Goal oriented error assessment: guaranteed bounds for general quantities and approximation of the dispersion error

Pedro Díez[†]

[†]Laboratori de Càlcul Numèric
E.T.S.E. de Camins, Canals i Ports de Barcelona
Universitat Politècnica de Catalunya
Campus Nord UPC
Jordi Girona 1, E-08034 Barcelona, SPAIN
pedro.diez@upc.edu

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ABSTRACT

The goal oriented error assessment techniques are essential to verify the numerical quality of the Finite Element computations. Practitioners are interested in determining a guaranteed and narrow enough interval in which yields the actual value of the quantity of interest. Thus, obtaining guaranteed error bounds of functional outputs is one of the real needs of the Computational Mechanics community.

One of the key ingredients of the techniques yielding guaranteed error bounds is an implicit residual error estimator. The Flux-free approach is a promising alternative to standard implicit residual time error estimators that require the equilibration of hybrid fluxes. The idea is to solve local error problems in patches of elements surrounding one node (also known as stars) instead of in single elements [1]. The resulting local problems are flux-free, that is the boundary conditions are natural and hence their implementation is straightforward. This allows precluding the computation and the equilibration of fluxes along the element edges (tractions in a mechanical context). The domain decomposition is performed using a partition of the unity strategy. The resulting estimates are much simpler from the implementation viewpoint, especially in the 3D cases, and provide upper bounds of the energy norm of the error (as well as the standard implicit residual estimators with equilibration of hybrid fluxes). In the past, the local flux-free problems have been solved using a finite element mesh inside each local subdomain. Consequently, the resulting estimates were asymptotic upper bounds (with respect to some reference solution) rather than exact upper bounds (with respect to the exact solution).

Some effort has been devoted to recover exact upper bound using the equilibrated hybrid fluxes approach. The idea is to solve the local problem using a dual formulation and to minimize the complementary energy [2]. Here, the same idea is employed to obtain exact upper bounds using the flux-free approach. The resulting estimates have similar features as their asymptotic version, while providing a guaranteed upper bound.

In the context of convection-diffusion-reaction problems, reference [3] provides a methodology to derive guaranteed bounds using the equilibrated hybrid fluxes approach. This technique uses as input of the error estimation procedure the Galerkin (not stabilized) finite element approximation of the problem. It is well known however, that Galerkin approximations to convection dominated problems are often corrupted by spurious node-to-node oscillations and in practical applications stabilitzation techniques are needed to properly approximate the solution. Here, following ideas given in [4], the methodology introduced in [3] is modified such that it accepts as input the stabilized solutions. Although the rationale of the proposed modifications is different to the original approach, it is worth noting that the implementation is similar and the same codes can be used for both purposes. Also on the same context, a new

methodology is introduced providing guaranteed bounds of the stabilized solutions using the flux-free domain decomposition.

In the context of wave problems (Helmoltz equation), practitioners are mostly interested in assessing the dispersion error, related with the error in the approximation of the wave number. In this case, the approach is completely different because 1) the quantity of interest is not expressed as a functional output and 2) the exact value of the wave number is paradoxically perfectly known as an input data but the numerical approximation of this quantity is unknown. A new and different strategy is devised to deal with this kind of problems, inspired in the a priori error analysis used in the same context.

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