Fuzzy Succesive Averaging Method in Multistage Optimization Systems

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Abstract

In this paper a new method which extends the classical metod of successive averages (MSA) used in flow assignment problems to the stochastic flows based on fuzzy agregation is developed. The link capacities assigment is of minimum risk type and the measurements on the traffic patterns flowing through these links are time limited (that is, the congestion processes are predicted and tracked using a minimum quantity of information about the system history). A regresison model for identification of the fuzzy parameters is considered. The application of this method to multistage decision making in stochastic flows assignment in digital communication networks is applied for the case of link capacities with Gaussian modulation.

1 Introduction and Motivation

To facilitate the system operations in general flow networks such as telecommunication burst switching, channel handover, dynamic link allocations and switching of flow commodities, multistage decision procedures are employed based on real-time measurements or estimated of the incomming flow patterns. The measurement of the traffic patterns in order to accommodate the incoming traffic to the available link capacities is a current limiting factor for traffic control in the multistage communication networks ([1]). The motivation for estimating the mean flow patterns comes from the actual routing/flow assignment protocols implemented in the burst-switched networks which need an extended description of the traffic parameters in order to cope with a large variety of information sources.

The measurement on the flow patterns provides only a fuzzy information about what these flows should be. The estimation of the parameters of a system requires the selection of a suitable class of functions or models in order to approximate the input-output behaviour of the system under analysis in the best possible manner. A new method which extends the classical metod of successive averages (MSA) to the stochastic flows is formulated as a fuzzy successive averaging process with the goal of controlling the system performance when fuzzy parameters of the flow patterns are available. The application of this method to multistage decision making problems as traffic flows in digital communication networks is analysed and simulation results are presented for the case of link capacities with Gaussian modulated constraints.

2 Predictive Dynamic Network User Equilibrium

2.1 Dynamic Network Model

We consider a flow network with multiple origins and multiple destinations (MOMD). The traffic is represented by a directed graph with nodes and directed links. The period of analyis is denoted by $[0, T]$. A time dependent origin-destination demand is assumed to be known and given as $d_{rs}(t)$ which denotes the traffic demand to the destination node *s* departing from the origin node *r* at time *t* ([4]). Let $P_{rs}(t)$ be the set of paths between OD pair *r-s* for those travellers leaving their origin node *r* at time *t*. Assuming that the traffic flow leaving origin *r* at time *t* via path *p* between OD pair *r-s* is $f_{rsp}(t)$, the flow conservation equations can be written as

$$d_{rs}(t) = \sum_{p \in P_{rs}(t)} f_{rsp}(t) \quad \forall r, s, t \tag{1}$$

Denote by $A_a(t)$ the cumulative arrival curve recording the number of bursts entering the link *a* at time *t* and $D_a(t)$ as the cumulative departure curve recording the number of bursts departing from link *a* at time *t*. Assuming that the network is empty at time $t = 0$, then we have $A_a(t) \geq D_a(t)$. The number of bursts crossing the link *a* at time *t* can be calculated as

$$x_a(t) = A_a(t) - D_a(t), \quad \forall a, t \tag{2}$$

Let l_a and n_a be respectively the length and number of virtual circuits on link *a*. The burst traffic density on link *a* at time *t* becomes

$$k_a(t) = \frac{x_a(t)}{l_a n_a}, \quad \forall a, t \tag{3}$$

A speed-density relationship is assumed for each link *a* as

$$u_a(t) = U_a(k_a(t)), \quad \forall a, t \tag{4}$$

where $u_a(t)$ is the average speed on link *a* at time *t* and $U_a(\cdot)$ is a particular speed-density relationship for the link *a*.

Two kind of capacities are defined in the model, namely, the *flow capacity* and the *spatial capacity*. The flow capacity, $C_a = \max_{k_a} \{k_a U_a(k_a)\}$ on link *a* is implicitly considered from the speed-density relationship for the link. The spatial capacity of a link *a* is defined by the saturation density of the link as K_a, where the quantity $l_a n_a K_a$ denotes the maximum number of bursts that can be virtually piped in the link *a*. Therefore, we have

$$x_a(t) \le l_a n_a K_a, \quad \forall a, t \tag{5}$$

Let $z = A_a(t)$ be the zth burst entering the link *a* at time *t*. From FIFO principle, this burst will be leaving the link *a* by time $D_a^{-1}(z)$ and hence, the travel time on link *a* for a burst entering the link at time t becomes

$$\tau_a(t) = D_a^{-1}(A_a(t)) - t \tag{6}$$

where $\tau_a(t)$ is the travel time a burst spent on link *a* when it enters the link at time *t*. The travel time is assumed to consist of two components: the propagation time along the link $\tau_a^c(t)$ and the queueing time at the end of the link $\tau_a^q(t)$. The propagation time is determined by the following equation

$$\int_t^{t+\tau_a^c(t)} \frac{d\omega}{u_a(\omega)} = l_a \tag{7}$$

where l_a is the length of the link *a* and the average speed u_a is obtained from equation (4). When a link reaches its spatial capacity, bursts from the immediate upstream link cannot enter the link and these bursts will then queue up at the end of the immediate upstream link, which will be discharging onto the link when its density falls below the saturation density value at a later time. The queueing time on a link *a* (if any) is determined by

$$\tau_a^q(t) = \tau_a(t) - \tau_a^c(t), \quad \forall a, t \tag{8}$$

Let $L_{rsp}(t) = (a_1, a_2, \ldots, a_m) \in P_{rs}(t)$ be a sequence of *m* consecutive links on the path *p* between OD pair *r-s* for bursts provided at time *t* and T_{rspa}, $\forall a \in L_{rsp}(t)$ be the time these bursts leave the link *a* while on the way to destination node *s*. We have

$$T_{rspa_i} = T_{rspa_{i-1}} + \tau_a(T_{rspa_{i-1}}), \quad i = 1, 2, \ldots, m \tag{9}$$

with $T_{rspa_0} = t$. The actual travel time on the path becomes

$$\eta_{rsp}(t) = T_{rspa_m} - t \tag{10}$$

where T_{rspa_m} is the arrival time at the destination node *s*, when a burst embarked onto the network at time *t* travels on path *p* between OD pair *r-s*.

If for any travellers between any OD pair leaving their origin at any instant, the actual travel times that these travellers experienced on any used routes are equal and minimal and the actual travel times that these travellers would experience on any unused routes are greater than or equal to the minimum actual travel time on used routes, then the network is in a dynamic equilibrium.

Define the function $[w]_+ = \max\{0, w\}$. *The predictive dynamic equilibrium conditions* are satisfied if and only if

$$f_{rsp}(t)\left[\eta_{rsp}(t) - \eta_{rsq}(t)\right]_+ = 0, \quad \forall r, s, t, p, q \tag{11}$$

The predictive user equilibrium conditions are equivalent to the following minimisation problem

$$\min_{\mathbf{f}} Z(\mathbf{f}) = \sum_{\forall r,s,p,q} \int_0^T f_{rsp}(t)\left[\eta_{rsp}(t) - \eta_{rsq}(t)\right]_+ dt \tag{12}$$

subject to

$$d_{rs}(t) = \sum_{p \in P_{rs}(t)} f_{rsp}(t), \quad \forall r, s, t \tag{13}$$

$$f_{rsp}(t) \ge 0, \quad \forall r, s, t, p \in P_{rs}(t) \tag{14}$$

where $\mathbf{f} = \left\{ f_{rsp}(t), \ \forall r, s, t, p \right\}$ is the vector of control variables in the problem. This is a multistage optimization problem for which various solution techniques exist.

2.2 Deterministic Successive Averaging Method

One of the possible techniques that can be used to solve the dynamic equilibrium problem is the method successive averages (MSA). At every iteration, for each OD pair at each instant, the path with minimum predictive travel time between the OD pair for travellers embarked at the instant is first determined by a time dependent shortest path algorithm, based on the network loading conditions in the previous iteration. The revised flow pattern is then calculated by spreading at each instant the traffic demand evenly among all the paths that have been obtained from all the previous iterations and the newly generated minimum paths obtained in the current iteration.

Let $\mathbf{f}^{(n)} = \left\{ f_{rspt}^{(n)};\ \forall r, s, t, p \in P_{rst}^{(n)} \right\}$ be the flow vector at the *n*th iteration, where $P_{rst}^{(n)}$ is the set of all paths obtained from all iterations so far and $f_{rspt}^{(n)}$, the corresponding path flow values. These path flows are then loaded onto the network and the auxiliary set of time-dependent shortest paths for all OD pair for all time intervals. We denote by $Y = \left\{ y_{rst},\ \forall r, s, t \right\}$ is the set of shortest paths for a burst travelling to the destination node *s* from the origin node *r* embarked at time *t*.

Proposition 1. If the auxiliary path is newly generated, i.e., $P_{rst}^{(n)} \cap y_{rst} = \varnothing$, the updated path flow vector is determined by

$$f_{rspt}^{(n+1)} = \begin{cases} \dfrac{n}{n+1} f_{rspt}^{(n)} \text{ if } p \in P_{rspt}^{(n)} \\ \dfrac{1}{n+1} d_{rst} \text{ if } p = y_{rst} \end{cases} \tag{15}$$

for all *r*, *s*, *t* and the set of used paths at the $(n+1)$th iteration is updated as $P_{rst}^{(n+1)} = P_{rst}^{(n)} \cup y_{rst}$. If the auxiliary path is an old path, i.e., $y_{rst} \in P_{rst}^{(n)}$, the updated path flow vector becomes

$$f_{rspt}^{(n+1)} = \begin{cases} \dfrac{n}{n+1} f_{rspt}^{(n)} & \text{if } p \neq y_{rst} \\ \dfrac{1}{n+1} f_{rspt}^{(n)} + \dfrac{1}{n+1} d_{rst} & \text{if } p = y_{rst} \end{cases} \tag{16}$$

and the set of used paths remains unchanged, i.e., $P_{rst}^{(n-1)} = P_{rst}^{(n)}$.

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The above mentioned procedure is repeated until certain convergence criteria are satisfied. Due to the discrete nature of the model, the duality gap can be employed as the convergence criteria:

$$G^{(n+1)} = \frac{\sum_{r,s,t,p \in P_{rst}^{(n)}} f_{rspt}^{(n)} \left| \eta_{rspt}^{(n+1)} - \eta_{rsy_{rst}t}^{(n+1)} \right|}{\sum_{r,s,t} d_{rst} \eta_{rsy_{rst}t}^{(n+1)}} \tag{17}$$

where $\eta_{rspt}^{(n+1)}$ is the travel time on a path *p* after the path flow vector at the *n*th iteration has been loaded onto the network and $\eta_{rsy_{rst}t}^{(n+1)}$ is the minimum travel time on path y_{rst} determined from the shortest path solution. The MSA algorithm is stopped when either $G^{(n+1)} < \varepsilon$ or $n > n_{\max}$.

3 Fuzzy Successive Averaging Method

We consider now the general framework where the flow assignment problem is regarded as a multistage optimization problem. We introduce the uncertainties on the demand patterns and the delay times experienced by the burst while travelling along the links of the communication network ([4]).

3.1 Fuzzy Multistage Optimization

Consider the standard notation in the control literature for a finite automaton $V = \{U, X, \delta\}$, where U, X are finite sets called the decision or control and the state spaces and $\delta\colon X \times U \to X$ is the state transtion map ([3], [5], [6]). The state equation is thus given by

$$x(t+1) = \delta(x(t),\ u(t)),\ t = 0,\ 1, \ldots, T-1 \tag{18}$$

with *T* the final time and $x(0) \in X$ is the initial state. In the context of this paper, the state is considered to be the number of bursts crossing the communication links at time *t* and the control are the decision variables in the flow assigment problem. A fuzzy control constraint μ is a fuzzy subset of U defined by the membership function $\mu\colon U \to [0,\ 1]$ and a fuzzy state target μ' is a fuzzy subset of *X* defined by the membership function $\mu'\colon X \to [0,\ 1]$. We assume the existence of the fuzzy control constraints $\{\mu_0,\ \mu_1, \ldots, \mu_{T-1}\}$, where μ_i is relevant to the control input U_i at time *i*, $0 \le i \le T-1$. Suppose also that a fuzzy goal μ'_T is imposed on the final state x_T. A fixed input sequence $\{u_0,\ u_1, \ldots, u_{T-1}\}$ corresponds to the fuzzy decision

$$U^T = U \times U \times \cdots \times U \tag{19}$$

given by the fuzzy interaction equation

$$\mu(u_0, u_1, \dots, u_{T-1}) = \mu_0(u_0) \wedge \mu_1(u_1) \wedge \cdots \wedge \mu_{T-1}(u_{T-1}) \wedge \mu'_T(x_T) \tag{20}$$

where we have written $u_i = u(i)$ and $x_T = x(T)$ is calculated for a given $x(0) \in X$ from the state equation. A *fyzzy decision* for the above problem is given by $(\bar{u}_0,\ \bar{u}_1, \dots, \bar{u}_{T-1}) \in U^T$ such that

$$\mu(\bar{u}_0,\ \bar{u}_1, \dots, \bar{u}_{T-1}) = \max_{u_0,\ u_1, \dots, u_{T-1} \in U^T} \mu(u_0,\ u_1, \dots, u_{T-1}) \tag{21}$$

Leting

$$S_k(x) = \max_{u_k, \dots, u_{T-1} \in U} \mu(u_k,\ u_{k+1}, \dots, u_{T-1}) \tag{22}$$

when the system starts in state x at time k and optimal control sequence is used, we have

$$S_k(x) = \max_{u_k \in U}\left\{\mu_k(u_k) \wedge \left\{\left[\max_{u_{k+1}, \dots, u_{T-1}} \left\{\mu_{k+1}(u_{k+1}) \wedge \right\}\right\} \wedge \cdots \wedge \mu_{T-1}(u_{T-1}) \wedge \mu'_T(x_T) \middle| x_{k+1} = \delta(x, u_k)\right\} \tag{23}$$

or equivalently

$$S_k(x) = \max_{u_k \in U}\left\{\mu_k(u_k) \wedge S_{k+1}(\delta(x, u_k))\right\} \tag{24}$$

$$S_T(x) = \mu'_T(x) \tag{25}$$

for $k = 0,\ 1, \dots, T-1$. The solution of this problem is of dynamic programming type and can be found by backward iteration in order to obtain the optimal multi-stage decision $(\bar{u}_0,\ \bar{u}_1, \dots, \bar{u}_{T-1})$.

3.2 Fuzzy Mapping Extension

Definition 1. A fuzzy mapping $\mathbf{F}: X \to Y$ is a fuzzy set on $X \times Y$ with membership function $\mu_{\mathbf{F}}(x, y)$.

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Definition 2. A fuzzy function $\mathbf{F}(x)$ is a fuzzy set on Y with membership function $\mu_{\mathbf{F}(x)}(y) = \mu_{\mathbf{F}}(x, y)$.

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Definition 3. Let $\mathbf{A}$ be a fuzzy subset on X defined by membership function $\mu_{\mathbf{A}}(x)$. The fuzzy set $\mathbf{F}(\mathbf{A})$ on Y is a fuzzy mapping of a fuzzy set defined as

$$\mu_{\mathbf{F}(\mathbf{A})}(y) = \vee_{x \in X}\left(\mu_{\mathbf{A}}(x) \wedge \mu_{\mathbf{F}}(x, y)\right), \text{ for all } y \in Y \tag{26}$$

where $\wedge$ stands for min and $\vee$ stands for max.

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Consider the fuzzy automaton $\tilde{V} = \{U, X, \delta, F(U), F(X)\}$, where U, X are finite sets called the control and the state spaces respectively and $\delta: X \times U \to X$ and $F(U)$, $F(X)$ are the sets of fuzzy controls and states, respectively. The state equation if given by

$$\mu_{x(t+1)}(x(t+1)) = \mu_{\delta(v)} = = \vee_{v \in V}\left(\mu_v(v) \wedge \mu_\delta(v, x(t+1))\right) \tag{27}$$

for all $x(t+1) \in X$ and $t = 0,\ 1, \dots, T-1$, with $V = X \times U$, $v = (x(t), u(t))$, $v \in V$ and $\mathbf{V}$ is a fuzzy set on V representing the fuzzy state $\mathbf{x}(t)$ with fuzzy control $\mathbf{u}(t)$ having the membership function

$$\mu_{\mathbf{V}}(x, u) = \mu_{\mathbf{x}(t)}(x) \wedge \mu_{\mathbf{u}(t)}(u) \tag{28}$$

Suppose further the existence of fuzzy constraints $\{\mu_0,\ \mu_1, \dots, \mu_{T-1}\}$, $\mu_i \in F(U)$, where μ_i is imposed on the input u_i, $0 \le i \le T-1$ and also that a fuzzy goal μ'_T is imposed on tne final state $\mathbf{x}(T)$.

3.3 Fuzzy Averaging as Aggregation Operation

Aggregation operations generally defined on fuzzy sets are operations by which several fuzzy sets are combined to produce a single set and are defined by a function

$$h: [0,\ 1]^n \to [0,\ 1] \tag{29}$$

for some $n \ge 2$. When applied to n fuzzy sets $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_n$ defined on X, h produces an aggregate fuzzy set $\mathbf{A}$ by operating on the membership grades of each $x \in X$ in the aggregated sets ([5]). Thus

$$\mu_{\mathbf{A}}(x) = h(\mu_{\mathbf{A}_1}(x),\ \mu_{\mathbf{A}_2}(x), \dots, \mu_{\mathbf{A}_n}(x)) \tag{30}$$

Fuzzy unions and intersections can be viewed as special aggregation operations that are symmetric, usually continuous and required to satisfy some additional boundary conditions. They produce aggregates of the n-tuple $a_1, a_2, \dots, a_n$ having values between $\min\{a_1, a_2, \dots, a_n\}$ and $\max\{a_1, a_2, \dots, a_n\}$. The fuzzy averaging operations are aggregation operations for which

$$\min\{a_1, a_2, \dots, a_n\} \le h(a_1, a_2, \dots, a_n) \le \le \max\{a_1, a_2, \dots, a_n\} \tag{31}$$

The standard max and min operations represent boundaries between the averaging operations and the fuzzy unions and intersections, respectively. One special class of averaging operation is represented by the *generalized means* defined by

$$h_\alpha(a_1,a_2,\dots,a_n)=\left(\frac{a_1^\alpha+a_2^\alpha+\dots+a_n^\alpha}{n}\right)^{1/\alpha} \qquad (32)$$

where $\alpha\in\mathbf{R}$, $\alpha\neq 0$, is a parameter by which different means can be obtained such that

$$\begin{aligned} h_{-\infty}(a_1,a_2,\dots,a_n) &= \min\{a_1,a_2,\dots,a_n\}\le \\ &\le h_\alpha(a_1,a_2,\dots,a_n)\le \\ &\le \max\{a_1,a_2,\dots,a_n\}= \\ &= h_\infty(a_1,a_2,\dots,a_n) \end{aligned} \qquad (33)$$

Proposition 2. For fixed arguments, the function h_α is monotonic increasing with α. For $\alpha\to 0$, the function h_α becomes the *geometric mean*

$$h_0(a_1,a_2,\dots,a_n)=(a_1a_2\cdots a_n)^{1/n} \qquad (34)$$

for $\alpha\to -1$, it becomes the *harmonic mean*

$$h_{-1}(a_1,a_2,\dots,a_n)=\frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+\dots+\frac{1}{a_n}} \qquad (35)$$

and for $\alpha\to 1$, it becomes the *arithmetic mean*

$$h_1(a_1,a_2,\dots,a_n)=\frac{a_1+a_2+\dots+a_n}{n} \qquad (36)$$

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When it is desirable to accomodate variations in the importance of individual aggregated sets, the function h_α can be generalized into *weighted generalized means* as defined by the formula

$$h_\alpha(a_1,a_2,\dots,a_n;\ w_1,w_2,\dots,w_n)=\left(\sum_{i=1}^n w_i a_i^\alpha\right)^{1/\alpha} \qquad (37)$$

where $w_i\ge 0,\ i\in\mathbf{N}_n$, are weights that express the relative importance of the aggregated sets satisfying the constraint

$$\sum_{i=1}^n w_i = 1 \qquad (38)$$

For fixed arguments and weights, the weighted generalized mean is monotonic increasing with α.

4 Bayesian Inference of Aggregation Weights

Since there is a flexibility in the choice of the weights in the weighted generalized mean, we would like to relate these weights to the relative ranking of the successive stages in the fuzzy multistage optimization problem. This ranking is intended to keep track on the splitting rates of the traffic and in order to do so, we introduce the assumption of the statistical dependencies of the splitting rates on the compund demand patterns (denoted in the subsequel by x_t) and on the delays within the links in the communication network. The model of these dependencies that we consider for a particulat link is the following

$$r_t = x_t w_t + \varepsilon_t,\ \ w_t = g_t w_{t-1} + \eta_t \qquad (39)$$

for $t=1,\dots,T$, where ε_t are i.i.d. random variables $N(0,\sigma^2)$, the η_t are i.i.d. $N(0,\lambda\sigma^2)$, with $\lambda,\ \sigma^2>0$ parameters that are to be estimated and ε_t and η_s uncorrelated noises for all time instants *t* and *s* ([7], [8]).. We assume that $\{g_t\}$ is a fixed, nonstochastic sequence of scalars . We assume that the parameter w_0 is unknown with its uncertainty expressed by a normal distribution $N(\hat{w}_{0|0},\ \sigma^2 R_{0|0})$, where the hyperparameters $\hat{w}_{0|0}$ and $R_{0|0}$ are known. Essentially, the Bayesian inference on the fuzzy aggregation weights is the problem of smoothing the regression parameter $w=(w_1,\dots,w_T)$ given an available data record $r^T=(r_1,\dots,r_T)$ ([2], [9], [10]) . The mean and variance of w_t given the information up to time *s* are denoted by $\hat{w}_{t|s}$ and $\hat{\sigma}^2_{t|s}$, respectively, i.e.,

$$\hat{w}_{t|s}=E\left[w_t\middle|r^s\right],\ \hat{\sigma}^2_{t|s}=E\left[(w_t-\hat{w}_{t|s})^2\middle|r^s\right] \qquad (40)$$

for $s=1,\dots,T$. The mean and variance of r_t given the information up to time *s* is given by $\hat{r}_{t|s}$ and $\hat{f}_{t|s}$, respectively, i.e.,

$$\hat{r}_{t|s}=E\left[r_t\middle|r^s\right],\ \hat{f}_{t|s}=E\left[(r_t-\hat{r}_{t|s})^2\middle|r^s\right] \qquad (41)$$

for $s=1,\dots,T$. Under the assumptions of the model, it is possible to derive the conditional distributions of w_t and r_t given the information up to *t*-1.

Proposition 3. If we assume that the posterior distribution of w_{t-1} given r^{t-1}, then we have

$$w_{t-1|r^{t-1},\lambda}\approx N(\hat{w}_{t-1,t-1,\lambda},\ \sigma^2 R_{t-1|t-1,\lambda}) \qquad (42)$$

$$w_{t|r^{t-1},\lambda} \approx N(\hat{r}_{t|t-1,\lambda},\ \sigma^2 R_{t|t-1,\lambda}) \quad (43)$$

$$r_{t|r^{t-1},\theta} \approx N(\hat{r}_{t|t-1,\lambda},\ \sigma^2 \hat{f}_{t|t-1,\lambda}) \quad (44)$$

where

$$\hat{w}_{t|t-1,\lambda} = g_t \hat{w}_{t-1|t-1,\lambda},\ \ R_{t|t-1,\lambda} = g_t^2 R_{t-1|t-1,\lambda} + \lambda \quad (45)$$

$$\hat{r}_{t|t-1,\lambda} = x_t \hat{w}_{t|t-1,\lambda},\ \ \hat{f}_{t|t-1,\lambda} = x_t^2 R_{t|t-1,\lambda} + 1 \quad (46)$$

■

Proposition 4. The Bayesian theorem allows to compute the conditional distribution of w_t given r^t and λ according to the following equations

$$w_{t|r^t,\lambda} \approx N(\hat{w}_{t|t,\lambda},\ \hat{\sigma}^2_{t|t,\lambda} = \sigma^2 R_{t|t,\lambda}) \quad (47)$$

$$\hat{w}_{t|t,\lambda} = \hat{w}_{t|t-1,\lambda} + R_{t|t-1,\lambda} x_t \hat{f}^{-1}_{t|t-1,\lambda}(r_t - x_t \hat{w}_{t|t-1,\lambda}) \quad (48)$$

$$R_{t|t,\lambda} = R_{t|t-1,\lambda} - R^2_{t|t-1,\lambda} x_t^2 \hat{f}^{-1}_{t|t-1,\lambda} \quad (49)$$

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These are Kalman filtering type of updating equations for the mean and variance of. The set $\theta = (\sigma^2, \lambda)$ of parameters that were left unspecified are computed using the maximization of the likelihood function expressed in terms of the conditional distribution of the flow splitting rates $r_{t|r^{t-1},\theta}$, $t = 1,\ldots,n$. Using the multiplicative theorem of the total probability we have

$$\begin{aligned} & l(\theta|r^n) \propto f(r_1,\ldots,r_n|\theta) \propto \\ & \propto f(r_1|\theta f(r_2|r^1,\theta)\cdots f(r_n|r^{n-1},\theta) \\ & \propto \left(\frac{1}{\sigma^2}\right)^{n/2} \prod_{t=1}^{n} (\hat{f}_{t|t-1,\lambda})^{-1/2} \times \\ & \times \exp\left[-\frac{1}{2\sigma^2}\sum_{t=1}^{n}\frac{(r_t - \hat{r}_{t|t-1,\lambda})^2}{\hat{f}_{t|t-1,\lambda}}\right] \end{aligned} \quad (50)$$

where $\hat{f}_{t|t-1,\lambda}$ and $\hat{r}_{t|t-1,\lambda}$ are produced by Kalman filter type of updating equations.

5 Conclusions

The input-output system whose parameters are modeled as random processes is the the switching board acting in a given node of a multihop communication network. The switching operation in a communication node is intrinsically a stochastic process and this has been the most important reason which motivated us to consider it as a regression model. In practice, the delay-throughput balance is the main criterion in various switching design architectures. Although there are some simulation techniques dealing with the estimation of the parameters of the delay pattern based on the long term average flows, these techniques do not work when they are applied to purely stochastic processes as it is the case with switching nodes working under bursty and congestion sensitive regimes.

The parameter estimation based on the Bayesian inference about the input-output behaviour of the switching operation considered here as the classical dynamic flow assignment is extendind the ability of the fuzzy aggregation to behave as successive time averaging. The dynamical performances of the mixture process generated by multiple bursty traffic patterns are to be analyzed in both as modelling issue and as a simulation task.

6 References

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