Switching Envelope Analysis for Statistical Aggregation of Traffic Flow Batches

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ABSTRACT

In this paper we develop a dynamic game theoretical framework for the large scale dynamic optimization models that are currently occuring in the planning of traffic streams in a switching network under the generic description of Trunk Group Management. The dynamic games approach of the switching process allows the minimization of transmission costs and shaping the time delays statistics under uncertainty about the traffic patterns. Further extensions of this approach include statistical information extraction/processing to aggregate the traffic streams in patterns that are determined on a competitive basis from batches of individual traffic streams.

1. Introduction and Motivation

Before the advent of Gigabit Ethernet, the spped of 100 Mbps was the maximum rate for Ethernet. For speeds exceeding 100 Mbps, multiple proprietary Fast Ethernet links are needed to be connected in parallel (e.g., Cisco Fast EtherChannel trunking) ([1]). The ATM technology had an edge with its 622 Mbps version. The Gigabit Ethernet and the proposed 802.3ad Link Aggregation/Truncking standard, Ethernet backbones of several gigabits per second can be built. The Ethernet now scales from 10 to 100 to 1000 Mbps. ATM speeds range from 25 to 622 Mbps. The Ethernet Link Aggregation proposal works for both switches and servers, whereas ATM does not allow server Link Aggregation. The 53 bythe ATM cell structure is less efficient than Ethernet frame structure. For 1 KB frames (Ethernet frames can range from 64 bytes to 1522 bytes), Ethernet protocol efficiency is 0.98, compared to ATM efficiency of 0.9 ([8]). The ATM attractiveness lies in its radically different approach of integrated LAN/WAN and voice/data traffic. The ATM Forum has created LAN Emulation (LANE) and a set of other technologies that enable a smooth migration from legacy LANs to a true ATM environment. If ATM is used only as a backbone technology (i.e., no ATM attached clients or servers), LANE is not required. The ATM has been designed by the service provider industry offers state-of-the-art quality of service (QoS) in the sense that the connections are specified in terms of their bit rates or bandwidth (CBR, VBR, UBR, ABR) ([8]). The Shared Ethernet offers a zero level QoS. The QoS is necessary if a network is overloaded and sporadic delays are a normal part of the network operation. For wide area networks (WANs), the links always run close to full capacity and the QoS becomes important. For applications in which the peak data rate is much larger than the average data rate, statistical multiplexing is used to share the link bandwidth most efficiently among the competing data streams. The amount of memory needed for efficient statistical multiplexing is highly dependent on the ratio of the link rate to the peak transmission rate of individual streams ([4]). The objective of this paper is to encapsulate in a closed form a dynamic game theory mathematical model for the issue of handling the traffic uncertainties which have to be handled by the communication network in order to preserve and guarantee the desired QoS level. The switching envelope method provides a statistical setting of the reference intervals for carrying out the complexity of uncertainties in the traffic streams when they are aggregated within the communication links.

2. Dynamic Network Planning Model

2.1. Notation and Assumptions

Let us consider $N : \{1, \ldots, |N|\}$, the set of nodes throughout the planning horizon. It is assumed that $|N|$ is a given parameter and the locations of the nodes are given in advance (the switching and crossconnect locations are known in advance and are predimensioned). The minimum total costs can be achieved by minimizing the total link associated costs. Let L : be the set of possible bidirectional links, a subset of $\bar{L} = \{(i,j)|\ i < j,\ i,j \in N\}$ which is the set of all bidirectional links, $(i,j) \equiv (j,i)$, because the links are bidirectional. Consequently,

$$\max\{|L|\} = \frac{|N|(|N|-1)}{2} \Rightarrow L = \bar{L} \tag{1}$$

Let us denote by $K : \{1, 2, \ldots, |K|\}$, the set of communicating origin/destination (OD) pairs of nodes or equivalently, commodities. Each commodity has a single origin node and a single destination node. Finally, let $T : \{1, 2, \ldots, |T|\}$ be the set of time periods in the planning horizon. The period t refers to the unit period from time point t to $(t+1)$. We assume that links and switching regimes can be installed during any point in time within a time period. The costs incurred during the period are assumed to take place at the beginning of the time period in which the changes take place. Let γ_k^t be the estimated traffic demand for commodity (origin-destination pair) k at period t. We assume that $F_{ij}(t)$ is the fixed cost of putting a switching regime on link (i,j) at period t and that $F_{ij}(t)$ values are nonincreasing over time, taking into account the time value of money. If $g_{ij}(t)$ is the fixed cost of installing a cable on link (i,j) at period t, this cost is incurred each time the link capacity is expanded. The cost of augmenting a unit capacity for link (i,j) at period t $c_{ij}(t)$ and maintaining it from period t through period $|T|$ $c_{ij}(t)$ has values that are decreasing over time, that is $c_{ij}(t) > c_{ij}(t+1), t \in T$. All $F_{ij}(t), g_{ij}(t), c_{ij}(t)$ are the present values of the associated dicounted costs. The decision variables used in formulating the model consist of the topological variable $y_{ij}(t)$ defined as

$$y_{ij}(t) = \begin{cases} 1, & \text{if a switching regime is installed on link } (i,j) \text{ at period } t \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

By definition, $y_{ij}(t) \leq y_{ij}(t+1)$. Let us consider $z_{ij}(t)$ as the aggregated capacity augmentation variable,

$$z_{ij}(t) = \begin{cases} 1, & \text{if the capacity of link } (i,j) \text{ is augmented during period } t \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

and $q_{ij}(t)$, a continuous variable which is the capacity of link (i,j) at period t. Link capacities are nondecreasing over time. Finally, let $x_{ijk}(t)$ be the directed flow of commodity k flowing from node i to j on link (i,j) or (j,i) at period t.

2.2. Dynamic Switching Model

There are three types of cost terms, that is, the fixed cost of the switching regimes, the fixed cost of links and the cost of flow shaping. Let TC be the total cost defined as

$$TC = \sum_{t\in T} \sum_{(i,j)\in L} (F_{ij}(t)(y_{ij}(t) - y_{ij}(t-1)) + g_{ij}(t)z_{ij}(t) + c_{ij}(t)(q_{ij}(t) - q_{ij}(t-1))) \tag{4}$$

or

$$TC = \sum_{t\in T} \sum_{(i,j)\in L} (f_{ij}y_{ij}(t) + g_{ij}(t)z_{ij}(t) + h_{ij}(t)q_{ij}(t)) \tag{5}$$

where we introduce the cost increments

$$f_{ij}(t) = F_{ij}(t) - F_{ij}(t+1) \geq 0,\ t \in T,\ (i,j) \in L \tag{6}$$

$$h_{ij}(t) = c_{ij}(t) - c_{ij}(t+1) \geq 0,\ t \in T,\ (i,j) \in L \tag{7}$$

The dynamic optimization problem can be stated as follows

Problem (P)

$$Z_T = \min\{\sum_{t\in T} \sum_{(i,j)\in L} (f_{ij}(t)y_{ij}(t) + g_{ij}(t)z_{ij}(t) + h_{ij}(t)q_{ij}(t))\} \tag{8}$$

subject to

$$\sum_{j\in N} x_{ijk}(t) - \sum_{j\in N} x_{jik}(t) = \begin{cases} \gamma_k(t), & \text{if } i = O_k \\ -\gamma_k(t), & \text{if } i = D_k,\ i \in N,\ k \in K,\ t \in T \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

$$\sum_{k\in K} (x_{ijk}(t) + x_{jik}(t)) \leq q_{ij}(t),\ (i,j) \in L,\ t \in T \tag{10}$$

$$q_{ij}(t) - q_{ij}(t-1) \geq 0,\ (i,j) \in L,\ t \in T \tag{11}$$

$$q_{ij}(t) \leq My_{ij}(t),\ (i,j) \in L,\ t \in T \tag{12}$$

$$q_{ij}(t) - q_{ij}(t-1) \leq Mz_{ij}(t),\ (i,j) \in L,\ t \in T \tag{13}$$

$$y_{ij}(t) - y_{ij}(t-1) \geq 0,\ (i,j) \in L,\ t \in T \tag{14}$$

$$y_{ij}(t),\ z_{ij}(t) = 0,1,\ (i,j) \in L,\ t \in T \tag{15}$$

$$x_{ijk}(t),\ x_{jik}(t) \geq 0,\ (i,j) \in L,\ k \in K,\ t \in T \tag{16}$$

where, O_k and D_k represent the origin and the destination of commodity k, and

$$M = \max_{t\in T}\{\sum_{k\in K} \gamma_k(t)\} \tag{17}$$

By definition, we consider that

$$q_{ij}^0 = y_{ij}^0 = 0,\ \forall(i,j) \in L \tag{18}$$

In the above formulation, the routing variables $\{x_{ijk}(t), x_{jik}(t)\}$ support the flow of each commodity and satisfy the flow conservation constraints (9) and the link capacity constraints (10). The constraints (11) and (14) and the expression (5), ensure that there is no capacity contraction and/or trunk switching regime disconnection.

2.3. Flow Balance Equations for Buffered Traffic

We introduce new flow balance equations where we assume that at each node $i \in N$ there a buffer of size B_i. Then

$$\sum_{j\in N} x_{jik}(t) - \sum_{j\in N} x_{ijk}(t) = \begin{cases} b_{ik}(t) - \gamma_k(t), & i = O_k,\ k \in K,\ t \in T \\ b_{ik}(t) + \gamma_k(t), & i = D_k,\ k \in K,\ t \in T \\ b_{ik}(t), & i \in N,\ k \in K,\ t \in T \end{cases} \tag{19}$$

where $\mathbf{b}_i(t)$ is the vector of traffic rates for the buffer of node i at time step t, that is,

$$\mathbf{b}_i(t) = [b_{i1}(t),\ b_{i2}(t),\ \ldots,\ b_{ik}(t)]^T \in R^{|K|} \tag{20}$$

with

$$0 \leq \delta \sum_{k\in K} b_{ik}(t) \leq B_i,\ \forall i \in N,\ \forall t \in T \tag{21}$$

for a time discretization step δ such that $t \to (t\delta, (t+1)\delta)$. Let us assume that we have a solution for the initial problem (P) for time t, $\{y_{i,j}(t), z_{ij}(t), q_{ij}(t), x_{ijk}(t), x_{jik}(t),\ (i,j) \in L,\ k \in K\}$. We would like ' to find M' such that (12) and (13) are meet jointly

$$\sum_{j\in N} x'_{ijk}(t) - \sum_{j\in N} x'_{jik}(t) = \sum_{j\in N} x_{ijk}(t) - \sum_{j\in N} x_{jik}(t) - b_{ik}(t) \tag{22}$$

It follows from (10) and (12) that

$$\sum_{k\in K} (x_{ijk}(t) + x_{jik}(t)) \le q_{ij}(t) \le M y_{ij}(t) \tag{23}$$

$$\sum_{k\in K} (x'_{ijk}(t) + x'_{jik}(t)) \le (M + \sum_{k\in K} b_{ik}(t)) y_{ij}(t) = (M + B_i) y_{ij}(t) \le M' y_{ij}(t) \tag{24}$$

which we obtain

$$M' = M + \max_{i\in N} B_i \tag{25}$$

3. Linear Input-Output Traffic Aggregation

In this section we present the main concepts related to the switching of the traffic flows in the ATM networks (considered as data micro/macro streams) when it is regarded as input-output aggregation process ([5]). Since the efective capacity of a switched link is scalled down according to the average value of the topological variable $y_{ij}(t)$ within a planning cycle, we will consider this variable as the only control variable for shaping the output traffic patterns ([6]).

3.1. Preliminaries

Suppose that an $n \times n$ input-output matrix $\mathbf{A}$ is to be aggregated from n data micro streams denoted by $N = \{1, ..., n\}$ to m macro streams, $M = \{1, ..., m\}$, with $m < n$. Let an $m \times n$ matrix $\mathbf{S}$ indicate which micro streams are to be combined, that is, for all $i \in M$ and $j \in N$, $s_{i,j} = 1$ if micro stream j is to be included in macro stream i and $s_{i,j} = 0$, otherwise. Thus, $\mathbf{S}$ is a 0-1 matrix with exactly one 1 in every column and at least one 1 in every row ($\mathbf{S}$ is a column stochastic matrix). Let an $n \times m$ matrix $\mathbf{T}$ indicate the proportional weights of each micro stream in its macro aggregate. For all $i \in M$ and $j \in N$, $t_{ji} \in [0, 1]$ if the micro stream j is included in macro stream i and $t_{ji} = 0$, otherwise. The sum of the weights of the micro streams assigned to a givean macro stream is assumed to be 1. Consequently, $\mathbf{T}$ is also column stochastic. The input-output aggregator is computed as the matrix $\mathbf{SAT}$. There are also other methods of matrix aggregation such as

1. Aggregation where $\mathbf{S}$ may contain any positive weights.
2. Aggregation chosen to optimize a particular objective function.

Other possibilities include the application of the above procedure to the $(\mathbf{I} - \mathbf{A})^{-1}$ matrix to obtain $\mathbf{S}(\mathbf{I}-\mathbf{A})^{-1}\mathbf{T}$ and then compute the aggregation of $\mathbf{A}$ as $\mathbf{I}-\mathbf{S}(\mathbf{I}-\mathbf{A})^{-1}\mathbf{T}$. We may also use the micro streams as data to estimate an aggregated matrix, using a variant of the econometric estimation techniques for estimating input-output models. Also, we may first aggregate the columns to produce a rectangular model with m macro streams and then convert the rectangular model to a square $m \times m$ model.

3.2. Functional Form of General Aggregators for Traffic Switching

Let us consider a general aggregator f mapping $n \times n$ input-output matrices into $m \times m$ input-output matrices, $m < n$. Denote the set of real $n \times m$ matrices by $M_{n,m}$. When $n = m$, $M_{n,m}$ is abbreviated as M_n. We consider an open input-output model with n streams given by

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{y} \tag{26}$$

where $\mathbf{x} \in R^n$ is an output vector, $\mathbf{y} \in R^n$ is a final demand vector and $\mathbf{A} \in M_n$ is an input-output matrix. For an input-output matrix $\mathbf{A} = (a_{ij})$, then the entry a_{ij} can be interpreted as the amount of commodity i necessary in the production of a unit of commodity j, given the technology represented by $\mathbf{A}$. Thus, $1 - \sum_i a_{ij}$ is the value added per unit of production of commodity j, which is assumed positive. In this event, the matrix $\mathbf{A}$ has nonnegative entries and column sums all less than 1. Such a matrix is usually called (strictly) column substochastic since a column stochastic matrix is a nonnegative one with column sums of 1.

Definition 1. By an *input-output* matrix we simply mean a square column substochastic matrix. The input-output equation

$$\mathbf{y} = (\mathbf{I} - \mathbf{A})\mathbf{x} \tag{27}$$

can be used to transform an output vector to a final demand. Conversely, since $\mathbf{A}$ is column substochastic, $(\mathbf{I} - \mathbf{A})$ must be nonsingular, so that $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}$ can be used to transform a final demand vector to an output vector. Since $\mathbf{A}$ is substochastic, the inverse matrix $(\mathbf{I} - \mathbf{A})^{-1}$ is nonnegative and $\mathbf{A}$ is irredicible, then $(\mathbf{I} - \mathbf{A})^{-1}$ is strictly positive

Definition 2. An *input-output matrix aggregator* is a function $f : M_n \to M_m$ that maps the $n \times n$ input-output matrices into $m \times m$ input-output matrices with $m < n$. The (k, l) element of the matrix $f(\mathbf{A})$ will be denoted by $f(\mathbf{A})_{kl}$. An input-output matrix $\mathbf{B}$ will be referred to as an *aggregation* of the input-output matrix $\mathbf{A}$ if it is the result of some aggregator applied to $\mathbf{A}$.

Considering that the input-output models are linear, one natural assumption is that the aggregator is linear. We call an input-output matrix $\mathbf{B}$ a *standard aggregation* of the input-output matrix $\mathbf{A}$ if $\mathbf{B}$ is the result of some standard aggregator applied to $\mathbf{A}$. The following theorem characterizes the functional form of the standard aggregators.

Theorem 1. An input-output aggregator $f : M_n \to M_m$ is standard if and only if f may be represented as

$$f(\mathbf{A}) = \mathbf{SAT} \tag{28}$$

in which $\mathbf{S} \in M_{m,n}$ is a 0-1 column stochastic matrix, $\mathbf{T} \in M_{n,m}$ is a column stochastic and $\mathbf{ST} = \mathbf{I} \in M_m$.

Remark 1. In the context of the theorem, the statement $\mathbf{ST} = \mathbf{I}$ simply means that the nonzero entries of $\mathbf{T}$ are contained among the positions indicated by the 1's of $\mathbf{S}^T$. If h maps N onto M, then the 0-1 matrix $\mathbf{S} \in M_{m,n}$ is a 0-1 column stochastic matrix with no row containing only 0s. The matrix $\mathbf{S}$ is called a *partitioning matrix* and the function h mapping N onto M given by $h(j) = i$ if $s_{ij} = 1$ is called the *function representation* of $\mathbf{S}$.

3.3. Properties of Input-Output Aggregation Process

The features of the input-output matrix $\mathbf{A}$ are, in general, preserved by a standard aggregator $\mathbf{B} = \mathbf{SAT}$.

Theorem 2. Suppose $\mathbf{S} \in M_{m,n}$ is a partitioning matrix, $\mathbf{T} \in M_{n,m}$ is column stochastic and $\mathbf{ST} = \mathbf{I}$. Then $\mathbf{TS}$ is a column stochastic, idempotent matrix of rank m.

Remark 2. The above theorem implies that the set of eigenvalues of $\mathbf{TS}$ includes 1 with multiplicity m and 0 with multiplicity $(n - m)$, since $\mathbf{TS}$ is idempotent. It can be established a close relationship between the standard aggregators and the notion of matrix similarity.

4. Dynamic Games in Switching Statistical Flow Batches

In this section, we apply the switching envelope method for dynamic games ([3], [7]) to describe the flow batches in the traffic aggregation mechanism. We associate with the traffic regimes two player A and B that are competing for network resources while having different goals. That is, the player A is interested in maximizing the thoughput of the traffic accross the switching system, while the player B wants to reduce the delays occuring in the switching system. Since the state of the switching system is not known apriori, we treat this competition as a game against nature.

4.1. Game Formulation of Input-Output Aggregation

Let us consider a rectangular game that is associated with the aggregation matrix defined in Section 3, that has the elements

$$a_{ij},\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n. \tag{29}$$

For player A there is an optimal strategy, in other words, relative frequencies

$$\mathbf{X}^* = [x_1^*, x_2^*, \ldots, x_m^*] \tag{30}$$

such that if he plays line (1) with the frequency x_1^*, line (2) with that of $x_2^*, \ldots$,line (m) with that of x_m^*, he is certain of at least making a profit equal to the value of the game. For player B there is an optimal strategy or relative frequencies

$$\mathbf{Y}^* = [y_1^*, y_2^*, \ldots, y_n^*] \tag{31}$$

such that if he plays column (1) with the frequency y_1^*, column (2) with that of $y_2^*, \ldots$, column (n) with that of y_n^*, he is certain of not losing more than the value v of the game. An optimal strategy for each player is formed from the solution of $m+n+2$ equations or inequalities, which are not strict, with $m+n+1$ unknowns $x_1, x_2, \ldots, x_m, y_1, y_2, \ldots, y_n, v$. The x_i and y_j terms are positive, but v can be any real number.

$$\begin{gathered}
x_1 + x_2 + \cdots + x_m = 1 \\
a_{11}x_1 + a_{21}x_2 + \cdots + a_{m1}x_m \geq v \\
a_{12}x_1 + a_{22}x_2 + \cdots + a_{m2}x_m \geq v \\
\vdots \\
a_{1n}x_1 + a_{2n}x_2 + \cdots + a_{mn}x_m \geq v \\
x_i \geq 0, \quad i = 1, 2, \ldots, m.
\end{gathered} \tag{32}$$

$$\begin{gathered}
y_1 + y_2 + \cdots + y_n = 1 \\
a_{11}y_1 + a_{12}y_2 + \cdots + a_{1n}y_n \leq v \\
a_{21}y_1 + a_{22}y_2 + \cdots + a_{2n}y_n \leq v \\
\vdots \\
a_{m1}y_1 + a_{m2}y_2 + \cdots + a_{mn}y_n \leq v \\
y_j \geq 0, \quad j = 1, 2, \ldots, n.
\end{gathered} \tag{33}$$

There is always a solution to the system formed by (32) and (33). Let

$$\mathbf{X}^* = [x_1^*, x_2^*, \ldots, x_m^*] \text{ and } \mathbf{Y}^* = [y_1^*, y_2^*, \ldots, y_n^*] \tag{34}$$

be one solution. If there are several, any linear form produced by the $\mathbf{X}^*$ terms, on the one hand, and by the $\mathbf{Y}^*$ terms, on the other, of which the coefficients are nonnegative and of a sum equal to 1 (convex weighting), is also a solution. Hence for A, there are, in this case, r optimal strategies with any two of which, for example $\mathbf{X}^{*'}$ and $\mathbf{X}^{*''}$, we form the infinity

$$\mathbf{X}^{*'''} = \lambda_1 \mathbf{X}^{*'} + \lambda_2 \mathbf{X}^{*''} \tag{35}$$

where $\lambda_1, \lambda_2 \geq 0,\ \lambda_1 + \lambda_2 = 1$. Similary, if there are s optimal strategies for B. There is only one value for v. If $\mathbf{X}^* = [x_1^*, x_2^*, \ldots, x_m^*]$ is an optimal strategy for A and

$$a_{1j}x_1^* + a_{2j}x_2^* + \cdots + a_{mj}x_m^* > v \tag{36}$$

then

$$y_j^* = 0 \tag{37}$$

but the converse is not always true. Similary, if $\mathbf{Y}^* = [y_1^*, y_2^*, \ldots, y_n^*]$ is an optimal strategy of B and

$$a_{i1}y_1^* + a_{i2}y_2^* + \cdots + a_{in}y_n^* < v \tag{38}$$

then

$$x_i^* = 0 \tag{39}$$

Remark 3. In the equations (30)-(31), the frequencies $x_1, x_2, \ldots, x_m$ and $y_1, y_2, \ldots, y_n$ are the normalized counterparts of the similar quantities from the input-output traffic aggregation model to the corresponding capacities of the input/output links in a switching node.

4.2. Bayes Strategy for Switching Envelope Estimation

If there are two states of nature and two final decisions, we take the cost of a draw, which is constant, as the unit of money. In addition, if consider r' and r'' two quantities that are not altered by a translation of the set of points representing the strategies ([7]), we can express the loss matrix in the following

$$E_1 \quad E_2 \begin{bmatrix} 0 & w_{12} \\ w_{21} & 0 \end{bmatrix} \tag{40}$$

with $\omega_{12} > 0$ and $\omega_{21} > 0$ by adding or removing a suitable number from each line. The knowledge of the two laws of probability ([2]) $\pi_{E_1}(X)$ and $\pi_{E_2}(X)$ which correspond to the states E_1 and E_2 of nature enable us to find the laws $g_1(z)$ and $g_2(z)$ of

$$z = \frac{\pi_{E_2}(X)}{\pi_{E_1}(X)} \tag{41}$$

Given an uncertain variable Z for which the law of probability is $g(z)$, we call $\bar{n}(-b, a)$ the expected number of values to be drawn before reaching or exceeding one of the limits. Here, $\varpi(-b, a)$ the probability of reaching or passing the higher bound (hence $1 - \varpi$ is the similar probability for the lower bound). The exact relation which determines $\bar{n}$ and ϖ as a function of $g(z)$ and of the bound is not known, but approximate values can be obtained in the following manner.

1. We seek the root which was not 0 in the equation in t:

$$\sum_z e^{tz} g(z) = 1 \tag{42}$$

2. We assume that
$$E = \sum_z zg(z) \tag{43}$$
3. If $-b < 0 < a$, we have
$$\varpi(-b,a) = \frac{1-e^{-bt}}{1-e^{-(a+b)t}} \tag{44}$$
$$\bar{n}(-b,a) = \frac{a\varpi(-b,a) + b[1-\varpi(-b,a]}{E} \tag{45}$$
4. If crossing bounds a and b costs W_a and W_b, respectively, with 1 atill the cost of the cost of drawing a value Z, the scoring rule counts, as an average
$$\bar{n}(-b,a) = \frac{a\varpi(-b,a) + b[1-\varpi(-b,a]}{E} \tag{46}$$
5. If we can determine $\rho_1(-b,a)$ and $\rho_2(-b,a)$ as loss depend on $g_1(z)$ and $g_2(z)$ respectively, then r' and r'' are respective solutions of the following two equations
$$\begin{aligned}\omega_{12} = 1 + r' + \omega_{12} \sum_{z\geq 0} g_1(z) + r'\omega_{21} \sum_{z\leq -b_0} g_2(z) \\ + \sum_{-b_0<z<0} [\rho_1(-b_0+z,-z)g_1(z) + r'\rho_2(-b_0+z,-z)g_2(z)]\end{aligned} \tag{47}$$
$$\begin{aligned}\omega_{21} = 1 + r'' + \omega_{12} \sum_{z\geq b_0} g_1(z) + r''\omega_{21} \sum_{z\leq 0} g_2(z) \\ + \sum_{0<z<b_0} [\rho_1(-z,b_0-z)g_1(z) + r''\rho_2(-z,b_0-z)g_2(z)]\end{aligned} \tag{48}$$
where
$$b_0 = \ln r'' - \ln r' \tag{49}$$
6. Conversely, if we obtain r' and r'', we can construct the corresponding table of loss. The two equations are linear in ω_{12} and ω_{21}.

We can also find r' and r'' by successive approximations. To do so, we take ρ for a *priory* probability of E_1 and $\rho(p)$ for the loss of Bayes's strategy against p ([9], [10]). The method consists of finding an increasing sequence of lower bounds $L''_n(p)$ and a decreasing sequence of upper bounds $L'_n(p)$, i.e.,
$$L''_n < \rho(p) < L'_n \tag{50}$$
which aproach $\rho(p)$. To do so, we use the formulas
$$L_n(p) = \min[\psi(p), 1 + \sum_X \{p\pi_{E_1}(X) + (1-p)\pi_{E_2}(X)\} L_{n-1}(\frac{p\pi_{E_1}(X)}{p\pi_{E_1}(X) + (1-p)\pi_{E_2}(X)})] \tag{51}$$
with
$$\psi(p) = \min[p\omega_{12}, (1-p)\omega_{21}] \tag{52}$$
Here, L'_n is obtained for the initial value $L'_0(p) = \psi(p)$ and L''_n by taking $L''_0(p) = 0$. Each function $L_n(p)$ has two corresponding values of p: p_1 and $p_2 < p_1$, which are two solutions of the question
$$\psi(p) = 1 + \sum_X \{p\pi_{E_1}(X) + (1-p)\pi_{E_2}(X)\} L_{n-1}(\frac{p\pi_{E_1}(X)}{p\pi_{E_1}(X) + (1-p)\pi_{E_2}(X)}) \tag{53}$$
then $\frac{(1-p_1)}{p}$ approaches r' and $\frac{(1-p_2)}{p_2}$ approaches r''.

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