEGRATION: IMPLICATIONS TO MAXWELL AND KORN INEQUALITIES

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## ABSTRACT

We will prove that in bounded and convex three-dimensional domains the Maxwell constants lie in between the Friedrichs and Poincare constants. Furthermore, in our efforts to prove these estimates, it turned out that in (piecewise) concave domains interesting new non-standard first Korn inequalities hold. We will discuss some disturbing consequences.

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# FINITE ELEMENT METHOD FOR NONLINEAR HELMHOLTZ EQUATION WITH HIGH WAVE NUMBER

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## ABSTRACT

A nonlinear Helmholtz equation (NLH) with impedance boundary condition at high frequency is considered. Stability estimates with explicit dependence on the wave number for the NLH in 2 and 3 dimensions are derived. Preasymptotic error estimates are proved for the linear finite element discretizations.

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# **PERFECTLY MATCHED LAYER METHOD FOR ELECTROMAGNETIC SCATTERING PROBLEMS IN LAYERED MEDIA**

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## **ABSTRACT**

This talk is to study the convergence of the perfectly matched layer (PML) method for electromagnetic scattering problems in layered media. The PML method is widely used in the engineering literature and very efficient to solve wave scattering problems. In 2010, Chen and Zheng first proved the exponential convergence of PML method for the Helmholtz scattering problem in a two-layered medium. Since the background materials in the upper and lower half spaces are different, the Green function of the scattering problem in layered media becomes very complicated. Their proof is very technical and depends on elaborates estimates for the Green function. In this work, we develop a new framework for the exponential convergence of the PML method and for the well-posedness of the approximate problem. The methodology is used to three-dimensional electromagnetic scattering problems in two-layer media.

# NUMERICAL ANALYSIS FOR THE OPTIMAL CONTROL OF THE TIME-DEPENDENT MAXWELL'S EQUATIONS

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## ABSTRACT

In this talk an optimal control problem of the full time-dependent Maxwell's equations and its numerical analysis are presented. We aim at finding the optimal current density and its time-dependent amplitude which steer the electric and magnetic fields to the desired ones. The mathematical analysis of the control problem which includes existence and regularity results is briefly discussed. Moreover, a mixed finite element discretization in space and a Crank-Nicolson scheme for the time-dependent Maxwell equation lead to a discretized optimal control problem that we peruse for the rest of the talk.

We can show convergence of the discretized problem to the continuous one up to subsequence. Further, due to the nonlinearity of the problem, we consider second order conditions of the problem and discuss rates of convergence for the optimal control problem.

# GRÖGER-MEYERS'S ESTIMATE AND JUSTIFICATION OF THE ENCLOSURE METHOD FOR THE MAXWELL SYSTEM.

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## ABSTRACT

The enclosure method is a direct reconstruction method designed to recover the shape of an inclusion (which can be acoustic, electromagnetic, elastic etc.) from its response to probing waves of the form of geometric optics. Compared to the well known reconstruction methods, linearity is not necessary. However, as it uses geometric optics solutions as probing signals, it provides only 'high frequency' features of the shapes as the support function, the distance function (from the surface of measurements to the surface of the inclusion) etc. This method is proposed by M. Ikehata and then developed and refined by several other authors as G. Nakamura, J. N. Wang, G. Uhlmann and many others. To justify it, some a priori geometrical conditions are imposed to the shapes. In addition, its extension to the full Maxwell system was an open issue.

In this talk, we explain how we can remove these geometrical conditions and extend the method to the full Maxwell system. One of the basic tools we use is the so called Gröger-Meyers's estimate which is well known for the scalar divergence form elliptic problems. This estimate means, in particular, that the solution operator is an isomorphism in the  $L^p$ -spaces with a range of the power  $p$  depending on the contrast of the materials defining the shape. We derive such an estimate for the full Maxwell system and use it to justify the enclosure method avoiding any geometrical condition.

This is a joint work with Manas Kar (from University of Jyväskylä), Rulin Kuan and Yi-Hsuan Lin (both from Taiwan National University). More details on these results can be found in the following references.

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# ADAPTIVE FEMS FOR INVERSE PROBLEMS

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## ABSTRACT

In this talk we shall review some adaptive finite element methods that are developed for solving linear and nonlinear ill-posed inverse problems, including the reconstruction of fluxes, Robin coefficients and conductivities from partial boundary measurements of some physical quantities. The adaptive reconstruction techniques are based on the output least-squares formulations with appropriate Tikhonov regularizations and the discretization of the forward PDEs using continuous piecewise linear finite elements. A posteriori error estimators are derived that involve the concerned state and adjoint variables and the recovered physical parameter in the corresponding inverse problem. Convergences of the adaptive FEMs will be addressed, and the numerical results are presented to demonstrate the convergence and efficiency of the adaptive FEMs. The main results of the talk were based on several joint works with Bangti Jin (UCL), Jingzhi Li (SUSTC) and Yifeng Xu (Shanghai Normal University), and were substantially supported by Hong Kong RGC grants (projects 14306814 and 405513).

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# MS 7: Numerical Methods for Time-Dependent Transportation and Optimal Control Problems

# OPTIMAL CONTROL OF INFINITE DIMENSIONAL BILINEAR SYSTEMS: APPLICATIONS TO THE HEAT, WAVE, AND SCHRÖDINGER EQUATIONS

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## ABSTRACT

In this paper we consider second order optimality conditions for bilinear optimal control problems governed by a strongly continuous semigroup operator, the control entering linearly in the cost function. We derive first and second order optimality conditions, taking advantage of the Goh transform. We then apply the results to the heat, wave, and Schrödinger equations. This is joint work with M.S. Aronna (EMAp/FGV, Rio de Janeiro) and J.F. Bonnans (INRIA Saclay and CMAP, École polytechnique).

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# **PROFILE LIKELIHOOD CALCULATION FOR TIME-DEPENDENT PDE CONSTRAINED PARAMETER ESTIMATION PROBLEMS**

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## **ABSTRACT**

Time-dependent partial differential equations are widely used to model processes in natural sciences and engineering. The parameters of the considered processes are often unknown and have to be determined from experimental data. Once the parameter identification process is performed, uncertainty analysis becomes important. A reliable but computationally intensive approach is to use profile likelihood based confidence intervals. In this talk an integration based profile likelihood calculation method is proposed and analysed. The practicality of the method is illustrated by application to a model of gradient formation in fission yeast.



# A DISCRETE HUGHES' MODEL FOR PEDESTRIAN FLOW ON GRAPHS

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## ABSTRACT

In this talk, we introduce a discrete time-finite state model for pedestrian flow on a graph in the spirit of the Hughes' dynamic continuum model [3, 1]. The pedestrians, represented by a density function, move on the graph choosing a route to minimize the instantaneous travel cost to the destination. The density is governed by a conservation law whereas the minimization principle is described by a graph eikonal equation (cf. [2]). We show that the discrete model is well-posed and the numerical examples reported confirm the validity of the proposed model and its applicability to describe real situations.

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# OPTIMAL FEEDBACK CONTROL OF NONLINEAR PARABOLIC EQUATIONS

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## ABSTRACT

Feedback control design plays a fundamental role in modern engineering. For an optimality-based formulation of the control problem, the Dynamic Programming Principle allows the characterization of the associated value function as the viscosity solution of a first-order, fully nonlinear Hamilton-Jacobi-Bellman equation. The equation is defined over the state-space of the controlled dynamical system and therefore, even control problems over low-dimensional dynamics lead to HJB equations of high complexity. In this talk, we present an approximation framework to compute (sub)optimal feedback controllers based on the solution of a Generalized HJB equation and a policy iteration algorithm. Problems arising from the feedback control of partial differential equations illustrate the effectiveness of our approach in a high-dimensional context.

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# **BINARY INTERACTION APPROXIMATION FOR MEAN-FIELD OPTIMAL CONTROL PROBLEMS**

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## **ABSTRACT**

In this talk I will present some recent developments on a class of numerical methods for the optimal control of multi-agent systems. Due to the high-dimensionality and the non-linearities of this type of problems, standard techniques usually fail or they are completely inefficient. In order to reduce the complexity of the problem, I will propose a general framework based on the approximation of the microscopic dynamics through a Boltzmann-like equation, showing that under a suitable scaling this is equivalent to the mean-field description of the original problem. Thereafter, I will present a class of algorithms, based on the simulation of the binary interactions of the Boltzmann dynamics, which allows to solve efficiently the mean-field optimal control problem. Several numerical examples will show the effectiveness of the proposed strategies, also for some of applications in the context of opinion formation and crowd motion.

# OPTIMAL CONTROL MODELS IN PEDESTRIAN DYNAMICS

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## ABSTRACT

Large pedestrian crowds often exhibit complex dynamics. There is a vast literature on different mathematical approaches ranging from the microscopic description of the individual dynamics to macroscopic equations for the evolution of the crowd. In this talk, we focus on optimal control models, which describe the evolution of a large pedestrian group trying to reach a specific target with minimal cost. We discuss different models regarding the cost functionals and PDE-constraints as well as the connection to the Hughes model for pedestrian flow. We propose a space-time method which is based on the Benamou and Brenier formulation of optimal transport problems and illustrate the dynamics with numerical simulations. This is a joint work with M.-T. Wolfram (University of Warwick).

## Contributed Talks 1

# DISCONTINUOUS GALERKIN ISOGEOMETRIC ANALYSIS OF ELLIPTIC DIFFUSION PROBLEMS ON SEGMENTATIONS WITH GAPS AND OVERLAPS

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## ABSTRACT

In this presentation, we derive error estimates for multipatch discontinuous Galerkin Isogeometric Analysis approximations  $a_{dG}(u_h, v_h) = \ell_{dG}(v_h)$ ,  $\forall v_h = \{v_{h,i}\}_{i=1}^N \in V_{dG,h}(\Omega)$ , to diffusion problems of the form  $-\nabla \cdot (\alpha \nabla u) = f$  in  $\Omega \subset \mathbb{R}^d$ ,  $d = 2, 3$ , and  $u = g_D$  on  $\partial\Omega$ . We suppose that  $\bar{\Omega} = \cup_{i=1}^N \bar{\Omega}_i$  has a multipatch representation, where every patch  $\bar{\Omega}_i = \Phi_i(\bar{\hat{\Omega}})$  is an image of the parameter domain  $\hat{\Omega}$ , that is  $(0, 1)^d$ , by the NURBS map  $\Phi_i : \hat{\Omega} \rightarrow \bar{\Omega}_i$ . Our numerical analysis covers low-regularity solutions, graded meshes, non-matching meshes, and segmentation crimes like non-matching interface parametrizations creating gap and overlap regions. This talk is based on the joint works with C. Hofer, U. Langer, A. Mantzaflaris, and S. Moore [1, 2, 3, 4].

This work was supported by the Austrian Science Fund (FWF) under the grant NFN S117-03.

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# DUAL-PRIMAL TEARING AND INTERCONNECTING METHODS FOR CONTINUOUS AND DISCONTINUOUS GALERKIN ISOGEOMETRIC ANALYSIS

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## ABSTRACT

In this talk, we construct and investigate fast solvers for large-scale linear systems of algebraic equations arising from isogeometric analysis (IgA) of diffusion problems with heterogeneous diffusion coefficient on multipatch domains. In particular, we investigate the adaption of the Dual-Primal Finite Element Tearing and Interconnecting (FETI-DP) method to IgA, called Dual-Primal Isogeometric Tearing and Interconnecting (IETI-DP) method. The use of open knot vectors is very crucial since in this case we can still distinguish between basis functions corresponding to the boundary and to the interior of the patches (subdomains). We consider the cases where we have matching and non-matching meshes on the interfaces. In the latter case we use a discontinuous Galerkin (dG) method to couple the different patches. This requires a special extension of the IETI-DP method to the dG-IgA formulation. We use ideas from the finite element case in order to formulate the corresponding IETI-DP method, called dG-IETI-DP. We design the dG-IETI-DP method in such a way that it can be seen as a IETI-DP method on an extended discrete interface space. These methods are highly suited for parallelization. We present numerical results for complicated two and three dimensional domains. We observe a quasi-optimal behavior of the condition number  $\kappa$  of the preconditioned system with respect to the mesh-size  $h$  and the patch-size  $H$ . More precisely, this condition number  $\kappa$  behaves like  $O((1 + \log(H/h))^2)$ , and is robust with respect to jumping diffusion coefficients.

# EXPERIENCES OF THE TIME-DISCONTINUOUS GALERKIN METHOD IN THE PROBLEMS OF STRUCTURAL MECHANICS

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## ABSTRACT

In this paper experiences on the use of the time-discontinuous Galerkin method  $dG(q)$  in integrating the constitutive models and the equations of motion is presented. For classical viscoplastic material models the implicit Euler (IE) scheme is very attractive due to its simplicity and good accuracy especially when practically relevant large time-steps are used [3, 4]. However, the situation is completely changed when viscoplastic constitutive models are coupled with damage models [1, 5] and the IE scheme severely overestimates the damage rate with increasing the time-step size. Time-discontinuous Galerkin schemes  $dG(q)$  [2], with  $q \geq 1$ , has shown to behave well also with coupled viscoplastic-damage constitutive models, although they are computationally more expensive per step than the IE. However, they have proven to have better contractive properties in the non-linear solution phase and thus facilitating the use of large time-steps, and therefore competitive in comparison to the IE scheme. Also experiences of the  $dG(q)$  scheme in integrating the equations of motion is given.

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# LOW-ORDER DPG-FEMS FOR LINEAR ELASTICITY

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## ABSTRACT

Since the design of pointwise symmetric stress approximations in  $H(\operatorname{div}, \Omega; \mathbb{S})$  is cumbersome, especially in  $3D$ , the discontinuous Petrov-Galerkin methodology promises a low-order symmetric stress approximation. In this talk, we use the ultraweak formulation and the practical dPG method [1, 2] to introduce three new methods with piecewise constant ansatz functions for the displacement and the stress variable and piecewise affine and continuous interface displacements and piecewise constant interface tractions. The minimal discrete test space is of lower order than those presented in [3, 4] and comprises piecewise (and, in general, discontinuous) symmetric parts of lowest-order Raviart-Thomas functions and piecewise affine functions. The methods differ from each other in the choice of norms and the occurrence of some constraint. These spaces allow for a direct proof of the discrete inf-sup condition and a complete a priori and a posteriori error analysis which is robust in the incompressible limit as  $\lambda \rightarrow \infty$ . Numerical experiments with uniform and adaptive mesh-refinings investigate  $\lambda$ -robustness and confirm that the third scheme is locking-free.

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# HIGH-QUALITY DISCRETIZATIONS FOR ELECTROMAGNETICS

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## ABSTRACT

We associate the degrees of freedoms of the electric and magnetic fields with primal and dual grids, respectively. Instead of vector presentations, we focus on differential forms, and discretize the model in spatial domain by the discrete exterior calculus [2] providing exact formulation for differential operators. The physical characterization of the discretization is presented by the discrete Hodge operators defining the connection between the primal and dual forms [1, 4]. The discretization is independent of the coordinate systems, and by the orthogonality of the primal and dual mesh elements, we ensure the diagonal construction of Hodge operators providing significant savings in computing time [3].

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# ALGORITHMIC SOLUTION OF SPARSE SENSORLOCATION PROBLEMS

Daniel Walter joint work with Ira Neitzel, Boris Vexler

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## ABSTRACT

In this talk we consider the elliptic partial differential equation

$$-\Delta y + q_1 y^3 = f(q_2) \text{ on } \Omega$$

$$y = 0 \text{ on } \partial\Omega$$

with a positive constant  $q_1$  and a parameter-dependent  $L^2(\Omega)$  function  $f(q_2)$ . However, we will also discuss more general situations with parameters  $q$  in  $\mathbb{R}^n$ . To fit the model to the reality, the true parameter  $q$  has to be estimated indirectly, e.g. from pointwise measurements of the state  $y$ . An important task is to find an optimal design  $\omega$  (optimal locations for the sensors and an optimal number of measurements) to obtain estimates which are more reliable. This is usually done by minimizing a suitable function on the eigenvalues of the linearized Fisher-information matrix  $C(\omega)$  over a set of possible experimental designs. In our approach, possible designs  $\omega$  are modelled by the space of finite Radon-measures, which allows to consider pointwise measurements (Dirac-Deltas) at arbitrary spatial points. The costs of the measurements are represented by the totalvariation norm. This leads to the optimal control problem min

$$\min_{\omega \in M(\Omega)} \text{tr}(C(\omega)^{-1}) + \beta \|\omega\|_{M(\Omega)}$$

$$s.t. \quad \omega \geq 0 \quad (P_\beta)$$

It can be shown that this problem has indeed solutions consisting of finitely many Dirac-Deltas. In this talk we will consider two ways to obtain such solutions algorithmically. We will first elaborate on a gradient-based optimization method including an additional step to remove support points and ensure convergence towards finitely supported solutions. As a second approach, we introduce a regularized problem min

$$\min_{\omega \in L^2(\Omega)} \text{tr}(C(\omega)^{-1}) + \beta \|\omega\|_{L^1(\Omega)} + \frac{\epsilon}{2} \|\omega\|_{L^2(\Omega)}^2$$

$$s.t. \quad \omega \geq 0 \quad (P_\beta^\epsilon)$$

For  $\epsilon \rightarrow 0$  the (unique) solutions  $\bar{\omega}_\epsilon$  of  $P_\beta^\epsilon$  converge to solutions of  $P_\beta$ . To obtain solutions of  $P_\beta$  we solve a sequence of regularized problems by a warm-started semismooth Newton method.

# IMPROVEMENT OF PARALLELISM OF E-SSOR PRECONDITION USING STRATEGY OF CACHE-CACHE ELEMENTS TECHNIQUE

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## ABSTRACT

We consider to solve efficiently a linear system of equations by preconditioned Krylov subspace methods on parallel computers. We want to improve parallel performance of Eisenstat's type of SSOR precondition [1] owing to adoption of strategy of Cache-Cache Element technique [2, 3]. So-called Eisenstat-SSOR precondition includes sequential process for gaining mathematical equivalence. Then convergence rate of E-SSOR precondition has slight disadvantage compared with that of incomplete Cholesky decomposition type of precondition. However, E-SSOR precondition has great advantage of less computation cost per one iteration. Then E-SSOR precondition is known to be efficient and robust for solving realistic problems.

Therefore, we propose parallelized E-SSOR precondition by means of adoption of Cache-Cache Elements technique. Through many numerical experiments, we make clear that parallel performance of E-SSOR precondition outperforms with other preconditions from the viewpoints of convergence rate and robustness.

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# MATHEMATICAL MODELS OF SPATIAL DISEASE PROPAGATION AND THEIR QUALITATIVE PROPERTIES

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## ABSTRACT

The large pandemics in the human history show that infectious diseases are able to cause widespread devastation. This is why we want to prevent their outbreak by all means, such as hygiene and vaccination. Beside virological investigations, mathematics also has efficient tools that help to understand the dynamics of epidemics.

The first mathematical model of epidemics in the form of a system of ordinary differential equations was constructed in 1927 by Kermack and McKendrick. This is the so-called SIR model. The model gives the connections between the number of individuals in three disjoint subpopulations: the susceptible ( $S$ ), the infective ( $I$ ) and the removed ( $R$ ) subpopulations. This model has been improved several times since then taking into account different other properties of the members of the population.

The above models assume that the population is homogeneous, that is they do not take into the account the different spatial positions of the individuals. There are several methods to bring also spatial dependence into the picture. For example, it is possible to allow the motion of the individuals in the population. This concept results in a system of reaction diffusion equations. Another possibility is to divide the original population into subpopulations according to some geopolitical considerations and connect them somehow into a network with some prescribed rules.

We follow a third approach given in the book of Jones and Sleeman (Differential Equations and Mathematical Biology, CRC Press, 2011). We transform the SIR model into a system of partial differential equations taking into the account the localised nature of the infections. We can show that this system may possess wave form solutions and under certain conditions the solution of the system must be monotone and nonnegative. We construct different numerical solution models for the continuous problem and investigate the above qualitative properties. We can give sufficient conditions for the mesh sizes that guarantee the qualitative properties a priori in the case of different problem settings. We show that epidemic waves can be stopped by reducing the number of susceptibles below a certain threshold. The results are demonstrated on several numerical test problems.

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## Contributed Talks 2

# DISCRETIZATION ERROR ESTIMATES FOR DIRICHLET CONTROL PROBLEMS IN POLYGONAL DOMAINS

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## ABSTRACT

In this talk, we discuss convergence results for finite element discretized Dirichlet control problems in polygonal domains. We investigate unconstrained as well as control constrained problems. In both cases, we discretize the state and the control by piecewise linear and continuous functions. The error estimates, which we obtain, mainly depend on the size of the interior angles but also on the presence of control constraints and the structure of the underlying mesh. For instance, considering non-convex domains, the convergence rates of the discrete optimal controls in the unconstrained case can even be worse than in the control constrained case.

The results were obtained in collaboration with Mariano Mateos, Johannes Pfefferer, and Arnd Rösch.

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# ON SOME MODELS OF THERMO-PIEZO-ELECTRO-MAGNETISM

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## ABSTRACT

Various linear models describing the interaction between thermodynamic, electromagnetic and elastic effects are discussed within a first order approach to the resulting coupled system. Well-posedness, i.e. existence, uniqueness of solutions and their continuous, causal dependence on the data, is established for associated initial boundary value problems covering a large class of materials.

This is in part a report on joint work with A. Mulholland, S. Trostorff and M. Waurick, [1].

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## Contributed Talks 3

# DOMAIN DECOMPOSITION AND UZAWA-TYPE ITERATIVE SOLUTION METHODS FOR VARIATIONAL INEQUALITIES

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Joint work with ERKKI LAITINEN, Department of Mathematical Sciences, University of Oulu, FINLAND

## ABSTRACT

We consider a variational inequality

$$u \in H_0^1(\Omega) : a(u, v - u) + \psi(|\nabla v|) - \psi(|\nabla u|) \geq \int_{\Omega} f(v - u) dx \quad \forall v \in H_0^1(\Omega), \quad (1)$$

where  $a(u, v) = \int_{\Omega} g_1(\nabla u) \cdot \nabla v dx + \int_{\Omega} g_2(u, \nabla u) v dx : H_0^1(\Omega) \times H_0^1(\Omega) \rightarrow R$  is a semilinear, continuous and uniformly monotone form and  $\psi(z) : L_2(\Omega) \rightarrow R \cup \{+\infty\}$  is a proper, convex and lower semicontinuous function. In the case of polygonal domain  $\Omega \subset R^2$  we construct a finite element approximation of (1) by using DDM with non-overlapping subdomains. Using Lagrange multipliers the mesh problem is transformed to a constrained saddle point problem

$$\begin{pmatrix} A & B^T \\ B & 0 \end{pmatrix} \begin{pmatrix} z \\ \lambda \end{pmatrix} + \begin{pmatrix} \partial\theta(z) \\ 0 \end{pmatrix} \ni \begin{pmatrix} g \\ 0 \end{pmatrix} \quad (2)$$

with a continuous and strictly monotone operator  $A$ , a full column rank matrix  $B$  and a proper, convex and lower semi-continuous function  $\theta$ . The operator  $A$  admits the representation  $Az = A(z, z)$ , where  $A(z, y)$  is a continuous operator, which is uniformly monotone in  $z$  for every  $y$ . Block relaxation-Uzawa method iterative method for solving problem (2)

$$\begin{aligned} A(z^{k+1}, z^k) + \partial\psi(z^{k+1}) + B^T \lambda^k &\ni f, \\ \frac{1}{\tau} D(\lambda^{k+1} - \lambda^k) - Bz^{k+1} &= g, \\ (z^0, \lambda^0) &\text{ is initial guess,} \end{aligned} \quad (3)$$

converges under the appropriate choice of a preconditioner  $D$  and parameter  $\tau > 0$ . Method (3) for mesh approximation of (1) allows a parallel and easy implementable algorithm which consists of point-wise projections and solving the systems of linear algebraic equations.

# ORTHOGONAL BASIS IN SPLINE-WAVELET DECOMPOSITION OF THE NUMERICAL FLOW

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## ABSTRACT

Wavelet decompositions are widely used, which strongly stimulates their further development. Unlike classical wavelet decompositions [1 - 4], the approach proposed in [5] does not require to construct a wavelet basis. On the other hand, the approach of [5] provides asymptotically optimal (with respect to an  $N$ -diameter of standard compact sets) spline-wavelet approximations [6]. In the computer realization of decompositions, the knowledge of a basis allows us to diminish the computation time essentially, but practical realizations of wavelet basis in functional spaces are often labor-consuming; therefore we discuss more useful and simple discrete orthogonal basis. We note that the discrete orthogonal wavelet bases have been considered earlier only for comb structure of the spline-wavelet decomposition (see [7]).

The aim of the paper is to present an orthogonal basis for discrete wavelets in the general structure of the spline-wavelet decomposition and estimate the time of computation of this decomposition by a concurrent computing system with computer communication surrounding taken into account.

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## Contributed Talks 4

# ON CONSERVATIVE SPATIAL DISCRETIZATIONS FOR QUASI-GASDYNAMIC SYSTEMS OF EQUATIONS

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## ABSTRACT

Quasi-gasdynamics (QGD) and quasi-hydrodynamic (QHD) systems of equations underlie the design of a class of finite-difference methods for solving gas- and fluid dynamic problems [1]. They are specific parabolic regularizations of the compressible Navier-Stokes systems of equations.

A multidimensional barotropic QGD system of equations in the form of mass and momentum conservation laws with a general gas equation of state  $p = p(\rho)$  with  $p'(\rho) > 0$  and a potential body force is considered [2]. For this system, two new symmetric spatial discretizations on nonuniform rectangular grids are constructed (in which the density  $\rho$  and velocity  $\mathbf{u}$  are defined on the basic grid, while the components of the regularized mass flux and the viscous stress tensor are defined on staggered grids). These discretizations involve nonstandard approximations for  $\nabla p(\rho)$ ,  $\operatorname{div}(\rho \mathbf{u})$  and  $\rho$ . As a result, a discrete total mass conservation law and energy inequality guaranteeing that the total energy does not grow with time can be derived. Importantly, these discretizations have the additional property of being well-balanced for equilibrium solutions [3]. For the simpler barotropic QHD system, the related simplifications of the constructed discretizations keep similar properties.

Importantly, the study is extended to the full (non-barotropic) multidimensional QGD system of equations, and the specific spatial discretization having the property of the non-decreasing total entropy is designed, cp. [4].

The work is supported by the RFBR, project no. 16-01-00048.

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# DISCRETE OPERATORS, FACTORIZATION, BOUNDARY VALUE PROBLEMS AND NUMERICAL ANALYSIS

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## ABSTRACT

We study special classes of discrete operators and equations in the space  $L_2(D \cap \mathbf{Z}^m)$  where  $D$  is a domain in  $m$ -dimensional space. We extract some classes of canonical domains like a whole space  $\mathbf{R}^m$ , a half-space  $\mathbf{R}_+^m$  or a cone  $C_+^a = \{x \in \mathbf{R}^m : x = (x', x_m), x_m > a|x'|, a > 0\}$  and describe invertibility conditions for model discrete operators in such domains. Sometimes for an invertibility one needs additional boundary conditions. We use a unified approach related to a factorization technique for operator symbols and give a comparison between discrete and continual cases for certain types of domains. Some preliminary results in this direction were obtained earlier [1, 2, 3, 4, 5, 6].

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## Contributed Talks 5

# EXACT FINITE-DIFFERENCE SCHEMES

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## ABSTRACT

The simplest exact difference scheme (EDS) for the Cauchy problem for a nonlinear ordinary differential equation

$$\frac{du}{dt} = f_1(u)f_2(t), \quad t > 0, \quad u(0) = u_0,$$

has the form [1]

$$\frac{y^{n+1} - y^n}{\tau} = \left( \frac{1}{y^{n+1} - y^n} \int_{y^n}^{y^{n+1}} \frac{du}{f(u)} \right)^{-1} \frac{1}{\tau} \int_{t_n}^{t_{n+1}} f_2(t) dt, \quad y^0 = u_0.$$

Finite difference scheme (FDS) is called *exact* if the error of approximation is equal to zero or  $y = u$  at the grid nodes.

This talk is devoted to construction of EDS for the IBVP for the wave equations; for multidimensional semilinear transport equation; for two-dimensional quasilinear convection-diffusion-reaction equations with the travelling-wave type solutions etc. Numerical experiments simulating solutions of semilinear transport equations with shock waves without smearing and non-physical oscillations will be presented. Some results on this topic may be found in [1, 2, 3, 4, 5, 6].

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# MULTIGRID SOLVER IN BOUT++

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## ABSTRACT

BOUT++ is a flexible, modular framework [1] for the solution of partial differential equations with plasma relevant differential operators and geometry. The code has a wide user base and widespread applications, mostly oriented towards pedestal and Scrape-Off Layer physics [2]. Turbulence and motion of coherent structures at the edge of magnetic fusion machines depend on the electrostatic potential, which is determined by inverting a generalization of the perpendicular Laplacian operator (represented by a second order nonlinear elliptic PDE). The multigrid solver for the second order nonlinear elliptic PDE allows for high performance simulations of 3D boundary plasma turbulence. In this talk, we introduce BOUT++ and consider the implementation and performance of the multigrid solver in BOUT++.

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## Contributed Talks 6

# NUMERICAL APPROXIMATION OF A 3D SINGULAR ELECTROMAGNETIC FIELDS BY A VARIATIONAL METHOD

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## ABSTRACT

We propose a new variational method to compute 3D Maxwell's equations in an axisymmetric singular 3D domain, generated by the rotation of a singular polygon around one of its sides, namely containing reentrant corner or edges. We consider the equations written in  $(r, \theta, z)$  and use a Fourier transform in  $\theta$  to reduce 3D Maxwell's equations to a series of 2D Maxwell's equations, depending on the Fourier variable  $k$ . The principle is to compute the 3D solution by solving several 2D problems, each one depending on  $k$ .

Let us denote by  $(\mathbf{E}_k, \mathbf{B}_k)$  the electromagnetic field for each mode  $k$ . Following [1] and [2], it can be proved that this solution can be decomposed into a regular and a singular part. The regular part can be computed with a classical finite element method. The singular part is more difficult to compute: it belongs to a finite-dimensional subspace. Its dimension is equal to the number of reentrant corners and edges of the 2D polygon that generates the 3D domain.

We will first consider the computation of  $\mathbf{x}_k^s$  and  $\mathbf{y}_k^s$ , the singular part of the electric and magnetic field. We will then derive the non stationary variational formulation to compute the total part of the solution. Following [2], this will require first to derive the system of equations solved by the singular parts for each  $k$ . Then to derive and solve the time-dependent variational formulation depending on  $k$ , for each mode. Finally to reconstruct the approximate 3D electromagnetic fields from each mode  $k$ . Numerical examples to illustrate our method will be shown.

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# FLUID-STRUCTURE INTERACTION SOLVERS OF NEWTON-KRYLOV TYPE WITH MULTIGRID PRECONDITIONING

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## ABSTRACT

In this work we study monolithic solvers of Newton-Krylov type for steady and time-dependent incompressible FSI problems. The coupling between fluid and structure is dealt with in a monolithic way, where stress balance and kinematic conditions are automatically satisfied across the fluid-solid interface. The deformation of the fluid domain is taken into account according to an Arbitrary Lagrangian Eulerian (ALE) scheme. The Newton linearization operator is implemented in an exact manner by the use of automatic differentiation tools. The ill-conditioning of the induced stiffness matrix makes the numerical solution of the entire fluid-structure system very challenging, in particular for steady-state configurations. We propose to solve the linearized system at each nonlinear iteration with a GMRES solver, preconditioned by a geometric multigrid algorithm with a domain decomposition smoother of additive Schwarz type. The domain decomposition in the smoothing step is driven by the natural splitting between fluid and solid domain. Numerical results of benchmark tests both for steady-state and time-dependent cases show agreement with the literature as well as an increased robustness with respect to standard choices.

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# TREFFTZ SPACE-TIME DISCONTINUOUS GALERKIN METHOD FOR THE WAVE EQUATION IN TIME DOMAIN

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## ABSTRACT

We present a new time-space discontinuous Galerkin method for one field wave equation which utilizes special Trefftz-type basis functions. The method is motivated by the class of Interior Penalty DG (IPDG), together with the classical work of Hulbert and Hughes. The existence and uniqueness of the scheme is proved via energy argument together with a special continuity of the bilinear form. Numerical experiments highlighting the performance of the method compared with standard polynomial spaces are presented.

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# NUMERICAL STUDY ON DEC AND CONTROLLABILITY TECHNIQUES IN TIME-PERIODIC SOLUTIONS OF WAVE EQUATION

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## ABSTRACT

This presentation discusses computational performance of a controllability algorithm for time-periodic solutions of a scalar wave equation. In recent works, Glowinski, Rossi and Pauly [1, 2] established theoretical framework and an algorithm with controllability technique for numerical time-periodic wave propagation. Since, method used in spatial discretization contributes the performance of the controllability algorithm, this presentation addresses comparison of two discretizations, the first one being based on finite elements and the second one on discrete differential forms or discrete exterior calculus (DEC). The discussion concentrates on scattering of a plane wave in a two-dimensional setup paying also attention on sensitivity of the numerical solution on quality of the computation grid.

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## Contributed Talks 7



# A TWO-DIMENSIONAL FAMILY OF TRANSFORMATIONS WITH VERY DIVERSE BEHAVIOUR

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## ABSTRACT

I will talk about a process called "map with memory" and a two dimensional family of transformations induced by it. The family exhibits most diverse behaviour: from absolutely continuous invariant measure to globally attracting fixed point and singular Sinaj-Ruelle-Bowen measure. I hope to show a multitude of pictures as the study is mostly based on computer experiments.

## REFERENCES

- [1] Paweł Góra, Abraham Boyarsky, Zhenyang Li and Harald Proppe, *Statistical and Deterministic Dynamics of Maps with Memory*, <http://arxiv.org/abs/1604.06991>

# DELAY-DEPENDENT STABILITY OF RUNGE-KUTTA TIME DISCRETIZATIONS FOR PARTIAL DIFFERENTIAL EQUATIONS WITH DELAY

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## ABSTRACT

This talk is concerned with the stability of difference methods for partial differential equations (PDEs) with time-delay. By using a variant of the classical second-order central differences to discretize the spatial derivatives, we first obtain a semi-discrete system, which is a set of ordinary differential equations in the time variable. Then, the time discretizations based on Runge-Kutta methods with a non-constrained mesh are applied, where an equi-stage interpolation procedure is employed to approximate the delay argument. We establish a general stability criterion which can guarantee that the fully discrete system completely preserves the delay-dependent stability of the PDE test problem under consideration. Some high order methods with certain linear or parabolic interpolation are proved to satisfy this criterion. Finally, numerical experiments are presented to confirm the theoretical results.

# HIGH ORDER REAL-SPACE TECHNIQUES FOR ATOMISTIC CALCULATIONS

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## ABSTRACT

It is known that atomistic calculations are limited by their expensive computational requirements, which are mostly numerical and not fundamental physically. In practice, the related physical models are represented mathematically by partial differential equations (PDEs), which should be then solved numerically, where several “real-space” methods have been used ranging from physically driven methods like those adopting atomic orbitals to low-order numerical methods like finite element (FEM) and finite difference (FDM) methods. Recently, high-order methods like spectral methods (SMs) start gaining more attention for atomistic calculations and many numerical methods have been developed accordingly. For such calculations, the abrupt boundary conditions are not a problem as the related physical quantities usually have no such conditions. However, there are many other “numerical” challenges. One of the main challenges is the associated singularities with Coulomb potential. This requires a sort of regularization, which can be deployed in different ways. Some of them involve physical alternations while others don’t. By proper regularization, the differential operators and source functions become smooth and at least  $C^2$  and hence single domain SMs are needed. This fact eliminates many of the challenges for implementing SMs. In this work, the recent developments will be reviewed and some particular applications will be presented.

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# Special Sessions

# OPERATIONAL MATRIX METHOD FOR SOLVING TWO POINT BOUNDARY VALUE PROBLEMS

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## ABSTRACT

The main purpose of this study is to give numerical solution of differential equations based on operational matrix method. Converting differential equations into algebraic expressions is the basic idea of this method. Since the simplicity of the computations, Haar wavelet is preferred. The properties of haar operational matrix method is presented. At the end, illustrative examples are included to show the efficiency and the applicability of this technique.

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# INTEGER POWERS OF CERTAIN COMPLEX PENTADIAGONAL 2–TOEPLITZ MATRICES

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## ABSTRACT

Arbitrary integer powers of special matrices have many applications in some research areas such as numerical analysis, difference equations, differential and delay differential equations, boundary value problems and high order harmonic filtering theory. Let  $K_n$  be Pentadiagonal  $n$ –square 2–Toeplitz matrix as following

$$K_n = \begin{bmatrix} a_1 & 0 & b_1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & a_2 & 0 & b_2 & 0 & 0 & \cdots & 0 & 0 & 0 \\ c_1 & 0 & a_1 & 0 & b_1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & c_2 & 0 & a_2 & 0 & b_2 & \cdots & 0 & 0 & 0 \\ 0 & 0 & c_1 & 0 & a_1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & b_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & a_2 & 0 & b_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & a_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & c_2 & 0 & a_2 \end{bmatrix}$$

here  $a_1, a_2 \in \mathbb{C}$  and  $b_1, b_2, c_1, c_2 \in \mathbb{C} \setminus \{0\}$ . We are studying a general expression for the entries of the  $s$ th ( $s \in \mathbb{Z}$ ) power of even order pentadiagonal 2-Toeplitz matrix. Firstly, motivated by [1], we apply  $K_n$  pentadiagonal 2-Toeplitz matrix to the characteristic polynomial and eigenvectors of this matrix in given [1]. Secondly, we obtain the eigenvalues and eigenvectors of  $K_n$  pentadiagonal 2-Toeplitz matrix. Then, the  $s$ th power of pentadiagonal 2-Toeplitz matrix we will get by using the expression  $K_n^s = L_n J_n^s L_n^{-1}$ , where  $J_n$  is the Jordan's form of  $K_n$  and  $L_n$  is the transforming matrix. Finally, some numerical examples are given.

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# NUMERICAL SOLUTION OF WAVE EQUATIONS BY REDUCED DIFFERENTIAL TRANSFORM METHOD WITH FIXED GRID SIZE

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## ABSTRACT

In this study, wave equation was solved through a method developed with a new method in order to find approximate solutions of partial differential equations that are quite difficult, time-consuming or do not have a solution and we encounter in many applied scientific fields. This method was created by adding fixed grid size algorithm to the reduce differential transform method existing in literature. The result of the wave equation obtained from this method was compared with the results obtained from analytical solution and variation iteration method.

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# REDUCED DIFFERENTIAL TRANSFORM METHOD WITH FIXED GRID SIZE FOR SOLVING TELEGRAPH EQUATIONS

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## ABSTRACT

Partial differential equations are equations that we encounter in many applied scientific fields and in whose solutions many numeric methods are used. In this study, while spread of electrical signals on transmission wires in electrodynamic was examined, we applied fixed grid size to the reduced differential transform method in order to solve telegraph equations explaining current on wires and we presented it. We compare the results obtained from this method with variation iteration method.

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# NEW OPERATIONAL MATRIX SCHEME FOR DIFFERENTIAL EQUATIONS

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## ABSTRACT

In this presentation, an efficient pseudo-spectral approach for the accurate numerical solution to the differential equation modelled for specific situation has been developed. The solution has the form of an finite series of the trial basis functions. The essential advantage of the scheme compared with the finite-difference method is related to the computational costs. The efficiency and validity of the proposed scheme are tested on some illustrative examples.

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# Daily timetables

## Sunday 31.7.

17:00 - 19:00 Registration: Hotel Alba, Ahlmaninkatu 4, 40100 Jyväskylä, Finland

## Monday 1.8.

9:00 - 9:45	Registration and coffee: Agora lobby, Mattilanniemi 2, 40100 Jyväskylä, Finland	
9:45 - 10:00	Opening: Pekka Neittaanmäki and Sergey Repin	
10:00 - 11:00	Plenary Lecture ( <i>chair: Pekka Neittaanmäki</i> ) <b>Raytcho Lazarov</b> Petrov-Galerkin Finite Element Method for Fractional Convection-Diffusion Equations	Auditorium 3
11:00 - 12:00	Plenary Lecture ( <i>chair: Rolf Stenberg</i> ) <b>Ragnar Winther</b> Geometric Decomposition of Finite Element Spaces	Auditorium 3
12:00 - 13:00	Lunch break	
13:00 - 14:00	Plenary Lecture ( <i>chair: Raytcho Lazarov</i> ) <b>Roland Glowinski</b> On the Motion of Rigid Solid Particles In Incompressible Viscous Elastic Liquids: A Numerical Approach	Auditorium 3
14:00 - 14:30	Coffee break	
14:30 - 17:30	Minisymposium 7 <b>Axel Kröner</b> Optimal control of infinite dimensional bilinear systems: applications to the heat, wave and Schrödinger equations <b>Romana Boiger</b> Profile likelihood calculation for time-dependent PDE constrained parameter estimation problems <b>Adriano Festa</b> A discrete Hughes' model for pedestrian flow on graphs <b>Dante Kalise</b> Optimal feedback control of nonlinear parabolic equations <b>Giacomo Albi</b> Binary interaction approximation for mean-field optimal control problems <b>Monika Wolfmayr</b> Optimal control models in pedestrian dynamics	Auditorium 3
	Minisymposium 5 <b>Venera Khoromskaia</b> Tensor-Structured Method for Fast Calculation of the Excitation Energies for Compact Molecules <b>Sergey Dolgov</b> A combination of alternating least squares and low-rank cross approximation for solution of parametric PDEs <b>Angelos Mantzaflaris</b> Using tensor decomposition in isogeometric analysis <b>Akwum Onwunta</b> Using tensor decomposition in isogeometric analysis <b>Kuniyoshi Abe</b> On Mister R Method for Solving Linear Equations with Symmetric Matrices <b>Max Winkler</b> Superconvergence Results for Neumann Boundary Control Problems on Locally Refined Meshes	Ag Alfa
17:30 - 19:00	Welcome reception: Hotel Alba, Ahlmaninkatu 4, 40100 Jyväskylä, Finland	

## Tuesday 2.8.

9:00 - 10:00	Plenary Lecture ( <i>chair: Stefan Sauter</i> ) <b>Markus Melenk</b> Directional $H^2$ -Matrices for Helmholtz Integral Operators	<b>Auditorium 3</b>
10:00 - 11:00	Plenary Lecture ( <i>chair: Stefan Sauter</i> ) <b>Jun Zou</b> Efficient Preconditioners for Edge Element Systems for Various Maxwell Equations	<b>Auditorium 3</b>
11:00 - 11:30	Coffee break	
11:30 - 12:30	Plenary Lecture ( <i>chair: Markus Melenk</i> ) <b>Carsten Carstensen</b> Axioms of Adaptivity: Rate Optimality of Adaptive Algorithms with Separate Marking	<b>Auditorium 3</b>
12:30 - 13:30	Lunch break	
13:30 - 15:30	Contributed talks 1 <b>Auditorium 3</b> <b>Ioannis Touloupoulos</b> Discontinuous Galerkin Isogeometric Analysis of Elliptic Diffusion Problems on Segmentations with Gaps and Overlaps <b>Christoph Hofer</b> Dual-Primal Tearing and Interconnecting Methods for Continuous and Discontinuous Galerkin Isogeometric Analysis <b>Reijo Kouhia</b> Experiences of the Time-Discontinuous Galerkin Method In the Problems of Structural Mechanics <b>Friederike Hellwig</b> Experiences of the Time-Discontinuous Galerkin Method In the Problems of Structural Mechanics	Minisymposium 1 <b>Ag Alfa</b> <b>Petr Vabishchevich</b> Numerical solution of boundary value problems with fractional boundary conditions  <b>Anatoly Alikhanov</b> A difference scheme for the tempered time fractional diffusion equation  <b>Fazal Haq</b> Numerical solution of fractional order model of HIV-1 infection of CD4 <sup>+</sup> T-cells by using Laplace Adomian Decomposition Method <b>Kotapally Harish</b> A pseudo spectral modified quasilinearization for fractional perturbation-differential equations
15:30 - 16:00	Coffee break	
16:00 - 17:30	Contributed talks 1 <b>Auditorium 3</b> <b>Sanna Mönkölä</b> High-Quality Discretizations for Electromagnetics  <b>Daniel Walter</b> Algorithmic Solution of Sparse Sensorlocation Problems  <b>Seiji Fujino</b> Improvement of Parallelism of E-SSOR Precondition Using Strategy of Cache-Cache Elements Technique <b>István Faragó</b> Mathematical Models of Spatial Disease Propagation and Their Qualitative Properties	Minisymposium 1 <b>Ag Alfa</b> <b>Joseph Pasciak</b> Numerical approximation of a variational problem on a bounded domain involving the fractional Laplacian <b>Aiguo Xiao</b> Dissipativity and contractivity of fractional-order systems and their numerical simulation <b>Sergey Piskarev</b> Fractional Equations and Difference Schemes  <b>Yücel Çenesiz</b> Approximate Analytical Solution of Sharma-Tasso-Olver and Burgers-Kdv Equations with Homotopy Analysis Method and Conformable Fractional Derivative
18:30 - 19:30	CMAM Editorial board meeting	

## Wednesday 3.8.

9:00 - 10:00	Plenary Lecture (chair: <i>Sergey Repin</i> ) <b>Yuri Kuznetsov</b> Nonconforming mixed FE methods on polyhedral meshes		Auditorium 3
10:00 - 11:00	Plenary Lecture (chair: <i>Sergey Repin</i> ) <b>Stefan Sauter</b> Wave-number explicit convergence analysis for Galerkin-type discretizations of the Helmholtz equation		Auditorium 3
11:00 - 11:30	Coffee break		
11:30 - 12:30	Contributed talks 2 <b>Thomas Apel</b> Discretization Error Estimates for Dirichlet Control Problems In Polygonal Domains  <b>Rainer Picard</b> On Some Models of Thermo-Piezo-Electro-Magnetism	Auditorium 3	Contributed talks 3 <b>Alexander Lapin</b> Domain Decomposition and Uzawa-Type Iterative Solution Methods for Variational Inequalities  <b>Yuri Demyanovich</b> Orthogonal Basis In Spline-Wavelet Decomposition of the Numerical Flow
12:30 - 13:30	Lunch break		
13:30 - 15:30	Minisymposium 2 <b>Maxim Frolov</b> Implementation of a functional-type a posteriori error estimate for Reissner-Mindlin plates <b>Stanislav Sysala</b> Reliable computational methods in limit analysis of elastic-perfectly plastic bodies <b>Johannes Pfefferer</b> Adapted Numerical Methods for the Poisson Equation with Non-Smooth Boundary Data and Emphasis On Non-Convex Domains <b>Marjaana Nokka</b> A posteriori error bounds for approximations of the stokes problem with nonlinear boundary conditions	Minisymposium 6 <b>Ag Alfa</b> <b>Eric Chung</b> Staggered discontinuous galerkin methods for Maxwell's equations  <b>Yifeng Xu</b> A convergent adaptive finite element method for cathodic protection  <b>Dirk Pauly</b> Non-standard partial integration: implications to Maxwell and Korn inequalities  <b>Haijun Wu</b> Finite element method for nonlinear Helmholtz equation with high wave number	Special sessions <b>Ag Beeta</b> <b>Muammer Ayata</b> Operational Matrix Method for Solving Two Point Boundary Value Problems  <b>Durmus Bozkurt</b> Reduced Differential Transform Method with Fixed Grid Size for Solving Telegraph Equations  <b>Sema Servi</b> Numerical Solution of Wave Equations by Reduced Differential Transform Method with Fixed Grid Size  <b>Galip Oturanc</b> Reduced Differential Transform Method with Fixed Grid Size for Solving Telegraph Equations
15:30 - 16:00	Coffee break		
16:00 - 18:30	Minisymposium 2 <b>Aud 3</b> <b>Svetlana Matculevich</b> A posteriori error estimates for a poroelastic medium  <b>Jan Valdman</b> A FEM approximation of a two-phase obstacle problem and its a posteriori error estimate  <b>Steffen Weisser</b> Reliable and efficient a posteriori error control for BEM-based FEM on polygonal meshes  <b>Stefka Dimova</b> Nonsymmetric and Nonstandard Galerkin Methods for Nonlinear Problems  <b>Irina Burova</b> Interval Estimation of Polynomial Splines of the Fifth Order	Minisymposium 6 <b>Ag Alfa</b> <b>Weiyang Zheng</b> Perfectly matched layer method for electromagnetic scattering problems in layered media  <b>Vera Bommer</b> Numerical analysis for the optimal control of the time-dependent Maxwell's equations  <b>Mourad Sini</b> Gröger-Meyers's estimate and justification of the enclosure method for the Maxwell system  <b>Jun Zou</b> Adaptive Fems for Inverse Problems	Special sessions <b>Ag Beeta</b> <b>Betul Ayse Koc</b> New Operational Matrix Scheme for Differential Equations
18:30 - 19:00	Guided walk to cruise Rhea		
19:00 - 23:00	Cruise and dinner (M/s Rhea, Jyväskylä Harbor, Satamakatu)		

# Thursday 4.8.

9:00 - 10:00	Plenary Lecture ( <i>chair: Yuri Kuznetsov</i> ) <b>Rold Stenberg</b> Wave-number explicit convergence analysis for Galerkin-type discretizations of the Helmholtz equation	<b>Auditorium 3</b>
10:00 - 11:00	Plenary Lecture ( <i>chair: Yuri Kuznetsov</i> ) <b>Boris Khoromskij</b> Rank-structured tensor approximation of multi-dimensional PDEs	<b>Auditorium 3</b>
11:00 - 11:30	Coffee break	
11:30 - 12:30	Contributed talks 4 <b>Alexander Zlotnik</b> On Conservative Spatial Discretizations for Quasi-Gas-dynamic Systems of Equations <b>Vladimir Vasilyev</b> Discrete Operators, Factorization, Boundary Value Problems and Numerical Analysis	<b>Auditorium 3</b>
	Contributed talks 5 <b>Piotr Matus</b> Exact finite-difference schemes <b>Kab Seok Kang</b> Multigrid Solver In BOUT++	<b>Ag Alfa</b>
12:30 - 13:30	Lunch break	
13:30 - 15:00	Minisymposium 3 <b>Marco Agnese</b> Fitted ALE scheme for two-phase Navier–Stokes flow <b>Manuel Borregales</b> Numerical convergence of iterative coupling for non-linear Biot's model <b>Radim Hosek</b> Convergent Finite Difference Scheme for Compressible Navier-Stokes in Three Spatial Dimensions	<b>Auditorium 3</b>
	Minisymposium 4 <b>Gallistl Dietmar</b> Approximation of polyharmonic eigenvalue problems <b>Arbaz Khan</b> Arnold-Winther mixed finite elements for Stokes eigenvalue problems <b>Patrick Kürschner</b> Inexact and preconditioned linear solves in iterative eigenvalue methods	<b>Ag Alfa</b>
15:00 - 15:30	Coffee break	
15:30 - 18:00	Minisymposium 3 <b>Denis Schadinskii</b> Conservation laws in blow-up problems for nonlinear parabolic equations <b>Nail Yamaleev</b> Entropy stable WENO spectral collocation schemes for the Navier-Stokes equations <b>Omar Lakkis</b> A Galerkin method for the Monge–Ampère problem with transport boundary conditions <b>Martin Lind</b> A priori feedback estimates for multiscale reaction-diffusion systems <b>Andreas Hahn</b> ALE-FEM for Incompressible Flows and Transport Problems	<b>Auditorium 3</b>
	Minisymposium 4 <b>Önder Türk</b> A stabilized finite element method for the two-field and three-field Stokes eigenvalue problems <b>Daniele Boffi</b> Adaptive Finite Element Schemes for Eigenvalue Problems: From Mixed Laplacian To Maxwell's Equations <b>Harri Hakula</b> A Posteriori Estimates for Eigenproblems Using Auxiliary Subspace Techniques <b>Antti Hannukainen</b> On A Priori Error Estimates for Eigenvalue Problems <b>Sergejs Rogovs</b> Maximum Norm Estimates with Application To Neumann Boundary Control Problems	<b>Ag Alfa</b>



Friday 5.8.

9:00 - 10:00	Plenary Lecture ( <i>chair: Jun Zou</i> ) <b>Roderick Melnik</b> Coupled Mathematical Models and Multiscale Phenomena at the Nanoscale with Their Applications	<b>Auditorium 3</b>
10:00 - 10:30	Coffee break	
10:30 - 12:30	Contributed talks 6 <b>Irina Raichik</b> Numerical Approximation of A 3D Singular Electromagnetic Fields By A Variational Method <b>Giorgio Bornia</b> Fluid-Structure Interaction Solvers of Newton-Krylov Type with Multigrid Preconditioning <b>Oluwaseun Lijoka</b> Trefftz Space-Time Discontinuous Galerkin Method for the Wave Equation In Time Domain <b>Tytti Saksa</b> Numerical Study on DEC and Controllability Techniques in Time-Periodic Solutions of Wave Equation	Contributed talks 7 <b>Ag Alfa</b> <b>Pawel Gora</b> A Two-Dimensional Family of Transformations with Very Diverse Behaviour <b>Chengming Huang</b> Delay-dependent stability of runge-kutta time discretizations for partial differential equations with delay <b>Fahhad Alharbi</b> High Order Real-Space Techniques for Atomistic Calculations
12:30 - 13:30	Lunch break	
13:30 - 14:00	Closing	

Saturday 6.8.

12:00	Optional Campus Excursion
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