Fuzzy Aggregation of Input-Output Service Level Dynamics in Multimedia Networks

Alexandru Murgu
Department of Mathematical Information Technology
University of Jyvaskyla, FIN-40351 Jyvaskyla, FINLAND
E-mail: murgu@julia.math.jyu.fi

ABSTRACT

In this paper we present a fuzzy interval modelling of the multistream traffic flows for the advanced multiservice systems in the third generation (3G) telecommunication networks. The fuzzy approach of the switching process of the traffic flows allows the minimization of the transmission costs and time delays under uncertainty about the traffic patterns by using an approach based on the multistream component rates input-output aggregation. In the multimedia service models these components are combined such that to meet the strict constraints deriving from the service level agreements (SLAs) in sharing the communication link capacities. The main motivation in considering the fuzzy interval modelling is the need of an adaptive multiplexing strategy which is able to track the relevant parameters characterizing the statistical calsses of traffic demand patterns.

1. Introduction

Before the advent of Gigabit Ethernet, 100 Mbps was the maximum rate for Ethernet. For speeds exceeding 100 Mbps, multiple proprietary Fast Ethernet links are needed to be connected in parallel (e.g., Cisco Fast EtherChannel trunking) ([1]). The ATM technology had an edge with its 622 Mbps version. The Gigabit Ethernet and the proposed 802.3ad Link Aggregation / Truncking standard, Ethernet backbones of several gigabits per second can be built. The Ethernet now scales from 10 to 100 to 1000 Mbps. ATM speeds range from 25 to 622 Mbps. The Ethernet Link Aggregation proposal works for both switches and servers, whereas ATM does not allow server Link Aggregation. The 53 bythe ATM cell structure is less efficient than Ethernet frame structure. For 1 KB frames (Ethernet frames can range from 64 bytes to 1522 bytes), Ethernet protocol efficiency is 0.98, compared to ATM efficiency of 0.9 ([6]). The ATM attractiveness lies in its radically different approach of integrated LAN/WAN and voice/data traffic. The ATM Forum has created LAN Emulation (LANE) and a set of other technologies that enable a smooth migration from legacy LANs to a true ATM environment. If ATM is used only as a backbone technology (i.e., no ATM attached clients or servers), LANE is not required. The ATM has been designed by the service provider industry offers state-of-the-art quality of service (QoS) in the sense that the connections are specified in terms of their bit rates or bandwidth (CBR, VBR, UBR, ABR) ([6]). The Shared Ethernet offers a zero level QoS. Over the past two years, many Ethernet innovations have narrowed the QoS gap between Ethernet and ATM significantly (VLANs and Layer 3 Switching). The QoS is necessary if a network is overloaded and sporadic delays are a normal part of the network operation. For wide area networks (WANs), the links always run close to full capacity and the QoS becomes important. The objective of this paper is to encapsulate in a closed form mathematical model the issue of handling the traffic incertainties which have to be handled by the communication network in order to preserve and guarantee the desired QoS level. The fuzzy interval modelling coupled with a statistical setting of the reference intervals provides a suitable vehicle for carrying out the complexity of uncertainties in the traffic flows when they are aggregated within the network.

2. Fuzzy Interval Arithmetic and Interval Matrices

2.1. Uncertainty, Randomness and Hybrid Numbers

Some observations obtained from a system are precise, while some are measurable only in statistical sense and other cannot be measured at all. The data obtained in this manner are hybrid and lead to hybrid operations and hybrid numbers suitable for dealing with large-scale systems. These concepts can be associated with the Monte Carlo method ([2]). A fuzzy is not a measurement, it is a function $\mu_{\mathbf{A}}(x$ and corresponds to both convex and normal fuzzy subsets. A fuzzy number is a subjective valuation assigned by one or more human operators. In the referential set R, we have a fuzzy number $\mathbf{A}$ with the memmbership function $\mu_{\mathbf{A}}(x)$ and a random variable L whose probability is given by the density function $f_L(x)$, where x is the value of L in R. Suppose that $f_L(x)$ has a convex shape and let the maximum value of $f_L(x)$ be

$$\max_x f_L(x) \tag{1}$$

Dividing the function $f_L(x)$ by its maximum value and define the new function $\mu_{\mathbf{L}}(x)$ which is convex and normal. By this process we substitute the fuzzy number $\mathbf{L}$ for the random variable L, which allows $\mathbf{L}$ to be added to any other fuzzy number by the operation of max-min convolution. Consequently,

$$\mu_{\mathbf{L}}(x) = \frac{f_L(x)}{\max\limits_x f_L(x)} \tag{2}$$

The addition with a fuzzy number $\mathbf{A}$, i.e., $\forall x, y, z \in R$

$$\mu_{\mathbf{A}(+)\mathbf{L}}(z) = \bigvee_{z=x+y} (\mu_{\mathbf{A}}(x) \wedge \mu_{\mathbf{L}}(y)) \tag{3}$$

Let us consider a fuzzy number $\mathbf{A} \subset R$. For a move to the right, $l > 0$, or a move to the left, $l < 0$, we write for $\forall \alpha \in [0,1]$

$$\mathbf{A}(+)l = \left[a_1^{(a)}, a_2^{(\alpha)}\right] (+) [l, l] = \left[a_1^{(a)} + l, a_2^{(\alpha)} + l\right] \tag{4}$$

or, $\forall x, y, z \in R$,

$$\mu_{\mathbf{A}(+)l}(z) = \bigvee_{z=x+y} (\mu_{\mathbf{A}}(x) \wedge \mu_l(y)) \tag{5}$$

where

$$\mu_l(y) = \begin{cases} 0, & y \neq l \\ 1, & y = l \end{cases} \tag{6}$$

If l is a random variable L with a probability density $f(l)$, then the fuzzy number $\mathbf{A}$ will move randomly according to the law $f(l)$. The couple $(\mathbf{A},\ l)$ is called a *hybrid number* and represents the addition of a fuzzy number to a random variable without altering the characteristics of either and without decreasing the amount of information available. A hybrid number can be represented as

$$\mathbf{A}_f = A(\mu,\ f) = \mathbf{A}\,[+]\,L \tag{7}$$

where L is the random variable with probability density $f(l)$. The probability density of $(A_\alpha(+)l)$ is the density of $L = l$, that is,

$$g(A_\alpha(+)l) = g\left(\left[a_1^{(\alpha)} + l,\ a_2^{(\alpha)} + l\right]\right) = f(l) \tag{8}$$

Consider two hybrid numbers in R, $(\mathbf{A}_1,\ L_1)$ and $(\mathbf{A}_2,\ L_2)$, where L_1 and L_2 have the densities $f_1(l_1)$ and $f_2(l_2)$, respectively. We define the addition by the *hybrid convolution*

$$(\mathbf{A}_1,\ L_1)\,[+]\,(\mathbf{A}_2,\ L_2) = (\mathbf{A}_1(+)\mathbf{A}_2,\ L_1(+)'L_2) \tag{9}$$

where $(+)$ represents the max-min convolution for addition and $(+)'$ represents the sum-product convolution for addition. We may also write for $\forall x, y, z \in R$,

$$\mu_{\mathbf{A}_1(+)\mathbf{A}_2}(z) = \bigvee_{z=x+y} (\mu_{\mathbf{A}_1}(x) \wedge \mu_{\mathbf{A}_2}(y)) \tag{10}$$

and

$$f(l) = \int\limits_R f_1(l-l_2) f_2(l_2) dl_2 = \int\limits_R f_1(l_1) f_2(l-l_2) dl_1 \tag{11}$$

A fuzzy number is a special case of a hybrid number, $\mathbf{A} = (\mathbf{A},\ 0)$, where 0 is the trivial random variable with probabilities

$$\Pr(l) = \begin{cases} 1,\ x=0 \\ 0,\ x \neq 0 \end{cases} \tag{12}$$

A random variable is also a special case of a hybrid number, $L = (0, L)$, where 0 is the trivial fuzzy number with certainty

$$\mu_0(x) = \begin{cases} 1,\ x=0 \\ 0,\ x \neq 0 \end{cases} \tag{13}$$

2.2. Mathematical Expectation of a Hybrid Number

For a function $\phi(x)$ in R that is nonnegative and monotonically increasing and a closed interval in $R, \left[a_1^{(\alpha)},\ a_2^{(\alpha)}\right]$ we have

$$\left[\phi(a_1^{(\alpha)}),\ \phi(a_2^{(\alpha)})\right] \subset R \tag{14}$$

and for $l \in R$

$$\left[\phi(a_1^{(\alpha)}+l),\ \phi(a_2^{(\alpha)}+l)\right] \subset R \tag{15}$$

If l is a value of the random variable L, the lower and the upper bounds of (15) depend only on l for a given level of confidence α and the mathematical expectations for each bound are calculated as follows

$$\left[\phi(a_1^{(\alpha)}+l),\ \phi(a_2^{(\alpha)}+l)\right] = \left[\int_{l_1}^{l_2} \phi(a_1^{(\alpha)}+l) f(l) dl,\ \int_{l_1}^{l_2} \phi(a_2^{(\alpha)}+l) f(l) dl\right] \tag{16}$$

Theorem 1. The membership function of the mathematical expectation of a hybrid number $(\mathbf{A},\ L)$ is the membership function of $\mathbf{A}$ shifted by the mathematical expectation of L.

For $k \in R,\ k \neq 0$ and $\forall x \in R$

$$\mu_{k\mathbf{A}}(x) = \mu_{\mathbf{A}}(x/k) \tag{17}$$

or $\forall \alpha \in [0,1]$

$$k \cdot A_\alpha = \begin{cases} \left[k \cdot a_1^{(\alpha)},\ k \cdot a_2^{(\alpha)}\right],\ k>0 \\ \left[k \cdot a_2^{(\alpha)},\ k \cdot a_1^{(\alpha)}\right],\ k<0 \end{cases} \tag{18}$$

If $f(l)$ is the density of L, then the density of $k \cdot L$ is given by

$$g(k \cdot L) = \frac{1}{k} f(l), \;\; k > 0 \tag{19}$$

For $k \neq 0$,

$$k \cdot (\mathbf{A}, \; 0) = (k \cdot \mathbf{A}, \; 0) \tag{20}$$

$$k \cdot (0, \; L) = (0, \; k \cdot L) \tag{21}$$

$$k \cdot (\mathbf{A}, \; L) = (k \cdot \mathbf{A}, \; k \cdot L) \tag{22}$$

If $L = L_1 + L_2$ and L_1 and L_2 have the same density $f(l)$, then

$$g(l) = \int\limits_R f(l - l_1) f(l_1) dl_1 = \int\limits_R f(l - l_2) f(l_2) dl_2 \tag{23}$$

2.3. Sheaf and Expectation of Fuzzy Numbers

We have n (finite) observations of the same phenomenon, each resulting in a fuzzy number $\mathbf{A}_i$, $i = 1, \; 2, ..., \; n$. The set of $\mathbf{A}_i$ in the same referential set E constitutes a *sheaf of fuzzy numbers.*

Theorem 2. Let a sheaf of n numbers $\mathbf{A}_i \in R$, $i = 1, \; 2, ..., \; n$, then for $\alpha \in [0, 1]$

$$A_{i,\alpha} = \left[a_{1,i}^{(\alpha)}, \; a_{2,i}^{(\alpha)} \right] \tag{24}$$

for the interval of confidence at the level α of $\mathbf{A}_i$. Defining the mean as

$$a_1^{m(\alpha)} = \frac{1}{n} \sum_{i=1}^{n} a_{1,i}^{(\alpha)} \tag{25}$$

$$a_2^{m(\alpha)} = \frac{1}{n} \sum_{i=1}^{n} a_{2,i}^{(\alpha)} \tag{26}$$

the mean interval of confidence at level α of the mean fuzzy number $\mathbf{A}^m$ is described as

$$A_\alpha^m = \left[a_1^{m(\alpha')} \;\; a_2^{m(\alpha)} \right] \tag{27}$$

Theorem 3. Consider a sheaf of n fuzzy numbers $\mathbf{A}_i \in R$, $i = 1, \; 2, ..., \; n$, and the corresponding probability law $p(i) = \Pr(i)$, for $i = 1, \; 2, ..., \; , n$. If $\bar{\mathbf{A}}$ represents the fuzzy subset defined as the level α by

$$\bar{A}_\alpha = \left[\sum_{i=1}^{n} a_{1,i}^{(\alpha)} p(i), \; \sum_{i=1}^{n} a_{2,i}^{(\alpha)} p(i) \right] = \left[\bar{a}_1^{(\alpha)}, \; \bar{a}_2^{(\alpha)} \right] \tag{28}$$

then $\bar{\mathbf{A}}$ is a fuzzy number called the *expected fuzzy number* of the sheaf, which is both convex and normal.

The results presented in this section are effectively used in handling the uncertainties in the traffic flows in the ATM networks based on the assumption that the reference point is subject the changes according to some stochastic process that emulated the instantaneous rates of these flows ([4]).

3. Linear Input-Output Traffic Aggregation

In this section we present the main concepts related to the switching of the traffic flows in the ATM networks (considered as data micro/macro streams) when it is regarded as an input-output aggregation process.

3.1. Preliminaries

Suppose that an $n \times n$ input-output matrix $\mathbf{A}$ is to be aggregated from n data micro streams denoted by $N = \{1, ..., n\}$ to m macro streams, $M = \{1, ..., m\}$, with $m < n$. Let an $m \times n$ matrix $\mathbf{S}$ indicate which micro streams are to be combined, that is, for all $i \in M$ and $j \in N$, $s_{i,j} = 1$ if micro stream j is to be included in macro stream i and $s_{i,j} = 0$, otherwise. Thus, $\mathbf{S}$ is a 0-1 matrix with exactly one 1 in every column and at least one 1 in every row ($\mathbf{S}$ is a column stochastic matrix). Let an $n \times m$ matrix $\mathbf{T}$ indicate the proportional weights of each micro stream in its macro aggregate. For all $i \in M$ and $j \in N$, $t_{ji} \in [0, 1]$ if the micro stream j is included in macro stream i and $t_{ji} = 0$, otherwise. The sum of the weights of the micro streams assigned to a givean macro stream is assumed to be 1. Consequently, $\mathbf{T}$ is also column stochastic. The input-output aggregator is computed as the matrix $\mathbf{SAT}$. There are also other methods of matrix aggregation such as

1. Aggregation where $\mathbf{S}$ may contain any positive weights.
2. Aggregation chosen to optimize a particular objective function.

Other possibilities include the application of the above procedure to the $(\mathbf{I} - \mathbf{A})^{-1}$ matrix to obtain $\mathbf{S}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{T}$ and then compute the aggregation of $\mathbf{A}$ as $\mathbf{I} - \mathbf{S}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{T}$. Alternatively, we may use the micro streams as data to estimate an aggregated matrix, using a variant of the econometric estimation techniques for estimating input-output models. Also, we may first aggregate the columns to produce a rectangular model with m macro streams and then convert the rectangular model to a square $m \times m$ model.

3.2. Functional Form of General Aggregators for Traffic Switching

We consider a general aggregator f mapping $n \times n$ input-output matrices into $m \times m$ input-output matrices, $m < n$. Denote the set of real $n \times m$ matrices by $M_{n,m}$. When $n = m$, $M_{n,m}$ is abbreviated as M_n. We consider an open input-output model with n streams given by

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{y} \tag{29}$$

where $\mathbf{x} \in R^n$ is an output vector, $\mathbf{y} \in R^n$ is a final demoand vector and $\mathbf{A} \in M_n$ is an input-output matrix. For an input-output matrix $\mathbf{A} = (a_{ij})$, then the entry a_{ij} can be interpreted as the amount of commodity i necessary in the production of a unit of commodity j, given the technology represented by $\mathbf{A}$. Thus, $1 - \sum_i a_{ij}$ is the value added per unit of production of commodity j, which is assumed positive. In this event, the matrix $\mathbf{A}$ has nonnegative entries and column sums all less than 1. Such a matrix is usually called (strictly) column substochastic since a column stochastic matrix is a nonnegative one with column sums of 1.

Definition 1. By an *input-output* matrix we simply mean a square column substochastic matrix. The input-output equation

$$\mathbf{y} = (\mathbf{I} - \mathbf{A})\mathbf{x} \tag{30}$$

can be used to transform an output vector to a final demand. Conversely, since $\mathbf{A}$ is column substochastic, $(\mathbf{I} - \mathbf{A})$ must be nonsingular, so that $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}$ can be used to transform a final demand vector to an output vector. Since $\mathbf{A}$ is substochastic, the inverse matrix $(\mathbf{I} - \mathbf{A})^{-1}$ is nonnegative and $\mathbf{A}$ is irredicible, then $(\mathbf{I} - \mathbf{A})^{-1}$ is strictly positive

Definition 2. An *input-output matrix aggregator* is a function $f : M_n \to M_m$ that maps the $n \times n$ input-output matrices into $m \times m$ input-output matrices with $m < n$. The (k, l) element of the matrix $f(\mathbf{A})$ will be denoted by $f(\mathbf{A})_{kl}$. An input-output matrix $\mathbf{B}$ will be referred to as an *aggregation* of the input-output matrix $\mathbf{A}$ if it is the result of some aggregator applied to $\mathbf{A}$.

Considering that the input-output models are linear, one natural assumption is that the aggregator is linear. We call an input-output matrix $\mathbf{B}$ a *standard aggregation* of the input-output matrix $\mathbf{A}$ if $\mathbf{B}$ is the result of some standard aggregator applied to $\mathbf{A}$. The following theorem characterizes the functional form of the standard aggregators.

Theorem 4. An input-output aggregator $f : M_n \to M_m$ is standard if and only if f may be represented as

$$f(\mathbf{A}) = \mathbf{SAT} \tag{31}$$

in which $\mathbf{S} \in M_{m,n}$ is a 0-1 column stochastic matrix, $\mathbf{T} \in M_{n,m}$ is a column stochastic and $\mathbf{ST} = \mathbf{I} \in M_m$.

Remark 2. In the context of the theorem, the statement $\mathbf{ST} = \mathbf{I}$ simply means that the nonzero entries of $\mathbf{T}$ are contained among the positions indicated by the 1's of $\mathbf{S}^T$. If h maps N onto M, then the 0-1 matrix $\mathbf{S} \in M_{m,n}$ is a 0-1 column stochastic matrix with no row containing only 0s. The matrix $\mathbf{S}$ is called a *partitioning matrix* and the function h mapping N onto M given by $h(j) = i$ if $s_{ij} = 1$ is called the *function representation* of $\mathbf{S}$.

3.3. Properties of Input-Output Aggregation Process

The features of the input-output matrix $\mathbf{A}$ are, in general, preserved by a standard aggregator $\mathbf{B} = \mathbf{SAT}$.

Theorem 5. Suppose $\mathbf{S} \in M_{m,n}$ is a partitioning matrix, $\mathbf{T} \in M_{n,m}$ is column stochastic and $\mathbf{ST} = \mathbf{I}$. Then $\mathbf{TS}$ is a column stochastic, idempotent matrix of rank m.

Remark 3. The above theorem implies that the set of eigenvalues of $\mathbf{TS}$ includes 1 with multiplicity m and 0 with multiplicity $(n - m)$, since $\mathbf{TS}$ is idempotent. It can be established a close relationship between the standard aggregators and the notion of matrix similarity.

4. Forecasting Model of the Aggregated Flows

We consider a forecasting model based on a fuzzy self-regression of the aggregated flows where the estimated parameters $\tilde{A}_i$ and the independent variable $\tilde{Y}_{t-i}$ are fuzzy numbers of $M - N$ form ([3], [5]).

4.1. Fuzzy Extension of ARMA Model

The classical n-th order forecasting model of self-regression ([7])

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_n Y_{t-n} + e_t \tag{32}$$

can be extended to fuzzy sets as follows

$$\tilde{Y}_t = \tilde{A}_1 \tilde{Y}_{t-1} + \tilde{A}_2 \tilde{Y}_{t-2} + \cdots + \tilde{A}_n \tilde{Y}_{t-n} + e_t \tag{33}$$

in which the awaiting-estimated parameters $\tilde{A}_i$, $i = 1, 2, ..., n$ and the independent variable $\tilde{Y}_t$ are all fuzzy numbers of $M - N$ form. Here, e_t is the error (noise).

Definition 3. $M(\varphi(\chi))$ is called a left (right) fuzzy regular function, if $M(\varphi(\chi))$ satisfies:

1. $\varphi(\chi)$ is a linear form of χ.
2. There exists $\chi_0 \in R^+$ such that $M(\varphi(\chi_0)) = 1$. $M(\varphi(\chi))$ is a monotone increasing continuous function in $(-\infty,\ \chi_0)$ (monotone continuous function on $(\chi_0,\ \infty)$.)

Definition 4. Suppose that $M(\cdot)$ and $N(\cdot)$ are a left and right fuzzy regular function, respectively. The fuzzy number $\tilde{A}$ is called a fuzzy number of $M - N$ if

$$\mu_{\tilde{A}}(\chi) = \begin{cases} M(\varphi(\chi)), & \chi \leq a \\ 1, & a < \chi < b \\ N(\varphi(\chi)), & \chi \geq b \end{cases} \tag{34}$$

Let us suppose that we have a linear function

$$\varphi(\chi) = \psi(\chi) = \frac{\chi - a}{\sigma} \tag{35}$$

for $a,\ \sigma > 0$.

Theorem 6. If $\tilde{A}_i$ and $\tilde{Y}_{t-i}$ are fuzzy numbers of $M - N$ form, that is,

$$\mu_{\tilde{A}_i}(a_i) = \begin{cases} M\left(\frac{a_i - c_i}{\sigma_{i1}}\right), & \text{if } c_{i1} \leq a_i \leq \alpha_{i1} \\ 1, & \text{if } \alpha_{i1} < a_i < \alpha_{i2} \\ N\left(\frac{a_i - \alpha_{i2}}{r_{i1}}\right), & \text{if } \alpha_{i2} \leq a_i \leq c_{i2} \end{cases} \tag{36}$$

and

$$\mu_{\tilde{Y}_{t-i}}(y_{t-i}) = \begin{cases} M\left(\frac{y_{t-i} - d_{i1}}{r_{i1}}\right), & \text{if } d_{i1} \leq y_{t-i} \leq \beta_{i1} \\ 1, & \text{if } \beta_{i1} < y_{t-i} < \beta_{i2} \\ N\left(\frac{y_{t-i} - \beta_{i2}}{r_{i2}}\right), & \text{if } \beta_{i2} \leq y_{t-i} \leq d_{i2} \end{cases} \tag{37}$$

then $\tilde{Y}_t = \sum_{i=1}^{n} \tilde{A}_i \tilde{Y}_{t-i}$ is a fuzzy number of $M - N$ form and the membership function of $\tilde{Y}$ is as follows

$$\mu_{\tilde{Y}_t}(y_t) = \begin{cases} M(-K_1/2) + \sqrt{(k_1/2)^2 - L_1(y_t)}, & \text{if } \sum\limits_{i=1}^{n} c_{i1} d_{i1} \leq y_t \leq \sum\limits_{i=1}^{n} \alpha_{i1}\beta_{i1} \\ 1, & \text{if } \sum\limits_{i=1}^{n} \alpha_{i1}\beta_{i1} < y_t < \sum\limits_{i=1}^{n} \alpha_{i2}\beta_{i2} \\ N(-K_2/2) + \sqrt{(k_2/2)^2 - L_2(y_t)}, & \text{if } \sum\limits_{i=1}^{n} \alpha_{i2}\beta_{i2} \leq y_t \leq \sum\limits_{i=1}^{n} c_{i2} d_{i2} \end{cases} \tag{38}$$

where

$$y_t = \sum_{i=1}^{n} y_{t-i} \tag{39}$$

$$k_1 = \frac{\sum\limits_{i=1}^{n} (c_{i1} r_{i1} + \sigma_{i1} d_{i1})}{\sum\limits_{i=1}^{n} \sigma_{i1} r_{i1}} \tag{40}$$

$$k_2 = \frac{\sum_{i=1}^{n} (d_{i2} r_{i2} + \sigma_{i2} \beta_{i2})}{\sum_{i=1}^{n} \sigma_{i2} r_{i2}} \tag{41}$$

$$L_1(y_t) = \frac{\sum_{i=1}^{n} (c_{i1} d_{i1}) - y_t}{\sum_{i=1}^{n} \sigma_{i1} r_{i1}} \tag{42}$$

$$L_2(y_t) = \frac{\sum_{i=1}^{n} (\alpha_{i2} \beta_{i2}) - y_t}{\sum_{i=1}^{n} \sigma_{i2} r_{i2}} \tag{43}$$

4.2. Model Coefficient Estimation

The main steps of the model estimation are as follows:

1. Form the self-related number sequence according to the observed number.
2. Calculate the self-related coefficient when i elapses, $i = 1, ..., N$ according to the formula

$$r_i = i \sum_t \tilde{Y}_{t-i} Y_t - \frac{\sum_t \tilde{Y}_{t-i} \sum_t \tilde{Y}_t}{\sqrt{\left[i \sum_t \tilde{Y}_{t-i}^2 - \left(\sum_t \tilde{Y}_{t-i} \right)^2 \right] \left[\left[i \sum_t \tilde{Y}_t^2 - \left(\sum_t \tilde{Y}_t \right)^2 \right] \right]}} \tag{44}$$

3. Take $r_p = \max \left\{ r_i \middle| \grave{i} = 1, 2, ..., N \right\}$. The model

$$\tilde{Y}_t = \tilde{A}_1 \tilde{Y}_{t-1} + \tilde{A}_2 \tilde{Y}_{t-2} + \cdots + \tilde{A}_p \tilde{Y}_{t-p} \tag{45}$$

is the best forecasting model of fuzzy self-regression.

References

[1] R. Breyer, S. Riley, *Switched, Fast and Gigabit Ethernet. Understanding, Building and Managing High-Performance Ethernet Networks*, Macmillan Technical Publishing, 1999.

[2] T. Katayama, S. Sugomoto, eds., *Statistical Methods in Control and Signal Processing*, Marcel Dekker, Inc., New York, 1997.

[3] A. Kaufmann, M.D. Gupta, *Introduction to Fuzzy Arithmetic. Theory and Applications*, Van Nostrand Reinhold Company, New York, 1985.

[4] A. Murgu, Fuzzy Clustering of Aggregated Flows for Traffic Control in ATM Networks, in *Proceedings of the 6th International Conference on Fuzzy Systems*, FUZZ-IEEE-97, pp. 235–240, July 1-5, 1997, Barcelona, Spain.

[5] A. Murgu, Adaptive Control for ATM Traffic Shaping with Neural Networks Based Receding Horizon Mappings, in *Proceedings of the 5th European Congress on Intelligent Techniques and Soft Computing*, EUFIT-97, pp. 2037–2045, September 8-12, 1997, Aachen, Germany.

[6] H. Saito *Teletraffic Technologies in ATM Networks*, Artech House, Boston, 1994.

[7] P. Strobach, *Linear Prediction Theory. A Mathematical Basis for Adaptive Systems*, Springer-Verlag, Berlin, 1990.