Multi-Class Service Dynamics Replication Using Evolutionary Selection Games

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Abstract. In this paper, we consider a mathematical approach describing the sequential averaging mapping for selection dynamics of multiple classes in digital data service systems which are generated as evolutionary games. The essential feature of this construction is the encoding of a behaviour discrimination for the long-term dominated strategies associated to the dynamic replicator games of competitive overlapping populations of data packets encountered in the common channels or switching nodes. The decision making process is structured as an averaging operator performing an aggregation task of interval-valued mappings.
Keywords: telecommunication systems, multiclass services, evolutionary strategies, replication games, dynamic aggregation.

1. Introduction

In many tehnical, economic and social applications, the decision process taking into account the uncertainties is best served if some evolutionary approaches are considered (Fogel, 2000). The replication by the way of biological or technical regeneration is an alternative determining the spread of a population of successuful solution strategies. The replication by imitation and enforcement of successful behaviors leads to pure strategies of cheap talk extension as in the game theory. If in the biological population, each individual is programmed to a decision rule that prescribe some base game, the action to be taken for each message is determined by the behaviour of the opponents. In this interpretation, the individuals are discriminating in their behavior and their actions are conditioned on their opponents' messages. When the evolutionary selection operates at this level of messages and decision rules, the long-term aggregate behavior produces strictly dominated strategies. If all possible decision rules are allowed, then the evolutionary selection satisfies a game theoretic rationality in a complex form involving nonsymmetric Nash equilibrium play. An example of such approaches in real life technical system include the widely acclaimed multiprotocol label switching (MPLS) strategies in digital communication networks (Ginsburg, 1996). In the digital switching networks, the delay sensitive applications lead to varying/variable classes of service designed to allow the service providers to offer a portfolio of service level agreements (SLA) (usually ranked according to a set of parameters understood as describing the quality of service). In the internet protocol (IP) managed networks, the connectionless feature operates on a "best-effort" principle where the packets are routed individually using the destination IP adress inside every packet. The routes to be followed by these packets are determined by the IP routing protocols such as the open shortest paths

(OSPs). The packets directed onto the shortest path make no distinction between the specific classes of data traffic flows. The consequence of this approach is the traffic aggregation on only few paths as the OSP does not take into account how much other clases traffic is traversing a particular link at any one time. The level of traffic aggregation is resposible for the quality of service degradation. The traffic flow behaviour discrimination is likely to enhance the network utilisation and service provisioning. Before the advent of Gigabit Ethernet, 100 Mbps was the maximum rate for Ethernet. For speeds exceeding 100 Mbps, multiple proprietary Fast Ethernet links are needed to be connected in parallel (e.g., Cisco Fast EtherChannel trunking). The ATM technology has an edge with its 622 Mbps version. The Gigabit Ethernet and the proposed 802.3ad Link Aggregation / Truncking standard, Ethernet backbones of several gigabits per second can be built. The Ethernet now scales from 10 to 100 to 1000 Mbps. The Ethernet Link Aggregation proposal works for both switches and servers, whereas ATM does not allow server Link Aggregation. The 53 bythe ATM cell structure is less efficient than Ethernet frame structure. For 1 KB frames (Ethernet frames can range from 64 bytes to 1522 bytes), Ethernet protocol efficiency is 0.98, compared to ATM efficiency of 0.9 (Ginsburg, 1996). The ATM attractiveness lies in its radically different approach of integrated LAN/WAN and voice/data traffic. The ATM has been designed by the service provider industry offers state-of-the-art quality of service (QoS) in the sense that the connections are specified in terms of their bit rates or bandwidth (CBR, VBR, UBR, ABR) (Ginsburg, 1996). The Shared Ethernet offers a zero level QoS. The QoS is necessary if a network is overloaded and sporadic delays are a normal part of the network operation. For wide area networks (WANs), the links always run close to full capacity and the QoS becomes important. The objective of this paper is to present an encapsulation method in the handling the traffic incertainties occuring in a network which is subject to multiclass competetive traffic patterns and to offer a way of preserving and guaranteeing the desired QoS level. The network state discovery mechanism based on a mobile agent style propagation of the interactions among the service nodes running the physical implementation of the MPLS strategies.

2. Multiclass Services as Evolutionary Selection Games

Suppose that we have a population of active services implemented in the network under consideration which are regarded as mobile agents which are randomly matched to play a symmetric two-player game. At each matching, we assume that an agent is programmed to issue one and the same of finitely many, distinct signals or messages before playing the game (Weibull, 1996) which tells to the remaining population of sessions that an agreement on the level of network resources sharing (simple/aggregated link capacities, processor sharing rates, etc.) has been reached. Let G represent the game performed using the pure strategy set $K = \{1, ..., m\}$ with the corresponding pure strategy payoff function π. Let M be the finite set of messages that can be issued by each service agreement instance. A decision rule is a function $f : M \to K$ which says that if the opponent message is $\mu \in M$, then use the pure strategy $i = f(\mu)$. Let F be the set of all such functions. A pure strategy in the associated game G_M is a pair $(\mu, f) \times F$ and the payoff to a pure strategy profile $((\mu, f), \nu, g))$ is $\pi\,[f(\nu),\ g(\mu)]$. Let p_h be the population share of mobile agents programmed to play the pure strategy $h = (\mu, f)$ (that is, reaching the service agreement level described by h. The vector $p = (p_h)$ is desined as the population state, a point on the unit simplex Δ_M of mixed strategies of the game. When the population state is $p \in \Delta_M$, the payoff of any strategy $h = (\mu, f)$ in the game is $u_M(e^h, p)$ and the average payoff is $u_M(p, p)$. The dynamics applied to the space Δ_M of mixed strategies of the game is given by

$$\dot{p}_h(t) = u_M(e^h - p(t), p(t))p_h(t) \tag{1}$$

where u_M is the aggregated dynamic operator. The interaction messages in the same subpopulation $\mu \in M$ differ only with respect to their decision rule $f \in F$, that is, when matched with any particular agent from the population, they will all describe the same interaction, which is the base game strategy. If the opponent decision rule is $g \in F$, then any agent from subpopulation μ faces the action $j = g(\mu)$. Let p^μ denote the population share of dynamic interactions of sender of type μ, that is, p^μ is the sum of all p_h such that $h = (\mu, f)$ for some $f \in F$. It is reasonable to decompose the matchings between interaction in the population of active sessions into batches, one batch being dedicated to each session interacting pair. For each action $i \in K$, the interaction $(\mu, \nu) \in M^2$ and the population state $p \in \Delta_M$ with $p^\mu > 0$, let us denote by $p_i^{\mu\nu} \in [0, 1]$ the share of the changing interactions in the subpopulation μ which take the action i when meeting a session of type ν.

Proposition 1. *The vector $p^{\mu\nu} = (p_i^{\mu\nu})_{i \in K}$ is a point on the unit simplex Δ and thus, $p^{\mu\nu}$ is the randomization action (mixed game strategy) facing any interaction of type ν when matched with an interaction of type μ.* □

Equivalently, $p^{\mu\nu} \in \Delta$ is the aggregated randomized action used by the subpopulation $\mu \in M$ against the subpopulation $\nu \in M$. Let us consider the dynamics defined on the mixed strategy simplex Δ in terms of growth rates for the population interaction shares associated with each pure strategy $i \in K$ of the game. This is given by

$$\dot{x}_i(t) = g_i(x(t))x_i(t) \tag{2}$$

where $g_i(x) \in R$ is the rate at which the pure strategy i replicates itself when the population is in the state x (a function with open domain X containing Δ). The population state remains in the simplex at all times if the weighted sum of growth rates $\sum_i g_i(x)x_i$ is constantly equal to zero.

Proposition 2. *The population state $x \in \Delta$ is stationay if and only if all pure strategies that are in use in x have zero growth rate, that is, if and only if $g_i(x) = 0$ for all $i \in C(x)$.* □

Geometrically speaking, the condition $g(x)^T x = 0$ means that the growth rate vector $g(x) \in R^k$ has to be orthogonal to the associated population vector $x \in \Delta$.

Remark 1. The stationarity feature of the aggregated pure strategies are usefl in order to prescribe stationary control rules in the aggregated multiservice classes of traffic flows under competition for network resources. □

3. Service Class Dynamics Replication

Suppose that the payoff function represents the fitness gain at a given time step, $t = 0,\ 1, ...$, where the fitness is simply taken to be the number of emerging interactions of the next generation of multiclass competitive traffic patterns. Suppose that each new interaction inherits in some extent its parents intended strategy and let $\alpha \geq 0$ be the background (lifetime) birthrate of these interactions regarded as mobile agents in social colonies (Goel and Richter-Dyn, 1974). If $p_i(t)$ is the number of agents in generation t who are programmed to perform the pure strategy $i \in K$, the associated population share is $x_i(t) = p_i(t)/p(t)$. Here, $p(t) = \sum_i p_i(t) > 0$ is the total population of interactions in the generation t. Each agent which is programmed to pure strategy i in the generation t gets $\alpha + u\left[e^i, x(t)\right]$ new agents where $e^i \in \Delta$ is the ith generation distribution over pure strategies in the game.

Proposition 3. *The interactive populations of traffic classes satisfy the following structural dynamics*

$$p_i(t+1) = \left(\alpha + u\left[e^i, x(t)\right]\right) p_i(t) \tag{3}$$

or equivalently, summing over all pure strategies $i \in K$, the aggregated dynamics

$$p(t+1) = \left(\alpha + u\left[x(t), x(t)\right]\right) p(t) \tag{4}$$

where $u\left[x(t), x(t)\right]$ is the average payoff to an interaction strategy in the generation t.

□

Proposition 4. *The population shares of discrete time replicator dynamics are given by*

$$x_i(t+1) = \frac{\alpha + u\left[e^i, x(t)\right]}{\alpha + u\left[x(t), x(t)\right]} x_i(t), \quad t = 0, 1, 2, ... \tag{5}$$

□

In the equation (5), all the interactions take place within one generation at a time. Suppose instead that the interaction generations overlap in time such that the births and death rates are $r \geq 1$ per time units. Each time, only the share $\tau = 1/r$ of the total population is involved, where $\tau \in (0,\ 1]$ is the the time duration between two successive population changes. Let us assume that the life-span of an interaction is a random multiple of the interval lenghts τ, that is, it is a geometrically distributed random variable with the mean value $\mu = 1$ and the variance $\sigma^2 = (1-\tau)/\tau = r-1$. Suppose that each interaction is programmed to be performed as a pure strategy i and once it is reproduced at time t, then it is replaced by $u\left[e^i, x(t)\right] + \beta \geq 0$ interactions, where $\beta \geq 0$ is the background (lifetime) birthrate. The population of interactive sessions has the following dynamics

$$p_i(t+1) = (1-\tau)p_i(t) + \tau\left(\beta + u\left[e^i, x(t)\right]\right) p_i(t) \tag{6}$$

where $p_i(t)$ is the number of interactions which at time t are programmed to strategy i and $p(t) > 0$ is the total number of interactions at that time and $x_i(t) = p_i(t)/p(t)$. Adding the population shares and using the bilinearity of the payoff function, we obtain the total population dynamics given by

$$p(t+\tau) = (1-\tau)p(t) + \tau\left(\beta + u\left[x(t), x(t)\right]\right) p(t) \tag{7}$$

Proposition 5. *The replication dynamics of population shares are obtained by dividing each side in (6) by the corresponding side in (7) as follows*

$$x_i(t+\tau) = \frac{1-\tau+\tau\left(\beta+u\left[e^i, x(t)\right]\right)}{1-\tau+\tau\left(\beta+u\left[x(t), x(t)\right]\right)} x_i(t) \tag{8}$$

□

Let us consider the stationary distribution of service class populations regarded as general mapping in the space of input-output matrices $m \times n$ called the steady mixing matrices ($m < n$) (the hybrid multiclass multistrategy dynamic interactions). We consider an open input-output model with n aggregated sessions given by

$$x = Ax + y \tag{9}$$

where $x \in R^n$ is an shaped dynamic population (output population), $y \in R^n$ is the final demand vector and $A \in R^{n\times n}$ is an input-output mixing strategy matrix. For an input-output matrix $A = (a_{ij})$, the entry a_{ij} can be interpreted as the amount of network resource share i necessary in allowing to obtain a unit of resource share j, given the specific mixing technology (for example, MPLS) represented by A. Thus, $1 - \sum_i a_{ij}$ is the value added per unit of commodity j (assumed positive). The matrix A has nonnegative entries and column sums all less than 1. Such a matrix is usually called (strictly) column substochastic since a column stochastic matrix is a nonnegative one with column sums of 1. The input-output equation

$$y = (I - A)x \tag{10}$$

can be used to transform an output population distribution vector into a finally shaped population vector. Conversely, since A is column substochastic, $(I - A)$ must be nonsingular, so that $x = (I - A)^{-1}y$ can be used to transform a final demand vector to an output vector. Since A is substochastic, the inverse matrix $(I - A)^{-1}$ is nonnegative and A is irredicible, then $(I - A)^{-1}$ is strictly positive.

4. Sequential Aggregation of Replicated Dynamic Classes

The methods for producing the input-output matrix A introduced are various and essentially are designed to produce averaging features of the behaviour discrimination uncertainties due to overlapping populations of data packets encountered in the common channels or switching nodes. In the framework of this paper, the dynamics in equation (8) can be regarded as the result of multiple swaps of populations replications as a particular type averaging operation (Fodor and Roubens, 1994). Let $R_1, ..., R_m$, $0 \le R_i \le 1$ be the set of rules leading to the reshaping of the output traffic patterns where m is the number of pure strategies in the initial evolutionary selection game. It is desired to substitute the vector of specific rules $(R_1, ..., R_m)$ by a simple valued aggregation rule R using a compounding operator M (the MPLS/IP traffic aggregator) generically defined as

$$M : \prod_{k=1}^{m} [0,\ 1]^k \to [0,\ 1] \tag{11}$$

subject to the constraint

$$R = M(R_1, ..., R_m) \in [0, 1] \tag{12}$$

Assuming that the condition of independence of irrelevant preferences of the population generations holds, if a profile of valued relations $(R_1, ..., R_m)$ is modified in such a way that the interactions paired comparisons among a set of alternatives (a, b) are unchanged, then the aggregation resulting from the original and the modified profiles should remain unchanged for the pair (a, b) (that is, a dynamic conservation constraint holds). We assume that $R(a, b)$ depends only on $R_1(a, b), ..., R_m(a, b)$ and it is a function of m arguments for every pair $(a, b) \in A \times A$. According to these intended features, the operator M will present a non-negative response to any increase of the arguments. This means that $M(x_1, ..., x_m)$ is monotonic (M-operator) and $x'_i > x_i$ implies that

$$M(x_1, ..., x'_i, ..., x_m) \geq M(x_1, ..., x_i, ..., x_m) \tag{13}$$

We consider that dynamic aggregation operator will satisfy the unanimity voting property, that is, if all R_i are identical, $M(R_i, ..., R_i)$ restitutes the common valued relation (Murgu, 2001, a). The ways to define the operator $M^{(m)}(x_1, ..., x_m)$ in terms of $M^{(m-1)}(x_1, ..., x_{m-1})$ use the associativity and decomposability properties.

Proposition 6. *The associativity property of the aggregation operator of only two arguments can be canonically extended to any finite number of arguments using the absorption dynamics into macrorules (macroclasses)*

$$M^{(3)}(x_1, x_2, x_3) = M^{(2)}(x_1, M^{(2)}(x_2, x_3)) = M^{(2)}(M^{(2)}(x_1, x_2), x_3) \tag{14}$$

$$M^{(m)}(x_1, ..., x_m) = M^{(2)}(M^{(m-1)}(x_1, ..., x_{m-1}), x_m) \tag{15}$$

□

Proposition 7. *The operator M is decomposable, that is, considering a sequence of functions $M^{(1)}(x_1)$, $M^{(2)}(x_1, x_2)$, ..., $M^{(m)}(x_1, ..., x_m)$, each function of the sequence has to satisfy the following equation*

$$M^{(m)}(x_1, ..., x_k, x_{k+1}, ..., x_m) = M^{(m)}(M_k, ..., M_k, x_{k+1}, ..., x_m) \tag{16}$$

with $k \in \{1, ..., m\}$, $M_k = M^{(k)}(x_1, ..., x_k)$. □

The generalized mean $f^{-1}\left[\frac{1}{m}\sum_i f(x_i)\right]$ is an easy and remarkable example of aggregation strategy representing an averaging operator which covers a wide spectrum of means including the arithmetic, quadratic, geometric, harmonic and root-power means. For example, the root-power corresponds to

$$M_m^{(\alpha)}(x_1, ..., x_m) = \left(\frac{1}{m}\sum_i x_i^{\alpha}\right)^{1/\alpha} \tag{17}$$

where $0 < |\alpha| < \infty$, which includes the arithmetic ($\alpha = 1$), quadratic ($\alpha = 2$) and the harmonic ($\alpha = -1$) means. The operator M is stable for any admissible positive linear transformation if

$$M(rx_1 + t, ..., rx_m + t) = rM(x_1, ..., x_m) + t \tag{18}$$

where $r > 0$, $rx_k + t \in [0,1]$ for all $k \in \{1, ..., m\}$ and $rM(x_1, ..., x_m) + t \in [0,1]$. The operator M is a generalized mean if and only if

$$M(x_1, ..., x_m) = \frac{1}{\alpha} \log \left[\sum_{i=1}^{m} \frac{e^{\alpha x_i}}{m} \right], \quad \alpha \neq 0 \tag{19}$$

$$M(x_1, ..., x_m) = \frac{1}{m} \sum_{i=1}^{m} x_i \tag{20}$$

The specific application of the equations (18) and (19) includes the determination of the associated parameters and data smoothing horizon fitted to describe in a best-effort manner the dynamic competition of multiclass services for network resources (Murgu, 2001, b). In the sequential window flow control, the first packets in a switch's input buffers first contend for the outputs, the losing input buffers that contain a second cell are allowed to contend a second time. Those that lose in the second time and have a third cell are allowed to contend a third time and so on, until N times are contended, where N is the depth of window. In parallel windowing (or batch window as it is called in this paper) all the cells in the windows of all buffers contend in a single round with the winners proceeding to the outputs, with the condition that only one cell can be selected at most from a buffer.

5. Concluding Remarks

The statistical multiplexing approach of traffic flows allows the minimization of the transmission costs and time delays based on the decomposition of the traffic patterns using the hierarchical sequential averaging mappings according to the value of the component rates as in (17). The digitally siwtched networks are designed to offer quality of service guaranteed applications. The MPLS/IP connections are specified by the bit rates or bandwidth for a given connection request (constant, variable, unspecified and available bit rates.

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