Minimum Cross Entropy Control in Stochastic Switching Systems

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Abstract — **In this paper, we consider a mathematical framework for switching systems of traffic streams based on the Minimum Cross Entropy (MCE) approach. Linear models of traffic buffering processes are providing the control means through traffic splitting/merging and input-output connectivity patterns. The probabilistic description of general queueing systems deals with the stochastic flows in the switching nodes.**

I. Introduction

Switching systems are central components in communications networks and serve two central purposes: they allow a reduction in overall network costs by reducing the number of transmission links required to enable a given population of users to communicate and they enable heterogeneity among terminals and transmission links, by providing a variety of interface types [4]. For applications where the peak data rate is much larger than the average data rate, the statistical multiplexing is used to share the link bandwidth more effectively among the competing data streams and it motivates an minimum cross entropy based solution of control strategies.

II. Probabilistic Switching Processes

The Little's Theorem implies that if $x(t)$ is the average intensity of the carried traffic, $\varrho^r(t)$ is the intensity of the rejected traffic and the incoming and outgoing traffic intensities are denoted by $\varrho^{in}(t) = \varrho(t)$ and $\varrho^{out}(t) = \varrho^c(t) + \varrho^r(t)$, respectively, then we have the following differential equation for the average traffic rates

$$\frac{dx(t)}{dt} = -\varrho^c(t) - \varrho^r(t) + \varrho^{in}(t) \quad (1)$$

III. Control Problem of Buffered Switching

The flow intensities can be approximated as nonlinear functions of the average number of the entities in the system [1] as $\varrho^{out}(t) = G[x(t)]$, $\varrho^r(t) = H[x(t)]G[x(t)]$ and $\varrho^c(t) = \{1 - H[x(t)]\}G[x(t)]$. The function $H(x)$ represents the *fraction* of outgoing traffic $\varrho^{out}(t) = G[x(t)]$ that is rejected, $0 \leq H[x(t)] \leq 1$. The control model is described as the minimization of the following multistep quadratic functional

$$J = \sum_{n=1}^{N} [H(\bar{x}_n)G(\bar{x}_n) - \bar{\varrho}_n^r]^2. \quad (2)$$

This control model will enable a given population of users to communicate with each other under heterogeneous traffic descriptors by keeping track on the deviations of average traffic rates from the long term average value of the traffic rate $\bar{x}_n$.

IV. Linear Prediction of Traffic Switching

The traffic patterns are shaped as through a dynamic connectivity pattern $x_{ij}(t)$ between input and output ports depending on the input traffic intensities $a_j(t)$. A linear prediction problem [2] can be formulated in order to find the functions $x_{ij}(t)$ by consideting a linear approximation equivalent $J_1^* = \max_{x_{ij}(t)\in\{0,1\}} \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}(t)$ subject to $\sum_{j=1}^{n} a_{ij}(t)x_{ij}(t) \leq b_i$, for $i = \overline{1,m}$, $\sum_{i=1}^{m} x_{ij}(t) \leq 1$, for $j = \overline{1,n}$ and $0 \leq x_{ij}(t) \leq 1$, for $i = \overline{1,m}$, $j = \overline{1,n}$. The streams $a_{ij}(t)$ are generating by the following splitting procedure [3]

$$a_{ij}(t) = a_j(t)f_{ij}(t), \quad t = \overline{0,T} \quad (3)$$

where $a_j(t)$ is the traffic rate in the link j at the moment t. The splitting funtion $f_{ij}(t)$ is interpreted as the dynamic routing probability, that is, $f_{ij}(t) \in [0,1]$, for $t = \overline{0,T}$, such that $\sum_{j=1}^{n} f_{ij}(t)a_j(t) \leq b_i$, $\sum_{i=1}^{m} f_{ij}(t) \leq 1$, for $i = \overline{1,m}$, $j = \overline{1,n}$, $t = \overline{0,T}$.

V. Minimum Cross Entropy Solution Based on Traffic Dissimilarities

The connectivity patterns $x_{ij}(t)$ are to be adjusted in order provide a best effort tracking of the statistical features of the incoming traffic streams and to prescribe a desired spectral profile of the output traffic flows [3]. The cross-entropy

$$H_c(p,q) := \int p(x)\log[p(x)/q(x)]dx \quad (4)$$

is a measure of the "information dissimilarity" [5] between the prior p.d.f. $q(x)$ (input traffic) and the posterior p.d.f. $p(x)$ (output traffic). If $S_p(f)$ and $S_q(f)$ are the output and input spectra of x, the spectra can be expressed in terms of the expectations of $|X_k|^2$ when the input/output p.d.f.'s of X are $P(X)$ and $Q(X)$. The linear prediction problem as cross entropy minimization has a solution of the form

$$H_c = -\frac{1}{2}\sum_{k=0}^{N-1}\log\frac{S_p(k)}{S_q(k)} + \frac{1}{2}\sum_{k=0}^{N-1}\left(\frac{S_p(k)}{S_q(k)} - 1\right) \quad (5)$$

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